Given a sequence of numbers $\{p_n\}$ in $[0,1]$, consider the following experiment. First, we flip a fair coin and then, at step $n \geq 2$, we turn the coin over to the other side with probability $p_n$. What can we say about the distribution of the empirical frequency of heads as $n \to \infty$?

We show that a number of phase transitions take place as the turning gets slower (i.e. $p_n$ is getting smaller), leading first to the breakdown of the Central Limit Theorem and then to that of the Law of Large Numbers. It turns out that the critical regime is $p_n = \text{const}/n$. Among the scaling limits, we obtain some well known special (Uniform, Gaussian, Semicircle and Arcsine) laws.

The talk is intended to a general audience and no expertise in probability is assumed!

This is joint work with S. Volkov (Lund, Sweden), to appear in JOTP.