

# Evolution Problems in Materials with Memory & Free Energy Functionals

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Challenging analytical problems are originated from applications; indeed, new materials are more and more widely studied since they are used in a variety of different environments. In particular, in recent years there has been a growing interest in smart material in general and, also, in materials with memory. Indeed, these materials exhibit the crucial physical property that their behaviour depends on time not only through the present time but also through their past history. This peculiarity leads, under the analytical point of view to study integro-differential model problems. This is the case of both rigid thermodynamics with memory as well as viscoelasticity.

The model of a rigid linear heat conductor with memory is considered and, specifically, an evolution problem which describes the time evolution of the temperature distribution is studied. The attention is focussed on the thermodynamical state of such a rigid heat conductor which, according to the model proposed by Fabrizio, Gentili and Reynolds in 1998 and the constitutive equations therein, depends on the history of the material. When initial and boundary conditions are assigned, the model evolution problem is represented by an integro-differential one whose kernel depends on time through the present time as well as the past history of the material. As a consequence, the choice of suitable expressions of the minimum free energy and of the thermal work turns out to be crucial in this study. Indeed, the functional spaces where solutions are looked for, are obtained by the requirement to be meaningful both under the physical as well as the analytic viewpoint. Then, on application of existence and uniqueness results recently proved, conditions which guarantee solution's exponential decay at infinity are obtained. Furthermore, some existence, and, possibly uniqueness, results recently obtained in joint work with V. Valente and G. Vergara Caffarelli, in the case, in turn, of a magneto-viscoelasticity problem, or of a singular viscoelasticity problem are considered. Again, the choice of suitable free energies plays a key role.