Math 1360
Exam 1 Review (Chapter 6-7.5)

Name ____________________________
Answers

1) Evaluate each of the following:

a) \( \int e^{\sqrt{x}} \, dx \) (Hint: Start with \( u \)-substitution for the square root of \( x \).)

\( u = \sqrt{x} \)
\( \frac{du}{dx} = \frac{1}{2\sqrt{x}} \)
\( 2\sqrt{x} \, du = dx \)

\[ 2 \int u e^u \, du \]
\( u = u \quad dv = e^u \, du \)
\( du = \, du \quad v = e^u \)

\[ 2[u e^u - \int e^u \, du] \]
\[ 2[u e^u - e^u] \]
\[ 2\sqrt{x} e^\sqrt{x} - 2e^\sqrt{x} + C \]

b) \( \int \sec^4 x \tan x \, dx \)

\( u = \tan x \)
\( \frac{du}{\sec^2 x} = dx \)

\[ \int_0^\pi \sec^4 x \tan^2 x \, du \]
\[ \int_0^\pi (1 + u^2) u^4 \, du \]
\[ \int_0^\pi u^4 + u^6 \, du \]
\[ \frac{u^5}{5} + \frac{u^7}{7} \bigg|_0^\pi \]

\( \tan^5 x + \tan^7 x \bigg|_0^\pi = \frac{1}{5} + \frac{1}{7} - (0) = \frac{7}{35} + \frac{5}{35} = \frac{12}{35} \)
c) \[ \int \frac{x-1}{x(x+2)^2} dx \]
\[ \frac{A}{x} + \frac{B}{x+2} + \frac{C}{(x+2)^2} = \frac{x-1}{x(x+2)^2} \]
\[ A(x+2)^2 + Bx(x+2) + Cx = x-1 \]
\[ A(x+2)^2 + Bx(x+2) + Cx = x-1 \]
\[ A + B = 0 \]
\[ 4A + 2B + C = 1 \]
\[ 4A = -1 \]
\[ A = -\frac{1}{4} \]
\[ B = \frac{1}{4} \]
\[ 4(\frac{-1}{4}) + 2(\frac{1}{4}) + C = 1 \]
\[ C = \frac{3}{2} \]

\[ \int \frac{-\frac{1}{4} + \frac{1}{4(x+2)} + \frac{3}{2(x+2)^2}}{dx} \]
\[ -\frac{1}{4} \ln |x| + \frac{1}{4} \ln |x+2| - \frac{3}{2(x+2)} + C \]

d) \[ \int \frac{dx}{\sqrt{25-x^2}} \]
\[ x = 5 \sin \theta \]
\[ dx = 5 \cos \theta d\theta \]
\[ \int \frac{5 \cos \theta d\theta}{\sqrt{25-25 \sin^2 \theta}} \]
\[ \int \frac{\cos \theta}{\sqrt{\cos^2 \theta}} d\theta = \int 1 d\theta = \theta = \sin^{-1}(\frac{x}{5}) + C \]
2) How many midpoint rectangles would you have to use in estimating the area under $f(x) = 2^x$ between $x = 0$ and $x = 2$ to get your answer accurate within 0.1? Hint: The derivative of $b^x$ is $(\ln b)b^x$. (Your answer will look messy—that's OK.)

$$|E_m| \leq \frac{k(2-0)^3}{24n^2} \leq 0.1$$

$f(x) = 2^x$

$f'(x) = \ln 2 \cdot 2^x$

$f''(x) = (\ln 2)^2 \cdot 2^x$ which is increasing so $|f''(x)|$ on $[0, 2] \leq 4(\ln 2)^2$

$$|E_m| \leq \frac{4(\ln 2)^2 \cdot 8}{24n^2} \leq 0.1 \rightarrow 2.53 \leq n$$

6.406 $\leq n^2$  
At least 3 rectangles

3) Evaluate the following integrals:

a) $\int_1^e \frac{1}{x} \, dx$

$$\lim_{t \to \infty} \int_1^t x^{-2} \, dx$$

$$\lim_{t \to \infty} -\frac{1}{x} \bigg|_1^t = \lim_{t \to \infty} -\frac{1}{t} + 1 = 1$$

b) $\int_0^8 x \, dx$

$$\lim_{t \to 0^+} \int_t^1 x \, dx = \lim_{t \to 0^+} \frac{3\ln 1}{1}$$

$$= \lim_{t \to 0^+} 8\ln 1 - 8\ln t$$

$$= 8[0 - (-\infty)] = \infty$$

divergent
4) Set up, but do not evaluate, an integral to find the following:

a) The area of the region in the first quadrant bounded by \( y=\tan x, \ y=1 \) and the \( y \)-axis.

\[
\int_0^{\frac{\pi}{4}} (1-\tan x) \, dx
\]

b) The volume that results when the region bounded by \( y=x \) and \( y=x^2 \) is rotated about the line \( x=4 \).

\[
2\pi \int_0^1 (4-x)(x-x^2) \, dx
\]

OR

\[
\pi \int_0^1 (4-y)^2 - (4-y^2)^2 \, dy
\]

c) The length of the curve that encloses the region bounded by \( y=x \) and \( y=x^2 \).

\[
\int_0^1 \sqrt{1+1^2} \, dx + \int_0^1 \sqrt{1+(2x)^2} \, dx
\]

d) The volume of the solid generated when the region bounded by \( y=\sin x \) and \( y=\cos x \) between \( x=0 \) and \( x=\frac{\pi}{4} \) is rotated about the \( y \)-axis.

Integrate \( \text{wrt } x \) : cut \( \perp \) to \( x \)-axis : shells

\[
2\pi \int_0^{\frac{\pi}{4}} x(\cos x - \sin x) \, dx
\]

e) The volume of the solid generated when the region in the first quadrant that is bounded by \( x=y^2 \), the \( y \)-axis and \( y=4 \) is rotated about \( y=5 \).

\[
2\pi \int_0^4 (5-y)y^2 \, dy
\]

OR

\[
\pi \int_0^4 (5-\sqrt{x})^2 - 1^2 \, dx
\]

f) The volume that results when the region bounded by \( y= -x^2+4 \) in the first quadrant is rotated about the \( x \)-axis.

Integrate \( \text{wrt } x \) : \( \perp \) to \( x \)-axis : disk

\[
\pi \int_0^2 (-x^2+4)^2 \, dx
\]

g) The surface area of the solid bounded by \( y=\sin x \), \( x=0, \ x=\frac{\pi}{2} \) and the \( x \)-axis is rotated about the \( x \)-axis.

\[
2\pi \int_0^{\frac{\pi}{2}} \sin x \sqrt{1+\cos^2 x} \, dx
\]