Chapter 21
The Electric Field 1: Discrete Charge Distributions

Conceptual Problems

13 ** Two point particles that have charges of $+q$ and $-3q$ are separated by distance $d$. (a) Use field lines to sketch the electric field in the neighborhood of this system. (b) Draw the field lines at distances much greater than $d$ from the charges.

**Determine the Concept** (a) We can use the rules for drawing electric field lines to draw the electric field lines for this system. In the field-line sketch we’ve assigned 2 field lines to each charge $q$. (b) At distances much greater than the separation distance between the two charges, the system of two charged bodies will "look like” a single charge of $-2q$ and the field pattern will be that due to a point charge of $-2q$. Four field lines have been assigned to each charge $-q$.

17 *** Two molecules have dipole moments of equal magnitude. The dipole moments are oriented in various configurations as shown in Figure 21-34. Determine the electric-field direction at each of the numbered locations. Explain your answers.

**Determine the Concept** Figure 21-23 shows the electric field due to a single dipole, where the dipole moment is directed toward the right. The electric field due two a pair of dipoles can be obtained by superposing the two electric fields.

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Chapter 21

Charge

23 • What is the total charge of all of the protons in 1.00 kg of carbon?

Picture the Problem We can find the number of coulombs of positive charge there are in 1.00 kg of carbon from \(Q = 6n_e\), where \(n_e\) is the number of atoms in 1.00 kg of carbon and the factor of 6 is present to account for the presence of 6 protons in each atom. We can find the number of atoms in 1.00 kg of carbon by setting up a proportion relating Avogadro’s number, the mass of carbon, and the molecular mass of carbon to \(n_e\). See Appendix C for the molar mass of carbon.

Express the positive charge in terms of the electronic charge, the number of protons per atom, and the number of atoms in 1.00 kg of carbon:

\[ Q = 6n_e \]

Using a proportion, relate the number of atoms in 1.00 kg of carbon \(n_e\), to Avogadro’s number and the molecular mass \(M\) of carbon:

\[ \frac{n_e}{N_A} = \frac{m_e}{M} \Rightarrow n_e = \frac{N_A m_e}{M} \]

Substitute for \(n_e\) to obtain:

\[ Q = \frac{6N_A m_e}{M} \]

Substitute numerical values and evaluate \(Q\):

\[ Q = \frac{6 \left( 6.022 \times 10^{23} \text{ atoms/mol} \right) (1.00 \text{ kg}) (1.602 \times 10^{-19} \text{ C})}{0.01201 \text{ kg/mol}} = 4.82 \times 10^7 \text{ C} \]

Coulomb’s Law

27 • Three point charges are on the x-axis: \(q_1 = -6.0 \, \mu\text{C}\) is at \(x = -3.0\) m, \(q_2 = 4.0 \, \mu\text{C}\) is at the origin, and \(q_3 = -6.0 \, \mu\text{C}\) is at \(x = 3.0\) m. Find the electric force on \(q_1\).
Picture the Problem $q_2$ exerts an attractive electric force $\vec{F}_{2,1}$ on point charge $q_1$ and $q_3$ exerts a repulsive electric force $\vec{F}_{3,1}$ on point charge $q_1$. We can find the net electric force on $q_1$ by adding these forces (that is, by using the superposition principle).

Express the net force acting on $q_1$:  
$$\vec{F}_1 = \vec{F}_{2,1} + \vec{F}_{3,1}$$

Express the force that $q_2$ exerts on $q_1$:  
$$\vec{F}_{2,1} = \frac{k|q_2||q_1|}{r_{2,1}^2} \hat{i}$$

Express the force that $q_3$ exerts on $q_1$:  
$$\vec{F}_{3,1} = \frac{k|q_3||q_1|}{r_{3,1}^2} (-\hat{i})$$

Substitute and simplify to obtain:  
$$\vec{F}_1 = \frac{k|q_2||q_1|}{r_{2,1}^2} \hat{i} - \frac{k|q_3||q_1|}{r_{3,1}^2} \hat{i}$$
$$= \frac{k|q_1|}{r_{2,1}^2} \left( \frac{|q_2|}{r_{2,1}} - \frac{|q_3|}{r_{3,1}} \right) \hat{i}$$

Substitute numerical values and evaluate $\vec{F}_1$:

$$\vec{F}_1 = \left(8.988 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2 \right)(6.0 \mu\text{C})(\frac{4.0 \mu\text{C}}{3.0 \text{m}}) - \frac{6.0 \mu\text{C}}{6.0 \text{m}} \hat{i} = \frac{0.5 \times 10^{-2}}{} \hat{i}$$

Five identical point charges, each having charge $Q$, are equally spaced on a semicircle of radius $R$ as shown in Figure 21-37. Find the force (in terms of $k$, $Q$, and $R$) on a charge $q$ located equidistant from the five other charges.

Picture the Problem By considering the symmetry of the array of charged point particles, we can see that the $y$ component of the force on $q$ is zero. We can apply Coulomb’s law and the principle of superposition of forces to find the net force acting on $q$.

Express the net force acting on the point charge $q$:

$$\vec{F}_q = \vec{F}_{\text{on x axis},q} + 2 \vec{F}_{\text{at 45°},q}$$
Express the force on point charge $q$ due to the point charge $Q$ on the $x$ axis:

$$\vec{F}_{\text{on axis, } q} = \frac{kqQ}{R^2} \hat{i}$$

Express the net force on point charge $q$ due to the point charges at 45°:

$$2\vec{F}_q = 2 \frac{kqQ}{R^2} \cos 45° \hat{i} = \frac{2}{\sqrt{2}} \frac{kqQ}{R^2} \hat{i}$$

Substitute for $\vec{F}_{\text{on axis, } q}$ and $2\vec{F}_q$ to obtain:

$$\vec{F}_q = \frac{kqQ}{R^2} \hat{i} + \frac{2}{\sqrt{2}} \frac{kqQ}{R^2} \hat{i}$$

$$= \frac{kqQ}{R^2} (1 + \sqrt{2}) \hat{i}$$

### The Electric Field

A point charge of 4.0 $\mu$C is at the origin. What is the magnitude and direction of the electric field on the $x$ axis at (a) $x = 6.0$ m, and (b) $x = -10$ m? (c) Sketch the function $E_x$ versus $x$ for both positive and negative values of $x$. (Remember that $E_x$ is negative when $\vec{E}$ points in the $-x$ direction.)

**Picture the Problem** Let $q$ represent the point charge at the origin and use Coulomb’s law for $\vec{E}$ due to a point charge to find the electric field at $x = 6.0$ m and $-10$ m.

(a) Express the electric field at a point $P$ located a distance $x$ from a point charge $q$:

$$\vec{E}(x) = \frac{kq}{x^2} \hat{r}_{P\theta}$$

Evaluate this expression for $x = 6.0$ m:

$$\vec{E}(6.0\text{ m}) = \frac{8.988 \times 10^9 \text{ N} \cdot \text{m}^2 / \text{C}^2 (4.0 \mu\text{C})}{(6.0 \text{ m})^2} \hat{i} = (1.0 \text{kN/C}) \hat{i}$$

(b) Evaluate $\vec{E}$ at $x = -10$ m:

$$\vec{E}(-10 \text{ m}) = \frac{8.988 \times 10^9 \text{ N} \cdot \text{m}^2 / \text{C}^2 (4.0 \mu\text{C})}{(10 \text{ m})^2} (-\hat{i}) = (-0.36 \text{kN/C}) \hat{i}$$
(c) The following graph was plotted using a spreadsheet program:

![Graph of Electric Field](image)

**41.** Two point charges \( q_1 \) and \( q_2 \) both have a charge equal to +6.0 nC and are on the y axis at \( y_1 = +3.0 \) cm and \( y_2 = -3.0 \) cm respectively. (a) What is the magnitude and direction of the electric field on the x axis at \( x = 4.0 \) cm? (b) What is the force exerted on a third charge \( q_0 = 2.0 \) nC when it is placed on the x axis at \( x = 4.0 \) cm?

**Picture the Problem** The diagram shows the locations of the point charges \( q_1 \) and \( q_2 \) and the point on the x axis at which we are to find \( \vec{E} \). From symmetry considerations we can conclude that the y component of \( \vec{E} \) at any point on the x axis is zero. We can use Coulomb’s law for the electric field due to point charges and the principle of superposition for fields to find the field at any point on the x axis and \( \vec{F} = q \vec{E} \) to find the force on a point charge \( q_0 \) placed on the x axis at \( x = 4.0 \) cm.
(a) Letting $q = q_1 = q_2$, express the $x$-component of the electric field due to one point charge as a function of the distance $r$ from either point charge to the point of interest:

$$\vec{E}_x = \frac{\vec{k}q}{r^2} \cos \theta \hat{i}$$

Express $\vec{E}_x$ for both charges:

$$\vec{E}_x = \frac{2\vec{k}q}{r^2} \cos \theta \hat{i}$$

Substitute for $\cos \theta$ and $r$, substitute numerical values, and evaluate to obtain:

$$\vec{E}(4.0 \text{ cm}) = 2\frac{\vec{k}q}{r^2} \frac{0.040 \text{ m}}{r} \hat{i} = 2\frac{\vec{k}(0.040 \text{ m})}{r^3} \hat{i}$$

$$= 2\left(8.988 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2\right)(6.0 \text{nC})(0.040 \text{ m}) \hat{i}$$

$$= \left[0.030 \text{ m}\right]^2 + (0.040 \text{ m})^2 \right]^{1/2}$$

$$= (34.5 \text{kN/C}) \hat{i} = (35 \text{kN/C}) \hat{i}$$

The magnitude and direction of the electric field at $x = 4.0 \text{ cm}$ is:

$$35 \text{kN/C @ 0°}$$

(b) Apply $\vec{F} = q \vec{E}$ to find the force on a point charge $q_0$ placed on the $x$ axis at $x = 4.0 \text{ cm}$:

$$\vec{F} = (2.0 \text{nC})(34.5 \text{kN/C}) \hat{i} = (69 \mu \text{N}) \hat{i}$$

47 Two point particles, each having a charge $q$, sit on the base of an equilateral triangle that has sides of length $L$ as shown in Figure 21-38. A third point particle that has a charge equal to $2q$ sits at the apex of the triangle. Where must a fourth point particle that has a charge equal to $q$ be placed in order that the electric field at the center of the triangle be zero? (The center is in the plane of the triangle and equidistant from the three vertices.)

**Picture the Problem** The electric field of 4th charged point particle must cancel the sum of the electric fields due to the other three charged point particles. By symmetry, the position of the 4th charged point particle must lie on the vertical centerline of the triangle. Using trigonometry, one can show that the center of an equilateral triangle is a distance $L/\sqrt{3}$ from each vertex, where $L$ is the length of the side of the triangle. Note that the $x$ components of the fields due to the base charged particles cancel each other, so we only need concern ourselves with the $y$ components of the fields due to the charged point particles at the vertices of the triangle. Choose a coordinate system in which the origin is at the midpoint of the base of the triangle, the $+x$ direction is to the right, and the $+y$ direction is upward.

**Deleted:** Note that the $x$ components of the electric field vectors add up to zero.
Express the condition that must be satisfied if the electric field at the center of the triangle is to be zero:

\[ \sum_{i=1}^{4} \vec{E}_i = 0 \]

Substituting for \( \vec{E}_1, \vec{E}_2, \vec{E}_3, \) and \( \vec{E}_4 \) yields:

\[
\frac{kq_1}{L} \cos 60^\circ \hat{j} + \frac{kq_2}{L} \cos 60^\circ \hat{j} - \frac{kq_3}{L} \hat{j} + \frac{kq_4}{y} \hat{j} = 0
\]

Solving for \( y \) yields:

\[ y = \pm \frac{L}{\sqrt{3}} \]

The positive solution corresponds to the 4th point particle being a distance \( L/\sqrt{3} \) above the base of the triangle, where it produces the same strength and same direction electric field caused by the three charges at the corners of the triangle. So the charged point particle must be placed a distance \( L/\sqrt{3} \) below the midpoint of the triangle.

**Point Charges in Electric Fields**

The acceleration of a particle in an electric field depends on \( q/m \) (the charge-to-mass ratio of the particle). (a) Compute \( q/m \) for an electron. (b) What is the magnitude and direction of the acceleration of an electron in a uniform electric field that has a magnitude of 100 N/C? (c) Compute the time it takes for an electron placed at rest in a uniform electric field that has a magnitude of 100 N/C to reach a speed of 0.01c. (When the speed of an electron approaches the speed of light \( c \), relativistic kinematics must be used to calculate its motion, but at speeds...
of 0.01c or less, non-relativistic kinematics is sufficiently accurate for most purposes.) \(d\) How far does the electron travel in that time?

**Picture the Problem** We can use Newton’s second law of motion to find the acceleration of the electron in the uniform electric field and constant-acceleration equations to find the time required for it to reach a speed of 0.01c and the distance it travels while acquiring this speed.

\(a\) Use data found at the back of your text to compute \(e/m\) for an electron:

\[
\frac{e}{m_e} = \frac{1.602 \times 10^{-19} \text{ C}}{9.109 \times 10^{-31} \text{ kg}} = 1.76 \times 10^{11} \text{ C/kg}
\]

\(b\) Apply Newton’s second law to relate the acceleration of the electron to the electric field:

\[
a = \frac{F_{\text{net}}}{m_e} = \frac{eE}{m_e}
\]

Substitute numerical values and evaluate \(a\):

\[
a = \frac{(1.602 \times 10^{-19} \text{ C})(100 \text{ N/C})}{9.109 \times 10^{-31} \text{ kg}}
= 1.759 \times 10^{13} \text{ m/s}^2
= \left[ 1.76 \times 10^{13} \text{ m/s}^2 \right]
\]

The direction of the acceleration of an electron is opposite the electric field.

\(c\) Using the definition of acceleration, relate the time required for an electron to reach 0.01c to its acceleration:

\[
\Delta t = \frac{v}{a} = \frac{0.01c}{a}
\]

Substitute numerical values and evaluate \(\Delta t\):

\[
\Delta t = \frac{0.01(2.998 \times 10^4 \text{ m/s})}{1.759 \times 10^{13} \text{ m/s}^2} = 0.1704 \mu\text{s}
= 0.2 \mu\text{s}
\]

\(d\) Use a constant-acceleration equation to express the distance the electron travels in a given time interval:

\[
\Delta x = v_i \Delta t + \frac{1}{2} a (\Delta t)^2
\]

or, because \(v_i = 0\),

\[
\Delta x = \frac{1}{2} a (\Delta t)^2
\]

Substitute numerical values and evaluate \(\Delta x\):

\[
\Delta x = \frac{1}{2} \left( 1.759 \times 10^{13} \text{ m/s}^2 \right) (0.1704 \mu\text{s})^2
= 0.3 \text{ m}
\]

\(\Delta x = \frac{1}{2} v_f \Delta t\)

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An electron starts at the position shown in Figure 21-39 with an initial speed \( v_0 = 5.00 \times 10^6 \) m/s at 45º to the x axis. The electric field is in the +y direction and has a magnitude of 3.50 \times 10^3 \) N/C. The black lines in the figure are charged metal plates. On which plate and at what location will the electron strike?

**Picture the Problem** We can use constant-acceleration equations to express the \( x \) and \( y \) coordinates of the electron in terms of the parameter \( t \) and Newton’s second law to express the constant acceleration in terms of the electric field. Eliminating the parameter will yield an equation for \( y \) as a function of \( x \), \( q \), and \( m \). We can decide whether the electron will strike the upper plate by finding the maximum value of its \( y \) coordinate. Should we find that it does not strike the upper plate, we can determine where it strikes the lower plate by setting \( y(x) = 0 \). Ignore any effects of gravitational forces.

Express the \( x \) and \( y \) coordinates of the electron as functions of time:

\[
x(t) = (v_0 \cos \theta) t
\]

and

\[
y(t) = (v_0 \sin \theta) t - \frac{1}{2} a_f t^2
\]

Apply Newton’s second law to relate the acceleration of the electron to the net force acting on it:

\[
a_y = \frac{F_{\text{net},y}}{m_e} = \frac{eE_y}{m_e}
\]

Substitute in the \( y \)-coordinate equation to obtain:

\[
y(t) = (v_0 \sin \theta) t - \frac{eE_y}{2m_e} t^2
\]

Eliminate the parameter \( t \) between the two equations to obtain:

\[
y(x) = (\tan \theta)x - \frac{eE_y}{2m_e v_0^2 \cos \theta} x^2 \quad (1)
\]

To find \( y_{\text{max}} \), set \( \frac{dy}{dx} = 0 \) for extrema:

\[
\frac{dy}{dx} = \tan \theta - \frac{eE_y}{m_e v_0^2 \cos \theta} x' = 0 \quad \text{for extrema}
\]

Solve for \( x' \) to obtain:

\[
x' = \frac{m_e v_0^2 \sin 2\theta}{2eE_y} \quad \text{(See remark below.)}
\]

Substitute \( x' \) in \( y(x) \) and simplify to obtain \( y_{\text{max}} \):

\[
y_{\text{max}} = \frac{m_e v_0^2 \sin^2 \theta}{2eE_y}
\]
Substitute numerical values and evaluate $y_{\text{max}}$:

$$y_{\text{max}} = \frac{(9.109 \times 10^{-31} \text{ kg})(5.00 \times 10^6 \text{ m/s})^2 \sin^2 45^\circ}{2(1.602 \times 10^{-19} \text{ C})(3.50 \times 10^3 \text{ N/C})} = 1.02 \text{ cm}$$

and, because the plates are separated by 2 cm, the electron does not strike the upper plate.

To determine where the electron will strike the lower plate, set $y = 0$ in equation (1) and solve for $x$ to obtain:

Substitute numerical values and evaluate $x$:

$$x = \frac{m_e v_0^2 \sin 2\theta}{eE_y}$$

Remarks: $x'$ is an extremum, that is, either a maximum or a minimum. To show that it is a maximum we need to show that $d^2y/dx^2$, evaluated at $x'$, is negative. A simple alternative is to use your graphing calculator to show that the graph of $y(x)$ is a maximum at $x'$. Yet another alternative is to recognize that, because equation (1) is quadratic and the coefficient of $x^2$ is negative, its graph is a parabola that opens downward.

General Problems

61 • Show that it is only possible to place one isolated proton in an ordinary empty coffee cup by considering the following situation. Assume the first proton is fixed at the bottom of the cup. Determine the distance directly above this proton where a second proton would be in equilibrium. Compare this distance to the depth of an ordinary coffee cup to complete the argument.

Picture the Problem Equilibrium of the second proton requires that the sum of the electric and gravitational forces acting on it be zero. Let the upward direction be the $+y$ direction and apply the condition for equilibrium to the second proton.

Apply $\sum F_y = 0$ to the second proton:

$$\vec{F}_e + \vec{F}_g = 0$$

or

$$\frac{kq^2}{h^2} - m_{p}g = 0 \Rightarrow h = \frac{kq_p^2}{m_p g}$$
Substitute numerical values and evaluate $h$:

$$h = \sqrt{\frac{8.988 \times 10^9 \text{ N} \cdot \text{m}^2}{(1.602 \times 10^{-10} \text{ C})^2} \left(\frac{1.673 \times 10^{-27} \text{ kg}}{9.81 \text{ m/s}^2}\right)} \approx 12 \text{ cm} \approx 5 \text{ in}$$

This separation of about 5 in is greater than the height of a typical coffee cup. Thus the first proton will repel the second one out of the cup and the maximum number of protons in the cup is one.

65 •• A positive charge $Q$ is to be divided into two positive point charges $q_1$ and $q_2$. Show that, for a given separation $D$, the force exerted by one charge on the other is greatest if $q_1 = q_2 = \frac{1}{2} Q$.

**Picture the Problem** We can use Coulomb’s law to express the force exerted on one charge by the other and then set the derivative of this expression equal to zero to find the distribution of the charge that maximizes this force.

Using Coulomb’s law, express the force that either charge exerts on the other:

$$F = \frac{kq_1 q_2}{D^2}$$

Express $q_2$ in terms of $Q$ and $q_1$:

$$q_2 = Q - q_1$$

Substitute for $q_2$ to obtain:

$$F = \frac{kq_1 (Q - q_1)}{D^2}$$

Differentiate $F$ with respect to $q_1$ and set this derivative equal to zero for extreme values:

$$\frac{dF}{dq_1} = \frac{k}{D^2} \left(q_1 (Q - q_1) \right) = \frac{k}{D^2} \left[q_1 (-1) + Q - q_1 \right] = 0 \text{ for extrema}$$

Solve for $q_1$ to obtain:

$$q_1 = \frac{1}{2} Q \Rightarrow q_2 = Q - q_1 = \frac{1}{2} Q$$
To determine whether a maximum or a minimum exists at \( q_1 = \frac{1}{2}Q \), differentiate \( F \) a second time and evaluate this derivative at \( q_1 = \frac{1}{2}Q \):

\[
\frac{d^2 F}{dq_1^2} = \frac{k}{D^2} \frac{d}{dq_1} [Q - 2q_1] = \frac{k}{D^2} (-2) < 0 \text{ independently of } q_1.
\]

\( \therefore q_1 = q_2 = \frac{1}{2}Q \) maximizes \( F \).

69  A rigid 1.00-m-long rod is pivoted about its center (Figure 21-42). A charge \( q_1 = 5.00 \times 10^{-7} \text{ C} \) is placed on one end of the rod, and a charge \( q_2 = -q_1 \) is placed a distance \( d = 10.0 \text{ cm} \) directly below it. (a) What is the force exerted by \( q_2 \) on \( q_1 \)? (b) What is the torque (measured about the rotation axis) due to that force? (c) To counterbalance the attraction between the two charges, we hang a block 25.0 cm from the pivot as shown. What value should we choose for the mass of the block? (d) We now move the block and hang it a distance of 25.0 cm from the balance point, on the same side of the balance as the charge. Keeping \( q_1 \) the same, and \( d \) the same, what value should we choose for \( q_2 \) to keep this apparatus in balance?

**Picture the Problem** We can use Coulomb’s law, the definition of torque, and the condition for rotational equilibrium to find the electrostatic force between the two charged bodies, the torque this force produces about an axis through the center of the rod, and the mass required to maintain equilibrium when it is located either 25.0 cm to the right or to the left of the mid-point of the rod.

(a) Using Coulomb’s law, express the electric force between the two charges:

\[
F = \frac{kq_1q_2}{d^2}
\]

Substitute numerical values and evaluate \( F \):

\[
F = \frac{(8.988 \times 10^9 \text{ N} \cdot \text{m}^2 / \text{C}^2) (5.00 \times 10^{-7} \text{ C})^2}{(0.100 \text{ m})^2} = 0.2247 \text{ N} = 0.225 \text{ N}
\]

(b) The torque (measured about the rotation axis) due to the force \( F \) is:

\[
\tau = F\ell
\]

Substitute numerical values and evaluate \( \tau \):

\[
\tau = (0.2247 \text{ N})(0.500 \text{ m}) = 0.1124 \text{ N} \cdot \text{m} = 0.112 \text{ N} \cdot \text{m}, \text{ counterclockwise.}
\]

(c) Apply \( \sum \tau_{\text{center of the rod}} = 0 \) to the rod:

\[
\tau - mg\ell' = 0 \Rightarrow m = \frac{\tau}{g\ell'}
\]
Substitute numerical values and evaluate \( m \):

\[
m = \frac{0.1124 \text{ N} \cdot \text{m}}{(9.81 \text{ m/s}^2)(0.250 \text{ m})} = 0.04583 \text{ kg} = 45.8 \text{ g}
\]

(d) Apply \( \sum \tau_{\text{center of the rod}} = 0 \) to the rod:

\[-\tau + mg \ell' = 0\]

Substitute for \( \tau \) to obtain:

\[-F \ell + mg \ell' = 0\]

Substituting for \( F \) gives:

\[-\frac{kq_1 q_2'}{\ell^2} + mg \ell' = 0 \Rightarrow q_2' = \frac{d^2 mg \ell'}{kq_1 \ell}\]

where \( q' \) is the required charge.

Substitute numerical values and evaluate \( q_2' \):

\[
q_2' = \frac{(0.100 \text{ m})^2(0.04582 \text{ kg})(9.81 \text{ m/s}^2)(0.250 \text{ m})}{(8.988 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(5.00 \times 10^{-7} \text{ C})(0.500 \text{ m})} = 5.00 \times 10^{-7} \text{ C}
\]

Two point charges have a total charge of 200 \( \mu \text{C} \) and are separated by 0.600 m. (a) Find the charge of each particle if the particles repel each other with a force of 120 N. (b) Find the force on each particle if the charge on each particle is 100 \( \mu \text{C} \).

**Picture the Problem** Let the numeral 1 denote one of the point charges and the numeral 2 the other. Knowing the total charge on the two spheres, we can use Coulomb’s law to find the charge on each of them. A second application of Coulomb’s law when the spheres carry the same charge and are 0.600 m apart will yield the force each exerts on the other.

(a) Use Coulomb’s law to express the repulsive force each point charge exerts on the other:

\[
F = \frac{kq_1 q_2}{r_{1,2}^2}
\]

Express \( q_2 \) in terms of the total charge and \( q_1 \):

\[
q_2 = Q - q_1
\]

Substitute for \( q_2 \) to obtain:

\[
F = \frac{kq_1 (Q - q_1)}{r_{1,2}^2}
\]
Substitute numerical values to obtain:

\[
120 \text{ N} = \frac{(8.988 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2) \left[ (200 \mu\text{C})q_1 - q_2^2 \right]}{(0.600 \text{ m})^2}
\]

Simplify to obtain the quadratic equation:

\[q_1^2 + (-200 \mu\text{C})q_1 + 4806 (\mu\text{C})^2 = 0\]

Use the quadratic formula or your graphing calculator to obtain:

\[q_1 = 27.9 \mu\text{C} \text{ and } 172 \mu\text{C}\]

Hence the charges on the particles are:

\[27.9 \mu\text{C} \text{ and } 172 \mu\text{C}\]

(b) Use Coulomb’s law to express the repulsive force each point charge exerts on the other when \(q_1 = q_2 = 100 \mu\text{C}\):

\[F = \frac{\kappa q_1 q_2}{r_{1,2}^2}\]

Substitute numerical values and evaluate \(F\):

\[F = \frac{(8.988 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2) \left[ (100 \mu\text{C})^2 \right]}{(0.600 \text{ m})^2} = 250 \text{ N}\]

Figure 21-46 shows a dumbbell consisting of two identical small particles, each of mass \(m\), attached to the ends of a thin (massless) rod of length \(a\) that is pivoted at its center. The particles carry charges of \(+q\) and \(-q\), and the dumbbell is located in a uniform electric field \(\vec{E}\). Show that for small values of the angle \(\theta\) between the direction of the dipole and the direction of the electric field, the system displays a rotational form of simple harmonic motion, and obtain an expression for the period of that motion.

**Picture the Problem** We can apply Newton’s second law in rotational form to obtain the differential equation of motion of the dipole and then use the small angle approximation \(\sin \theta \approx \theta\) to show that the dipole experiences a linear restoring torque and, hence, will experience simple harmonic motion.

Apply \(\sum \tau = I \alpha\) to the dipole:

\[\tau = pE \sin \theta = I \frac{d^2 \theta}{dt^2}\]

where \(\tau\) is negative because acts in such a direction as to decrease \(\theta\).
For small values of $\theta$, $\sin \theta \approx \theta$ and:

$$-pE\theta = I \frac{d^2 \theta}{dt^2}$$

Express the moment of inertia of the dipole:

$$I = \frac{1}{2} ma^2$$

Relate the dipole moment of the dipole to its charge and the charge separation:

$$p = qa$$

Substitute for $p$ and $I$ to obtain:

$$\frac{1}{2} ma^2 \frac{d^2 \theta}{dt^2} = -qaE \theta$$

or

$$\frac{d^2 \theta}{dt^2} = -\frac{2qE}{ma} \theta$$

the differential equation for a simple harmonic oscillator with angular frequency $\omega = \sqrt{\frac{2qE}{ma}}$.

Express the period of a simple harmonic oscillator:

$$T = \frac{2\pi}{\omega}$$

Substitute for $\omega$ and simplify to obtain:

$$T = \frac{2\pi}{\sqrt{\frac{ma}{2qE}}}$$

79 ** An electron (charge $-e$, mass $m$) and a positron (charge $+e$, mass $m$) revolve around their common center of mass under the influence of their attractive coulomb force. Find the speed $v$ of each particle in terms of $e$, $m$, $k$, and their separation distance $L$.

**Picture the Problem** The forces the electron and the proton exert on each other constitute an action-and-reaction pair. Because the magnitudes of their charges are equal and their masses are the same, we find the speed of each particle by finding the speed of either one. We’ll apply Coulomb’s force law for point charges and Newton’s second law to relate $v$ to $e$, $m$, $k$, and their separation distance $L$.

Apply Newton’s second law to the positron to obtain:

$$\frac{ke^2}{L^2} = m \frac{v^2}{\frac{1}{2} L} \Rightarrow \frac{ke^2}{L} = 2mv^2$$
Solving for $v$ gives:

$$v = \sqrt{\frac{ke^2}{2mL}}$$

During a famous experiment in 1919, Ernest Rutherford shot doubly ionized helium nuclei (also known as alpha particles) at a gold foil. He discovered that virtually all of the mass of an atom resides in an extremely compact nucleus. Suppose that during such an experiment, an alpha particle far from the foil has an initial kinetic energy of 5.0 MeV. If the alpha particle is aimed directly at the gold nucleus, and the only force acting on it is the electric force of repulsion exerted on it by the gold nucleus, how close will it approach the gold nucleus before turning back? That is, what is the minimum center-to-center separation of the alpha particle and the gold nucleus?

**Picture the Problem** The work done by the electric field of the gold nucleus changes the kinetic energy of the alpha particle—eventually bringing it to rest. We can apply the work-kinetic energy theorem to derive an expression for the distance of closest approach. Because the repulsive Coulomb force $\vec{F}_e$ varies with distance, we’ll have to evaluate $\int \vec{F}_e \cdot d\vec{r}$ in order to find the work done on the alpha particles by this force.

Apply the work-kinetic energy theorem to the alpha particle to obtain:

$$W_{net} = \int_{\infty}^{r_{ap}} \vec{F}_e \cdot d\vec{r} = \Delta K$$

or, because

$$\int_{\infty}^{r_{ap}} \vec{F}_e \cdot d\vec{r} = -\int_{\infty}^{r_{ap}} k(\frac{2e)(79e)}{r^2} dr$$

and

$$K_f = 0,$$

$$-158ke^2 \int_{\infty}^{r_{ap}} \frac{dr}{r^2} = -K_i$$

Evaluating the integral yields:

$$-158ke^2 \left[ \frac{1}{r} \right]_{\infty}^{r_{ap}} = -\frac{158ke^2}{r_{min}} = -K_i$$

Solve for $r_{min}$ and simplify to obtain:

$$r_{min} = \frac{158ke^2}{K_i}$$
Substitute numerical values and evaluate $r_{\text{min}}$:

$$r_{\text{min}} = \frac{158 \left( 8.988 \times 10^9 \frac{N \cdot m^2}{C^2} \right) \left( 1.602 \times 10^{-19} C \right)^2}{5.0 \text{ MeV} \times \frac{1.602 \times 10^{-20} \text{ J}}{\text{eV}}} = 4.6 \times 10^{-14} \text{ m}$$

87  *** In Problem 86, there is a description of the Millikan experiment used to determine the charge on the electron. During the experiment, a switch is used to reverse the direction of the electric field without changing its magnitude, so that one can measure the terminal speed of the microsphere both as it is moving upward and as it is moving downward. Let $v_u$ represent the terminal speed when the particle is moving up, and $v_d$ the terminal speed when moving down. (a) If we let $u = v_u + v_d$, show that $q = 3\pi \rho u / E$, where $q$ is the microsphere’s net charge. For the purpose of determining $q$, what advantage does measuring both $v_u$ and $v_d$ have over measuring only one terminal speed? (b) Because charge is quantized, $u$ can only change by steps of magnitude $N$, where $N$ is an integer. Using the data from Problem 86, calculate $\Delta u$.

**Picture the Problem** The free body diagram shows the forces acting on the microsphere of mass $m$ and having an excess charge of $q = Ne$ when the electric field is downward. Under terminal-speed conditions the sphere is in equilibrium under the influence of the electric force $F_e$, its weight $mg$, and the drag force $F_d$. We can apply Newton’s second law, under terminal-speed conditions, to relate the number of excess charges $N$ on the sphere to its mass and, using Stokes’ law, to its terminal speed.

(a) Apply Newton’s second law to the microsphere when the electric field is downward:

$$F_e - mg - F_d = ma_y$$

or, because $a_y = 0$,

$$F_e - mg - F_{d,\text{terminal}} = 0$$

Substitute for $F_e$ and $F_{d,\text{terminal}}$ to obtain:

$$qE - mg - 6\pi \rho v_u = 0$$

or, because $q = Ne$,

$$NeE - mg - 6\pi \rho v_u = 0$$
Solve for \( v_u \) to obtain:

\[
v_u = \frac{NeE - mg}{6\pi\eta r}
\]  \( (1) \)

With the field pointing upward, the electric force is downward and the application of Newton’s second law to the microsphere yields:

\[ F_{\text{d, terminal}} - F_e - mg = 0 \]

or

\[ 6\pi\eta rv_d - NeE - mg = 0 \]

Solve for \( v_d \) to obtain:

\[
v_d = \frac{NeE + mg}{6\pi\eta r}
\]  \( (2) \)

Add equations (1) and (2) and simplify to obtain:

\[
u = v_u + v_d = \frac{NeE - mg}{6\pi\eta r} + \frac{NeE + mg}{6\pi\eta r} = \frac{NeE}{3\pi\eta r} = \frac{qE}{3\pi\eta r}
\]

Measuring both \( v_u \) and \( v_d \) has the advantage that you don’t need to know the mass of the microsphere.

\( (b) \) Letting \( \Delta u \) represent the change in the terminal speed of the microsphere due to a gain (or loss) of one electron we have:

\[
\Delta u = v_{N+1} - v_N
\]

Noting that \( \Delta v \) will be the same whether the microsphere is moving upward or downward, express its terminal speed when it is moving upward with \( N \) electronic charges on it:

\[
v_N = \frac{NeE - mg}{6\pi\eta r}
\]

Express its terminal speed upward when it has \( N + 1 \) electronic charges:

\[
v_{N+1} = \frac{(N + 1)eE - mg}{6\pi\eta r}
\]

Substitute and simplify to obtain:

\[
\Delta u = \frac{(N + 1)eE - mg}{6\pi\eta r} - \frac{NeE - mg}{6\pi\eta r} = \frac{eE}{6\pi\eta r}
\]

Substitute numerical values and evaluate \( \Delta u \):

\[
\Delta u = \frac{(1.602 \times 10^{-19} \text{ C})(6.00 \times 10^4 \text{ N/C})}{6\pi(1.8 \times 10^{-12} \text{ Pa} \cdot \text{m})(5.50 \times 10^{-7} \text{ m})} = 52 \mu\text{m/s}
\]
Because the positive solution corresponds to the 4th charge being at the center of the triangle, it follows that:

\[ y = -\frac{L}{\sqrt{3}} \]

Substitute numerical values and evaluate \( \Delta x \):

\[
\Delta x = \frac{1}{2} \left[ (0.01) \left(2.998 \times 10^8 \text{ m/s} \right) \right] (0.1704 \mu s) = 0.3 \text{ m}
\]