Discovering Triangle Congruency Theorems

Teacher Instructions

http://mste.illinois.edu/courses/ci303fa01/students/kethoms1/unitplan/day4.html

Grade Level - This lesson should be taught in a high school geometry course, comprised mostly of freshmen or sophomores. This day's plan follows the teaching of the triangle sum theorem and the triangle inequality in our unit on triangle properties. Students should also be familiar with making conjectures, testing their hypotheses, and coming to their own conclusions from previous math experiences.

Time - 90 minute (or two 50 minute) class periods

Main Idea - The students will use fettuccine noodles to discover the truth of different conjectures about triangle congruencies. They will develop an understanding of the triangle congruency theorems that will be used throughout geometry. A problem involving needing to find congruent triangles will be posed and students will begin with the most basic case and work toward the actual congruency theorems in solving this problem.

Preparation and Materials - The teacher should measure, cut, and color enough pieces of fettuccine for each student to have at least two each of three different sized pieces (7 cm, 9 cm, and 12 cm). Each length should be a different color. Each student should receive a Ziploc bag with the following: two pieces of each color (size) of fettuccine, several pieces of uncolored noodles, and angle diagrams (two of each size), which can be found here (or see attached). (Note: make extra fettuccine pieces if you plan to use this lesson for several classes as some of the noodles WILL get broken.) Glue, construction paper, and markers should be provided to each group of students. Students should have notebooks and pencils to take notes and/or record their observations.

Lazy Lawrence Situation - Lawrence works for a company that makes roof trusses, triangular pieces that support simple roofs. His job is to ensure that each and every roof truss that is made at the company is exactly the same size as all the others. Because Lawrence is so lazy and likes to avoid as much work as possible, he wants to find the easiest possible way to do his job. Help Lawrence find the easiest way to show that two triangular trusses are equal in measure with as little work as possible.

Plan

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<td>Present and explain the Lazy Lawrence situation on an overhead projector. Ask students their opinion of what Lawrence could do to solve this problem. Review what it means for two objects to be congruent.</td>
<td>Listen; take notes; answer and ask questions. Think about how Lawrence could measure two roof trusses to see if they are exactly the same. Share those ideas with classmates.</td>
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congruent. Write several ideas of solutions to the problem on the board.

Encourage the development of different ideas, but move on toward measuring the roof trusses. Ask if Lawrence would have to measure each side, each angle, or some combination of these to do his job.

Organize ideas and thoughts by writing down ideas and conjectures. The class as a whole should decide that the simplest case needing to be explored is when one side or one angle of two different triangles is the same.

Write the conjecture "If two triangles have one side of equal measure, then the triangles are congruent." on the overhead projector. Pass out fettuccine and materials while explaining that the class will now be testing this conjecture with models of triangular trusses made from fettuccine noodles.

Listen and take notes. Divide into pairs to begin exploring the truth of the statement written on the overhead projector. Think about whether or not this statement is true.

Explain and demonstrate using two pieces of blue fettuccine to attempt to create two different triangles.

In pairs, determine whether one can create two different triangles that both have a blue side length. Students should understand that measuring one side of two roof trusses does not prove that they are the same.

Trace a counterexample on the overhead projector or large piece of construction paper. Label with a large "S" crossed out to show that the method could not be used by Lawrence. **Click here** (or see attached) for an example of a display sheet.

Trace out the counterexample onto a piece of construction paper and write the conclusion on the bottom. Label the sheet with the letter "S." Listen and take notes.

Write the next simplest case to be tested on the board: "If two triangles have one angle of equal measure, then the triangles are congruent." Explain and demonstrate using the prepared angle measures from their Ziploc bags to lay uncolored fettuccine on.

Discover (in pairs) that two different sized triangles can be created when one angle is fixed. Students should understand that measuring one angle of two triangles does not prove that they are the same.

Create another display sheet or overhead with two different triangles each having a congruent angle. Label this sheet with the letter "A." **Click here** (or see attached) for an example.

Trace out this counterexample on construction paper and label this with the letter "A."

Move on to measuring two parts of each triangle. Guide students as a class to determine the different combinations of

Use fettuccine to discover in pairs that measuring two parts of triangles does not guarantee they are the same. These three
angles and sides that are possible to try. Using shorthand notation, these are AA, SS, and AS. Walk around the room and help students as they use the prepared angles and side lengths to discover that none of these methods work either.

Move on to measuring three parts of a triangle. The class should find the six possible ways of measuring the two triangles: SSS, SSA, SAS, SAA, ASA, and AAA. If students are familiar with combinations and probability, a nice connection could be made here. During this lesson, we will explore SSS, SAS, SAA and ASA. Again, each of these hypotheses should be explored in the student pairs and the conclusion should be reached that only one and the same triangle can be made for each of the SSS, SAS, SAA and ASA cases.

Make a final conclusion involving Lazy Lawrence: he can either measure all three sides of the trusses, two angles and one side, or two sides and one angle. It should be stressed that the sides and angles in ASA and SAS must be in that order. If students think they don't need to be, have them explore this hypothesis if there is time.

Teacher Tips - It may be useful to have students create the display sheets for some or all of the cases. The teacher could choose one pair of students whose triangles are especially good demonstrators of a particular conjecture. Posters could be displayed around the room for a reminder of what students discovered during this lesson, as triangle congruency theorems will be used throughout the geometry class.

One thing we have found students to have trouble understanding is the concept of pre-measured angles being only angles, without fixed side lengths. Students may have difficulty understanding that they can extend or even shorten the length of the noodles in an angle and still have the same angle measure. To reduce this confusion, uncolored fettuccine should be used for angles and sides that have no fixed length and the colored fettuccine should be used for fixed side lengths. This should be explained to students at the start of their experimenting.

When students are experimenting with their fettuccine and coming up with ideas, the teacher should challenge their conclusions, whether right or wrong. Students should be asked how they can be certain that different methods will not work for Lazy Lawrence.
This should not be done in the spirit of making students feel wrong, but in making them
take ownership of their conclusions. If some students finish before others, they could be
encouraged to work ahead or to come up with and test other conjectures. Any conjecture
that a student comes up with (such as perhaps measuring one side of one triangle and one
angle of another) could and should be tested. In any case, developing students' reasoning
skills is an important component of this activity.

In many geometry classes, the SSS, ASA, and SAS triangle congruency rules, like
other theorems, are simply given to students who accept them without question. This
causes problems in understanding their usefulness. This lesson plan allows students to
own these theorems since they will have basically discovered them on their own. A
formal introduction to the SSS, ASA, and SAS theorems may be given after this lesson so
that students understand that these really are theorems used in formal geometry, but
references to Lazy Lawrence and what works for him can be used throughout this
geometry class.

**Assessment** - To assess student understanding during the activity, the teacher or any
teaching assistants should circle the room and discuss with students how they are doing.
Due to the many class and group discussions and hands-on activity, students should
frequently participate and ask questions. The teacher should be able to gauge their
understanding of triangle congruency theorems from their participation in class as well as
the display sheets students will make. In addition, the notebook where students have
written their ideas, conjectures, and conclusions should be collected at the end of the
unit. For homework, students should write in their journal their conclusions from the
day's activity, that is, the easiest way for Lazy Lawrence to measure his roof trusses.

**Special Needs Students** - Since students are working in pairs, students who may need
help forming triangles or understanding directions should be paired with others willing to
help them. Instructions in this lesson are normally given orally by the teacher, but a
paper copy could be distributed to some or all students. Student instructions can be found
[here](#), (or see attached) where the entire lesson could followed with limited teacher
directions if needed by some students. Diverse learning styles are addressed in this
lesson since students are exposed to working with a partner, the teacher, and the entire
class and various manipulatives and learning tools are used throughout.

**Extending Student Thinking** – If you have students that finish early or need more of a
challenge, you could provide the students with paper, ruler, and protractor and challenge
them to create two triangles that prove that SSA is not a theorem of congruency. This
lesson also leads naturally into lessons of using SAS, AAS, SSS, and ASA to write proofs
for triangle congruency.

**Standards** - The main NCTM Standards addressed in this lesson are Geometry and
Reasoning & Proof. For Geometry, students at the high school level should be able to
make conjectures and solve problems involving geometric objects, as they do in this
lesson. Students establish the validity of their own conjectures in this lesson by finding
the easiest way to measure a roof truss. In NCTM's *Principles and Standards 2000*, it is
stressed that reasoning and proof are not activities that are reserved for special lessons but
should be an integral part of all math curriculum. In this lesson, students are greatly
developing their reasoning skills by coming up with their own ideas and testing them
through a hands-on activity. This activity should lead students who may not have been
introduced to the idea of a formal proof yet to understanding the processes involved in
proving whether or not something is true.

References - Eggleton, Patrick J. "Triangles a la Fettuccine: A Hands-on Approach to
National Council of Teachers of Mathematics (NCTM). *Principles and Standards for
Display Sheet Example (Side)

The following is an example of a display sheet to be made after exploring keeping one side of two triangles fixed to determine if the two triangles must then be congruent.

Conclusion: If one side of each of two triangles has the same length, the two triangles are not necessarily congruent.
Display Sheet Example (Angle)

The following is an example of a display sheet to be made after exploring keeping one angle of two triangles fixed to determine if the two triangles must then be congruent.

Conclusion: If one angle of each of two triangles has the same length, the two triangles are not necessarily congruent.
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Student Instructions

Main Idea - In this lesson, you will use fettuccine noodles to discover the truth of different conjectures about triangles. You will be given a problem involving needing to find whether or not two triangles are exactly the same. You will begin with the most basic case and work toward a final conclusion in solving this problem. What do we call two objects that are exactly the same measure?

Materials - Your teacher will provide you with a Ziploc bag with the following: two pieces each of different colored fettuccine noodles, several pieces of uncolored noodles, and angle diagrams, which can also be found by clicking [here](#). Each different color of fettuccine are a different length (the blue are 7 cm, the red are 9 cm, the green are 12 cm). You will also need glue, construction paper, and markers for this activity. As always, you should have a notebook and a pencil to take notes and/or record your observations.

Instructions

- Listen and take notes as your teacher presents the problem of [Lazy Lawrence](#) to the class. Think about how Lawrence could determine if two roof trusses are exactly the same. Share those ideas with classmates.
- Organize your ideas and thoughts by writing them down in your notebook. If Lawrence were to measure parts of the roof trusses, what is the simplest thing he could do? If one side of each triangle is the same, does that mean the triangles are the same all around? What about one angle?
- With a partner, try to find whether or not measuring only one side of two different triangles is enough to see that they are the same. Use two pieces of blue fettuccine to try to find two different triangles that both have a blue side length. Can you do it? Trace your triangles onto a sheet of construction paper and write your conclusion on the bottom. Label this sheet with an S for side.
- Now try making two different triangles that both have an angle measure of 40 degrees. Using the angle diagrams, lay plain fettuccine on the paper and try to make two different triangles. The side lengths of the 40 degree angles can be whatever you like, since only the angle must be the same. Did you make two different triangles? Make another display sheet showing what you have found. Label this sheet with an A for angle.
- Next, we'll measure two parts of different triangles to see if they are the same all around. We can measure two sides, two angles, or one side and one angle of each triangle.
  - Try making two different triangles that both have a green side and a blue side. Can you do it? Make a sheet displaying your conclusions and label it with SS for side-side.
Now try making two different triangles that both have one 40 degree angle and one 60 degree angle. Can you do it? Make a sheet displaying your conclusions and label it with **AA** for angle-angle.

Now try making two different triangles that both have one 40 degree angle and one blue side. Can you do it? Make a sheet displaying your conclusions and label it with **SA** for side-angle.

- Now we need to see if Lawrence could measure three aspects of different roof trusses to see if they are the same. What are the different combinations we could measure? We could measure all three sides, all three angles, two sides and one angle, or one side and two angles. The different cases we will explore today are SSS (side-side-side), SAS (side-angle-side), and ASA (angle-side-angle).
  - First, try making two different triangles that both have one blue side, one green side, and one red side. Can you do it? Make a sheet displaying your conclusions and label it with **SSS** for side-side-side.
  - Now try making two different triangles that both have one blue side, one green side, and one 40 degree angle. The angle should be in between the sides. Can you do it? Try again using other angles and sides, as long as two sides and the angle between them are the same. Make a sheet displaying your conclusions and label it with **SAS** for side-angle-side.
  - Finally, try making two different triangles that both have one 40 degree angle, one 60 degree angle, and one blue side. The blue side should be in between the two angles. Can you do it? Try again using other angles and sides, as long as two angles and the side between them are the same. Make a sheet displaying your conclusions and label it with **ASA** for angle-side-angle.

- What is your final conclusion about Lazy Lawrence? Must he measure all the sides and angles of each roof truss to do his job correctly, or can he get by with only measuring some angles and sides? Write your conclusions in your notebook.

Angle Diagrams

Use these angle diagrams as templates for making angles. Print out this sheet, cut out each angle, and lay any length of fettuccine noodles on the rays.