Lecture today: Chapter 36 (more diffraction)
   1) Example double slit diffraction
   2) Diffraction Gratings
   3) X-ray diffraction

Announcements:
   - tomorrow (Th) office hours 10-10:30 (due to campus visitor)
   - HW 10 correction: Question 4b, use $\lambda = 700$nm. (corrected HW10 online)

Last lecture:
   - The Michelson Interferometer (Application of interference)
   - talked about the difference between interference and diffraction
   - Diffraction from a single slit
     general minima - dark fringes: $a \sin \theta = m\lambda$ for $m=\pm1, \pm2, \pm3,\ldots$
     for the case of $D \gg a$ (Fraunhofer or far-field diffraction)
   - Intensity in single-slit diffraction

$$I = I_0 \left[ \frac{\sin(\alpha)}{\alpha} \right]^2$$
where
$$\alpha = \frac{1}{2} \phi = \frac{\pi a}{\lambda} \sin \theta$$

- Diffraction by double slit
the intensities of the fringes produced by double-slit interference are modified by
diffraction of the light passing through each slit (as discussed in Chapter 36).

With diffraction effects taken into account, the intensity of a double-slit interference pattern is given by

$$I(\theta) = I_0 \cos^2 \beta \left( \frac{\sin \alpha}{\alpha} \right)^2 \quad \text{(double slit),}$$

$$\beta = \frac{\pi d}{\lambda} \sin \theta$$

$$\alpha = \frac{\pi a}{\lambda} \sin \theta.$$  

Here $d$ is the distance between the centers of the slits and $a$ is the slit width.
Note
(1) The interference factor $\cos^2 \beta$ is due to the interference between two slits with slit separation $d$
(2) The diffraction factor $[(\sin \alpha)/a]^2$ is due to diffraction by a single slit of width $a$

We get the two-slit pattern, but modified by the single-slit intensity as an envelope. Instead of all peaks being of the same height, they get weaker at larger angles.
**Example:** In a double-slit experiment, the wavelength \( \lambda \) of the light source is 405 nm, the slit separation \( d \) is 19.44 mm, and the slit width \( a \) is 4.050 mm. Consider the interference of the light from the two slits and also the diffraction of the light through each slit. How many bright interference fringes are within the central peak of the diffraction envelope?

\[
\begin{align*}
\lambda &= 405\text{ nm} = 405 \times 10^{-9} \text{ m} \\
d &= 19.44\text{ mm} = 19.44 \times 10^{-3} \text{ m} \\
a &= 4.050\text{ mm} = 4.05 \times 10^{-3} \text{ m}
\end{align*}
\]

The limits of the central peak are the first minima in the diffraction pattern (single slit):

\[
a \sin \theta = m \lambda, \quad m = 1
\]

Interference \( \Rightarrow \) given by double slit for \( a \ll \lambda \)

**Bright:** \( d \sin \theta = m' \lambda, \quad m' = 0, 1, 2, \ldots \)

**Dark:** \( d \sin \theta = (m' + \frac{1}{2}) \lambda \)

Need bright fringes \( \Rightarrow \) solve for \( m' \)

\[
m' = \frac{d \sin \theta}{\lambda} = \frac{ds \sin \theta}{\lambda \sin \theta} = \frac{19.44}{4.05} = 4.8
\]

\( \Rightarrow \) need to truncate to integer: \( m' = 4 \)

\( m' = 0, m' = 1, m' = 2, m' = 3, m' = 4 \) \( \Rightarrow \) total of 9 bright fringes!

9 bright fringes fit within the central peak of the diffraction envelope.
2) Diffraction Gratings
One of the most useful tools in the study of light and of objects that emit and absorb light is the diffraction grating.

Diffraction grating: Many Slits
A diffraction grating consists of a large number of \( N \) slits each of width \( a \) and separated from the next by a distance \( d \).

Systems of narrow slits are of tremendous practical importance in spectroscopy, the determination of the particular wavelength of light coming from a source.

Assume that each slit is narrow in comparison to the wavelength, so its diffraction pattern spreads out nearly uniformly.

Here, 8 slits with \( d \) = separation distance between slits. Constructive interference occurs when

\[
d \sin \theta = m \lambda \quad \text{for} \ m = 0, \pm 1, \pm 2, \pm 3, \pm 4, \pm 5, \ldots
\]

The maxima in the pattern occur at the same position as for two slits with the same spacing. To this extent the pattern resembles the two-slit pattern.

But what happens between the maxima? In the two-slit pattern, there is exactly one intensity minimum located midway between each pair of maxima - phase difference between waves from the two sources is \( \pi, 3 \pi, 5 \pi, \ldots \). In the 8-slit pattern, these are also minima because the light from adjacent slits cancel out in pairs, but these are not the only minima.
The figure below shows the interference pattern for 2, 8, and 16 equally spaced narrow slits.

We notice that the principal maxima become sharper and narrower as \( N \) increases.

**The observation can be explained as follows:**
Suppose an angle \( \theta \) which initially gives a principal maximum is increased slightly, if there were only two slits, then the two waves will still be nearly in phase and produce maxima which are broad. However, in grating with a large number of slits, even though \( \theta \) may only be slightly deviated from the value that produces a maximum, it could be exactly out of phase with light wave from another slit far away. Since grating produces peaks that are much sharper than the two-slit system, it gives a more precise measurement of the wavelength \( \lambda \).

By making the slits really close together, the maxima become more separated. If the light falling on the slits contains more than one wavelength (color), there will be more than one pattern, separated more or less according to wavelength, although all colors have a maximum at \( m = 0 \).

This means that the different orders make rainbows—separating wavelengths into a spectrum, with the separation being greater for greater order \( m \).
Example:
The wavelengths of the visible spectrum are approximately 380 nm (violet) to 750 nm (red).
a) Find the angular limits of the first-order visible spectrum produced by a plane grating with 600 slits per millimeter when white light falls normally on the grating.
b) Do the first order and second order spectra overlap? What about the 2nd and 3rd orders?

d sinθ = mλ

θ = sin⁻¹(mλ/d)

(a)
distance between slits is  
\[ d = \frac{1 \text{ mm}}{600 \text{ slits}} = 1.67 \times 10^{-6} \text{ m} \]

Violet light for 1st (m=1) order occurs at
\[ \theta = \arcsin \left( \frac{\lambda}{d} \right) = \arcsin \left( \frac{3.8 \times 10^{-7}}{1.67 \times 10^{-6}} \right) = 13.2° \]

Red light for 1st order occurs at
\[ \theta = \arcsin \left( \frac{\lambda}{d} \right) = \arcsin \left( \frac{7.5 \times 10^{-7}}{1.67 \times 10^{-6}} \right) = 26.7° \]

(b) recalculate for m = 2 and m = 3.

Violet light for 2nd order occurs at 27.1°
Violet light for 3rd order occurs at 43.0°

Red light for 2nd order occurs at 63.9°
Red light for 3rd order occurs at 90° (because 3λ/d > 1)

The 1st-order spectrum extends from 13.2-26.7°. The 2nd-order spectrum extends from 27.1-63.9° while the 3rd order is from 43-90°.

Answer: 1st-order spectrum does not overlap with 2nd-order spectrum, but 2nd-order spectrum overlaps with 3rd-order spectrum.

One can show that, of two diffraction gratings, the grating with the larger value of N is better able to distinguish between wavelengths because its diffraction lines are narrower and so produce less overlap.

Grating Spectrographs (Application)
A diffraction grating can be used to disperse light into a spectrum. The greater the number of slits, the better the resolution.
- used a lot in light/fluorescence microscopy
3) X-ray diffraction

X rays are electromagnetic radiation whose wavelengths are of the order of
1 Å = 10^{-10} m.

X-rays are produced when electrons escaping from a heated filament are accelerated by a
potential difference and strike a metal target.

They were discovered by Wilhelm Röntgen in 1895. He found that X-rays could pass
straight through his hand and cast shadows of his bones on the fluorescent screen. He
spent several weeks privately investigating the rays before publishing his results at the
end of the year. (First X-ray image of his wife's hand) (Later X-ray diffraction images of
DNA led to the discovery of the DNA double helix.)

A standard optical diffraction grating cannot be used to discriminate between different
wavelengths in the x-ray wavelength range.

Example:
λ = 10^{-10} m
d = 3000 nm
find angle at which first order maximum occurs:

\[ \theta = \sin^{-1} \left( \frac{m \lambda}{d} \right) = \sin^{-1} \left( \frac{(1)(0.1 \text{ nm})}{3000 \text{ nm}} \right) = 0.0019^\circ. \]

This is too close to the central maximum to be practical. A grating with d \approx \lambda is desirable,
but, because X-ray wavelengths are about equal to atomic diameters, such gratings cannot
be constructed mechanically.

1912, it occurred to German physicist Max von Laue that a crystalline solid, which
consists of a regular array of atoms, might form a natural three-dimensional “diffraction
grating” for X-rays.

When X-rays pass through a crystal, the crystal behaves like a diffraction grating,
causing x-ray diffraction

An X-ray which reflects from the surface of a substance has travelled less distance than
an X-ray which reflects from a plane of atoms inside the crystal. The penetrating X-ray
travels down to the internal layer, reflects, and travels back over the same distance before
being back at the surface. The distance travelled depends on the separation \( d \) of the layers
and the angle at which the X-ray entered the material. For this wave to be in phase with
the wave which reflected from the surface it needs to have travelled a whole number of
wavelengths while inside the material. Bragg expressed this in an equation now known as
Bragg's Law, the criterion for intensity maxima for x-ray diffraction
$2d \sin \theta = m\lambda \quad \text{for } m = 1, 2, 3, \ldots$

$m$ is an integer determined by the order given,
$\lambda$ is the wavelength of x-rays, and moving electrons, protons and neutrons,
$d$ is the spacing between the planes in the atomic lattice, and
$\theta$ is the angle between the incident ray and the scattering planes.