Today:
- Motion of charged particles in magnetic (and electric) fields
- Next Week Wednesday – EXAM 2 (study guide online)

Last time we introduced the magnetic field and talked about the magnetic force on a moving charged particle

\[ \vec{F}_B = q \vec{v} \times \vec{B} \quad \text{(vector/cross product)} \quad [T] \quad \text{Units: Tesla=T} \]

Example: The Figure shows a uniform magnetic field \( \vec{B} \) directed into the plane of the paper (x). A particle with negative charge moves in the plane. Which of the three paths - 1, 2, or 3 - does the particle follow?

Answer: path 3!

So what is the force on a charged particle moving through a region in space where both electric and magnetic fields are present? Now both fields exert forces on the particle. The total force \( \vec{F} \) is the vector sum of the electric and magnetic forces:

\[ \vec{F} = \vec{F}_E + \vec{F}_B = q \vec{E} + q \vec{v} \times \vec{B} = q \left( \vec{E} + \vec{v} \times \vec{B} \right) \]

Today we will study the consequences and applications of this equation that is the motion of charged particles in magnetic (and electric) fields.

The relevant concept we will be using:
In analyzing the motion of a charged particle in electric and magnetic fields, we will apply Newton's 2nd law of motion

\[ \sum \vec{F} = m \vec{a} \]

with the net force given by

\[ \sum \vec{F} = q \left( \vec{E} + \vec{v} \times \vec{B} \right) \]

We will see that often forces such as gravity can be neglected and we will see that in situations governed by this equations, charged particles will move in curved particle trajectories.
Consequences and Applications

**Particle Trajectories:**
Magnetic fields are very good at making charged particles go around circles! A beam of charged particles will move in a circle at constant speed when they are sent into it perpendicular to a magnetic field.

For \( \vec{v} \) 90° to \( \vec{B} \), A force at 90° to \( \vec{v} \) causes a change in direction but no change in speed

\[ \Rightarrow \text{Uniform Circular Motion} \]

**Centripetal Acceleration:**
\[ a_{\text{rad}} = \frac{v^2}{r} \]

Question: How can we calculate the radius of this circular motion?

\[ \sum F = q \left( \vec{E} + \vec{v} \times \vec{B} \right) \]

becomes in case of the magnitude:

\[ m \frac{v^2}{r} = |q| vB \sin(90) = |q| vB \]

hence after some rearranging the radius of the circular motion is given by:

\[ r = \frac{mv}{|q|B} \]

Let's make sure that the units work out:

\[
\begin{bmatrix}
\text{kg} & \text{m} \\
\text{s} & \text{C} \cdot \text{T}
\end{bmatrix}
= \begin{bmatrix}
\text{kg} & \text{m} \\
\text{s} & \text{C} \cdot \text{A} \cdot \text{m}
\end{bmatrix}
= \begin{bmatrix}
\text{kg} & \text{m} \\
\text{s} & \text{C} \cdot \text{s} \cdot \text{m}
\end{bmatrix}
= \begin{bmatrix}
\text{kg} & \text{m} \\
\text{s} & \text{C} \cdot \text{s} \cdot \text{m}
\end{bmatrix}
= [m]
\]
**Example:** What is the mass of a $10 \, \mu C$ charge if with speed $4.0 \times 10^6 \, \text{m/s}$ it travels on a $1 \, \text{mm}$ radius circle in a magnetic field $B = 0.5 \, \text{T}$?

We can also obtain the period for this circular motion:
From Physics 1 we know that the period $T$ of circular motion is:

$$T = \frac{2\pi r}{v}$$

with

$$r = \frac{mv}{|q|B}$$

we get

$$T = \frac{2\pi m}{|q|B}$$

which is independent of speed $v$!

Now we can define the frequency

$$f = \frac{1}{T} = \frac{|q|B}{2\pi m}$$
**Magnetic Bottles: Helical paths**

When the particle’s velocity is not 90° to \( \vec{B} \), the particle spirals on a helix, i.e. if there is a component of the charged particle’s velocity in the direction of the magnetic field, the path will be helical.

- By using a non-uniform field of the appropriate design, you can trap charged particles in a “magnetic bottle”.

![Diagram of magnetic fields and particle trajectories](image1)

<table>
<thead>
<tr>
<th>(a)</th>
<th>(b)</th>
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- a) The Van Allen radiation belts around the earth. Near the poles, charged particles from these belts can enter the atmosphere, producing the aurora borealis ("northern lights") and aurora australis ("southern lights")
- b) A photograph of the aurora borealis
Mass spectrometer that can be used to measure the mass of an ion.

Example 2: Uniform circular motion of a charged particle in a magnetic field

An ion of mass $m$ (to be measured) and charge $q$ is produced in source $S$. The initially stationary ion is accelerated by the electric field due to a potential difference $V$. The ion leaves $S$ and enters a separator chamber in which a uniform magnetic field is perpendicular to the path of the ion. A wide detector lines the bottom wall of the chamber, and the causes the ion to move in a semicircle and thus strike the detector. Suppose that $B = 80.000 \text{ mT}$, $V = 1000.0 \text{ V}$, and ions of charge $q = +1.6022 \times 10^{-19} \text{ C}$ strike the detector at a point that lies at $x = 1.6254 \text{ m}$. What is the mass $m$ of the individual ions?

![Diagram of mass spectrometer and ion's path]

\[
\begin{align*}
\text{Example 2} \\
R &= \frac{x}{2} \\
q &= 1.6022 \times 10^{-19} \text{ C} \\
V &= 1000.0 \text{ V} \\
B &= 80.000 \text{ mT} \\
x &= 1.6254 \text{ m} \\
V &= 1000.0 \text{ V} \\
\text{Use conservation of energy:} \\
K_e + U_a &= K_f + U_d \\
O + qV &= \frac{1}{2}mv^2 + O \\
v &= \sqrt{\frac{2qV}{m}} \\
m &= \frac{RqIB}{2qV} \\
\text{Rewrite in atomic mass units:} \\
m &= \frac{(1.6254)^2(1.6022 \times 10^{-19})}{8(1000)} \\
m &= 3.3863 \times 10^{-25} \text{ kg} \\
m &= 203.93 \text{ u}
\end{align*}
\]
Crossed Fields:
Both an electric field and a magnetic field can produce a force on a charged particle. When the two fields are perpendicular to each other, they are said to be crossed fields. Here we shall examine what happens to charged particles - namely, electrons - as they move through crossed fields.

Velocity Filter
In a beam of charged particles produced by a heated cathode or a radioactive material, not all particles move with the same speed. Many applications, however, require a beam in which all the particle speeds are the same. Using crossed fields particles of a specific speed can be selected as follows:

We can use the magnetic force in conjunction with the electric force to filter out particles of a certain velocity (or just determine velocity).

\[
\vec{F}_E = q\vec{E} \\
\vec{F}_B = q\vec{v} \times \vec{B}
\]

When the forces are equal,

\[qE = q\nu B\]

there is no deflection and

\[\nu = E/B.\]

By moving a slit that blocks particles except that go through the hole, you can pick out different velocities.

(a) Schematic diagram of velocity selector
(b) Free-body diagram for a positive particle
Discovery of the Electron:
This idea of selecting particle velocities by using crossed fields was used in one of the landmark experiments in physics by J. J. Thomson (1856-1940) at the end of the 19th century measured the ratio of charge to mass for the electron. In 1897, J.J. Thompson used a velocity filter to determine the ratio of the charge to mass of particles emitted from a cathode.

Charged particles (which we now know as electrons) are emitted by a hot filament at the rear of the evacuated tube and are accelerated from rest by an applied potential difference $V$. The speed $v$ of the electron is determined by this accelerating potential $V$:

The gained kinetic energy equals the lost electric potential $K_i + U_i = K_f + U_f$
since we have here no initial kinetic energy and no final potential energy $U_i = K_f$
using $U = qV$ with $e =$ magnitude of electron charge $eV = \frac{1}{2}mv^2$

hence $v = \sqrt{\frac{2eV}{m}}$

substitute into $v = E/B$ (if there is no deflection) we obtain for the ratio of charge to mass for the electron

$e \frac{E}{m} = \frac{1}{2}vB$

He found that, regardless of cathode material, the ratio was always constant, and thus discovered the electron as a universal particle. Note your book gives you an expression in case of a deflection $y$ from center with plate length $L$ using $V=EL$ for a uniform electric field

$e \frac{y}{m} = \frac{2yE^2}{L^2B^2}$

Do problems 29 to 32 on sheet!