PES 1120: Homework assignment 2 [24 points]

Handed out: Friday February 7
Due in: Wednesday February 12, at the start of class 6:05pm
Worth 3% of your total grade. Show all reasoning and workings.

Part 1: fully graded questions [18 points]

Question 1 [12 points]
Negative charge $-Q$ is distributed uniformly around a quarter-circle of radius $R$ that lies in the first quadrant, with the center of curvature at the origin.

a) [1 point] In which direction does the electric field point at the origin, which is marked with $O$ on the diagram? Explain your reasoning.

b) [1 point] Add variables to your diagram that can be used to construct an integral to add the fields caused by each little section of charge $dq$.

c) [1 point] Write an expression for the electric charge $dq$ in terms of $d\theta$ (use $s = R\theta$) in that short section of the charge distribution.

d) [3 points] Write expressions for the non-zero components (both $x$- and $y$-components) of the electric field at $O$ caused by the designated charge $dq$ in part b).

e) [3.5 points] Write the integral with the appropriate limits and complete the integral. Show that you get $E_x = E_y = Q/(2\pi \epsilon_0 R^2)$.

f) [1.5 points] Find an algebraic expression for the magnitude and direction of the electric field at the origin. For the direction of the electric field compare your result to your answer in part a).

g) [1 point] If $R \to \infty$, what is the limit of the expression for the electric field found in part f)? Why does this make sense?

Hint: This question is similar to the worked sample problem on p.588-589 of the 9th edition and on p.641-642 of the 10th edition of your textbook.
Question 2 [6 points]
Given a parallelepiped shown below, the electric field are both horizontal and uniform over the two parallelepiped’s faces in question. The faces are inclined at 33.0° from the horizontal. $\vec{E}_1$ has magnitude $1.50 \times 10^4 N/C$, $\vec{E}_2$ and has a magnitude of $5.50 \times 10^4 N/C$.

a) [4.5 points] What is the net flux through the parallelepiped?

b) [1.5 points] How is the electric field produced? Due charges within the parallelepiped produce the electric field or do charges outside contribute as well? How can you tell?

Part 2: completion grade only [6 points]
In this section you must fully complete 3 problems from the textbook to score 6 points in total. If you do everything correct in a problem, you will get 2 points for it. If you get something wrong but make a reasonable attempt, you will get 1 point for a problem. No attempt or an unconvincing attempt will score zero.

To get the full 2 points you must:
- Write down all known and unknown variables clearly
- Draw a diagram
- Include all mathematical and logical steps, including assumptions
- Get the correct answer with the correct units and significant figures

Please grade this section yourself using the answers at the back of the book. The grader will check that you have not cheated by awarding yourself too many points.

This week’s questions are:
- Chapter 22, Problem 31 [2 points]
- Chapter 22, Problem 37 [2 points]
- Chapter 22, Problem 83 [2 points]
PES 1120  Homework # 2

Part 1  Question 1

a) 

The ring is negatively charged. Therefore, the \( E \)-field must point towards the ring. \( E \) must be at \( \theta = 45^\circ \) due to symmetry.

b) 

Note: we took already care of the negative charge by having \( dE \) point \( \theta \) towards \( dq \).

c) 
\[ dq = \lambda \, ds \] 
\[ s = R \theta \Rightarrow ds = R \, d\theta \text{ since } R \text{ is a constant} \]
\[ dq = \lambda R \, d\theta \] 
\[ dE = \frac{1}{4 \pi \varepsilon_0} \frac{dq}{R^2} = \frac{1}{4 \pi \varepsilon_0} \frac{\lambda R \, d\theta}{R^2} = \frac{1}{4 \pi \varepsilon_0} \frac{\lambda \, d\theta}{R} \]
\[ dE_x = dE \cos \theta = \frac{1}{4 \pi \varepsilon_0} \frac{\lambda}{R} \cos \theta \, d\theta \]
\[ dE_y = dE \sin \theta = \frac{\lambda}{R} \sin \theta \, d\theta \]

d) 
\[ E_x = k \frac{\lambda}{R} \int_0^{\pi/2} \frac{\cos \theta \, d\theta}{R^2} = k \frac{\lambda}{R} \sin \frac{\pi}{2} = k \frac{\lambda}{R} (1 - 0) = \frac{k \lambda}{R} \]
\[ E_y = k \frac{\lambda}{R} \int_0^{\pi/2} \frac{\sin \theta \, d\theta}{R^2} = k \frac{\lambda}{R} (1 - 0) = \frac{k \lambda}{R} \]

Use \( \lambda = \frac{|Q|}{s} = R \theta_{\text{max}} = \frac{Q}{R \theta_{\text{max}}} = \frac{2Q}{R \pi R} \) with \( k = \frac{1}{4 \pi \varepsilon_0} \)
\[ E_x = E_y = \frac{1}{4 \pi \varepsilon_0} \frac{2Q}{R \pi R} = \frac{Q}{2 \pi \varepsilon_0 R^2} \]
1. \( E = \sqrt{E_x^2 + E_y^2} = \frac{Q}{2\pi \varepsilon_0 R^2} \)

   direction: \( \theta = \tan^{-1} \left( \frac{E_y}{E_x} \right) = \tan^{-1}(1) = 45^\circ \)

   This result agrees with part a) as \( E \) must point towards the negatively charged quarter-circle.

   Also \( E \) is at \( 45^\circ \) as expected from symmetry.

2. \( E = \frac{Q}{2\pi \varepsilon_0 R^2} \) as \( R \to \infty \); \( E = 0 \)

   \( E = 0 \) for \( R \to \infty \) makes sense, since at infinity the \( E \)-field must drop to 0.

Other way to solve for \( E \):

   Rotate quarter-line \( 90^\circ \) clockwise

   \[ \text{d}E' = k \frac{dq}{R^2} = k \frac{\lambda d\theta}{R} \]

   \[ E' = E_x' \Rightarrow \text{d}E' = \text{d}E' \cos \theta' \]

   \[ E_x' = k \frac{\lambda}{R} \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \cos \theta' \text{d}\theta' = k \frac{\lambda}{R} \sin \theta' \]

   \[ E_x' = k \frac{\lambda}{R} \left( \frac{1}{\sqrt{2}} - (-\frac{1}{\sqrt{2}}) \right) = \frac{2}{\sqrt{2}} k \frac{\lambda}{R} \]

   \[ E_x = k \frac{\lambda}{R} \left( \frac{1}{\sqrt{2}} - (-\frac{1}{\sqrt{2}}) \right) = \frac{2}{\sqrt{2}} k \frac{\lambda}{R} = \frac{\sqrt{2}}{2} k \frac{\lambda}{R} \]

   Answer for part f) \( \Rightarrow E' = E_x' = \frac{\lambda}{\sqrt{2}} k \frac{\lambda}{R} = 1.41 k \frac{\lambda}{R} = \frac{\sqrt{2}}{2} k \frac{\lambda}{R} \)

   To get \( E_x \) and \( E_y \) from going \( \theta \) in \( E \):

   e) \[ E_x = E' \cos (\theta') = E' \cos (45^\circ) = \frac{1}{\sqrt{2}} \sqrt{\frac{Q}{2\pi \varepsilon_0 R^2}} = \frac{Q}{2\pi \varepsilon_0 R^2} \]

   \[ E_y = E' \sin (\theta') = E' \sin (45^\circ) = \frac{1}{\sqrt{2}} \sqrt{\frac{Q}{2\pi \varepsilon_0 R^2}} = \frac{Q}{2\pi \varepsilon_0 R^2} \]
Question 2

"Back" \[ \vec{E}_2 \]

"Front" \[ \vec{E}_1 \]

\[ E_2 = 5.50 \times 10^4 \, \text{N/C} \]
\[ E_1 = 1.50 \times 10^4 \, \text{N/C} \]

(3 sig. figs)

a) Fields pass through two faces of the parallipiped.

"Front" \[ \text{Area that } \vec{E}_1 \text{ is coming out} \]

"Back" \[ \text{Area that } \vec{E}_2 \text{ is coming out} \]

\[ A_1 = A_2 = (0.055 \, \text{m})(0.066 \, \text{m}) = 0.00363 \, \text{m}^2 = 3.63 \times 10^{-3} \, \text{m}^2 \]

For both areas the field is uniform \[ \Rightarrow \text{flux } \Phi = \vec{E} \cdot \vec{A} \]

Total flux \[ \Phi = \Phi_1 + \Phi_2 \]

For \( A_1 \):

\[ \vec{A}_1 \]

\[ \alpha = 90 - \theta = 90^\circ - 33^\circ = 57^\circ \]

\[ \Phi_1 = \vec{E}_1 \cdot \vec{A}_1 = \frac{E_1 A \cos (57)}{C} = \frac{(1.5 \times 10^4)(3.63 \times 10^{-3}) \cos (57)}{C} \]

\[ \Phi_1 = 29.66 \, \text{Nm}^2 \]

For \( A_2 \):

\[ \vec{A}_2 \]

\[ \alpha' = 90 + \theta = 90^\circ + 33^\circ = 123^\circ \]

\[ \Phi_2 = \vec{E}_2 \cdot \vec{A}_2 = \frac{E_2 A \cos (123)}{C} = \frac{(5.5 \times 10^4)(3.63 \times 10^{-3}) \cos (123)}{C} \]

\[ \Phi_2 = -108.7 \, \text{Nm}^2 \]

Total flux: \[ \Phi = \Phi_1 + \Phi_2 = 29.66 - 108.7 = -79.077 \, \text{Nm}^2 \]

\[ \Phi = -79.1 \, \text{Nm}^2 \]

\[ 1 = 0.25 \]

sig. figs

\[ -0.25 \]

sig. figs
b) \( \vec{E}_1 \) and \( \vec{E}_2 \) cannot be produced only by a charge inside.

If only a negative charge distribution would be inside then field lines would only go towards the parallel piped.

\[ \rightarrow - Q \leftrightarrow \]

But \( \vec{E}_1 \) is coming out. This is why.
Part 2:

Chapter 22, Problem 31

a) \( \lambda = \frac{q}{\text{dotted length}} = \frac{-q}{L} = \frac{-9.23 \times 10^{-15}}{0.0815} = -5.19 \times 10^{-14} \text{ C/m} \)

b) \( E \) must point along the \( \mathbb{E} \)-x-direction.

d\( E = dE_x = k \frac{dq}{r^2} = k \frac{dq}{(L+a-x)^2} \)

d\( dq = \lambda dx \) with \( \lambda = \frac{-q}{L} \) (Took care of the sign) (Since \( E \) is in the \( \mathbb{E} \)-x-direction)

\( dE = k \frac{\lambda dx}{(L+a-x)^2} \)

\( E = k \lambda \int_0^L \frac{dx}{(L+a-x)^2} = k \lambda \int_0^L \frac{\frac{L}{a(L+a)} - \frac{1}{a(L+a)}}{a(L+a)} = k \lambda \frac{\frac{L}{a(L+a)} - \frac{1}{a(L+a)}}{a(L+a)} = k \lambda \frac{\frac{L}{a(L+a)} - \frac{1}{a(L+a)}}{a(L+a)} = k \frac{q}{a(L+a)} \)

Magnitude: \( E \sim 8.99 \times 10^9 \text{ N/m}^2 \), \( a = 0.128 \text{ m}, L = 0.0815 \text{ m} \), \( q = 4.23 \times 10^{-15} \text{ C} \)

\( E = (8.99 \times 10^9) \left[ \frac{4.23 \times 10^{-15}}{0.128(0.0815 + 0.120)} \right] = 0.001573 \text{ N/C} \)

\( E = 1.57 \times 10^{-3} \text{ N/C} \)

c) Direction is in the negative x-direction.

d) \( a = 50 \text{ m} \Rightarrow a \gg L \)

\( E = (8.99 \times 10^9) \left( \frac{4.23 \times 10^{-15}}{50(0.0815 + 50)} \right) = 1.52 \times 10^{-8} \text{ N/C} \)

\( E \sim 1.52 \times 10^{-8} \text{ N/C} \)

e) \( E = k \frac{q}{r^2} = 1.52 \times 10^{-8} \text{ N/C} \)

Some magnitude as in d) as expected!
Chapter 22, problem 37

For disk: \( E_{\text{disk}} = \frac{\varepsilon_0}{2} \left( 1 - \frac{2}{(R^2 + z^2)^{1/2}} \right) \)

For ring use: from \( R/2 \) to \( R \) (Integral we did in class)

\[
E = \frac{\varepsilon_0}{4} \left[ \frac{(z^2 + R^2)^{1/2}}{R} \right]
\]

\( E_{\text{ring}} = \frac{\varepsilon_0}{\delta z} \left( \frac{z}{(R^2/2 + z^2)^{1/2}} - \frac{z}{(R^2 + z^2)^{1/2}} \right) \)

Relative difference:

\[
\frac{E_{\text{disk}} - E_{\text{ring}}}{E_{\text{disk}}} = \frac{\varepsilon_0}{2} \left[ \frac{2}{(R^2 + z^2)^{1/2}} \right]
\]

\[
= \frac{2R}{R^2/4 + 4R^2 z^2}
\]

\[
= \frac{2}{(1 + 4)^{1/2}}
\]

For \( z = 2R \)

\[
= \frac{1 - 0.970143}{1 - 0.89927} = 0.2828
\]

The E-field magnitude would be decreased by \( 28\% \).
Chapter 22, Problem 8.3

\[ \vec{P} = (3.00 \hat{x} + 4.00 \hat{y}) (1.24 \times 10^{-30} \text{ cm}) \]

\[ \vec{E} = 4000 \text{ N/C} \hat{z} \]

a) \[ U = -\vec{P} \cdot \vec{E} = -\left[ (3.00)(4000) \hat{z} \cdot \hat{x} \right] + (3.00)(0) \hat{z} \cdot \hat{j} \times 1.24 \times 10^{-30} \]

\[ U = -1.49 \times 10^{-26} \text{ J} \]

b) \[ \vec{T} = \vec{P} \times \vec{E} = 4(4000) \times 1.24 \times 10^{-30} \]

\[ \vec{T} = -(4 \hat{k}) (4 \times 1.24 \times 10^{-30}) (4 \times 10^3) = -1.98 \times 10^{-26} \hat{k} \]

c) \[ W = \Delta U = U_f - U_i \]

\[ U_i = -1.49 \times 10^{-26} \text{ J} \text{ from part a) } \]

\[ U_f = -\vec{P}_f \cdot \vec{E} = -\left[ (4 \hat{x} + 3 \hat{y}) \times (4000 \hat{x}) \right] \times 1.24 \times 10^{-30} \]

\[ = 4(4000) \times 1.24 \times 10^{-30} = 1.98 \times 10^{-26} \text{ J} \]

\[ U_f - U_i = 1.98 \times 10^{-26} \text{ J} - (-1.49 \times 10^{-26} \text{ J}) = 3.47 \times 10^{-26} \text{ J} \]

\[ W = 3.47 \times 10^{-26} \text{ J} \]
Chapter 22, Problem 31

31. (a) The linear charge density is the charge per unit length of rod. Since the charge is uniformly distributed on the rod,

\[ \lambda = \frac{q}{L} = \frac{-4.23 \times 10^{-15} \text{ C}}{0.0815 \text{ m}} = -5.19 \times 10^{-14} \text{ C/m}. \]

(b) We position the x axis along the rod with the origin at the left end of the rod, as shown in the diagram.

Let \( dx \) be an infinitesimal length of rod at \( x \). The charge in this segment is \( dq = \lambda \, dx \). The charge \( dq \) may be considered to be a point charge. The electric field it produces at point \( P \) has only an \( x \) component, and this component is given by

\[ dE_x = \frac{1}{4\pi\varepsilon_0} \frac{\lambda \, dx}{(L + a - x)^2}. \]

The total electric field produced at \( P \) by the whole rod is the integral

\[ E_x = \frac{\lambda}{4\pi\varepsilon_0} \int_0^L \frac{dx}{(L + a - x)^2} = \frac{\lambda}{4\pi\varepsilon_0} \left[ \frac{1}{L + a - x} \right]_0^L = \frac{\lambda}{4\pi\varepsilon_0} \left( \frac{1}{L + a} - \frac{1}{a} \right) \]

\[ = \frac{\lambda}{4\pi\varepsilon_0} \frac{L}{a(L + a)} = -\frac{1}{4\pi\varepsilon_0} \frac{q}{a(L + a)}, \]

upon substituting \( -q = \lambda L \). With \( q = 4.23 \times 10^{-15} \text{ C}, L = 0.0815 \text{ m} \) and \( a = 0.120 \text{ m} \), we obtain \( E_x = -1.57 \times 10^{-3} \text{ N/C} \), or \(|E_x| = 1.57 \times 10^{-3} \text{ N/C}|.\)

(c) The negative sign in \( E_x \) indicates that the field points in the \(-x\) direction, or \(-180^\circ\) counterclockwise from the \(+x\) axis.

(d) If \( a \) is much larger than \( L \), the quantity \( L + a \) in the denominator can be approximated by \( a \), and the expression for the electric field becomes

\[ E_x = -\frac{q}{4\pi\varepsilon_0 a^3}. \]

Since \( a = 50 \text{ m} \Rightarrow L = 0.0815 \text{ m} \), the above approximation applies, and we have \( E_x = -1.52 \times 10^{-8} \text{ N/C} \), or \(|E_x| = 1.52 \times 10^{-8} \text{ N/C}|.\)

(e) For a particle of charge \( -q = -4.23 \times 10^{-15} \text{ C}, \) the electric field at a distance \( a = 50 \text{ m} \) away has a magnitude \(|E_x| = 1.52 \times 10^{-8} \text{ N/C} |.\)
Chapter 22, Problem 37

37. We use Eq. 22-26, noting that the disk in figure (b) is effectively equivalent to the disk in figure (a) plus a concentric smaller disk (of radius $R/2$) with the opposite value of $\sigma$. That is,

$$E_{(b)} = E_{(a)} - \frac{\sigma}{2\varepsilon_0} \left( 1 - \frac{2R}{\sqrt{(2R)^2 + (R/2)^2}} \right)$$

where

$$E_{(a)} = \frac{\sigma}{2\varepsilon_0} \left( 1 - \frac{2R}{\sqrt{(2R)^2 + R^2}} \right).$$

We find the relative difference and simplify:

$$\frac{E_{(a)} - E_{(b)}}{E_{(a)}} = \frac{1 - 2/\sqrt{4 + 1/4}}{1 - 2/\sqrt{4 + 1}} = 0.0299 = 0.283$$

or approximately 28%.

Chapter 22, Problem 83

83. (a) From Eq. 22-38 (and the facts that $\hat{i} \cdot \hat{i} = 1$ and $\hat{j} \cdot \hat{i} = 0$), the potential energy is

$$U = -\vec{p} \cdot \vec{E} = -\left[ (3.00\hat{i} + 4.00\hat{j}) (1.24 \times 10^{-26} \text{ C} \cdot \text{m}) \right] \cdot \left[ (4000 \text{ N/C}) \hat{i} \right]$$

$$= -1.49 \times 10^{-26} \text{ J}.$$

(b) From Eq. 22-34 (and the facts that $\hat{i} \times \hat{i} = 0$ and $\hat{j} \times \hat{i} = -\hat{k}$), the torque is

$$\vec{\tau} = \vec{p} \times \vec{E} = \left[ (3.00\hat{i} + 4.00\hat{j}) (1.24 \times 10^{-26} \text{ C} \cdot \text{m}) \right] \times \left[ (4000 \text{ N/C}) \hat{i} \right]$$

$$= (-1.98 \times 10^{-26} \text{ N} \cdot \text{m}) \hat{k}.$$

(c) The work done is

$$W = \Delta U = \Delta (-\vec{p} \cdot \vec{E}) = (\vec{p}_f - \vec{p}_i) \cdot \vec{E}$$

$$= \left[ (3.00\hat{i} + 4.00\hat{j}) - (-4.00\hat{i} + 3.00\hat{j}) \right] (1.24 \times 10^{-26} \text{ C} \cdot \text{m}) \left[ (4000 \text{ N/C}) \hat{i} \right]$$

$$= 3.47 \times 10^{-26} \text{ J}.$$