Today:
- Start Rotation (10.1-10.4)
- Quiz 4 at the end of class 15 min
- problem 3 on HW 7 correction: You need to know that right before the collision the 5.6 kg box has a velocity of 11.25 m/s to the right.

At the beginning of this semester we discussed kinematics of bodies moving in a straight line. First we talked about measuring distance. Then we defined velocity and acceleration. Then we developed the equations of motion to describe motion with constant acceleration. We used these equations to describe motion of falling objects, how high bodies go when thrown up, and more.

Now it is time to do the same thing for rotational motion.

**Rotational Motion**
Spinning or rolling of rigid bodies. A rigid body is a body that does not change shape as it spins. We will start with spinning bodies first and then eventually include rolling.

**Circular Objects** (wheels)
If you know how to do wheels you can do any rigid body

We can follow two points on a spinning wheel. Follow two points, \( A \) and \( B \). As the wheel spins they rotate around on a circle. They don’t go around the same circle. For one revolution, once around the circle, the distance traveled by \( A \) is

\[ A : 2 \pi r_A \]

and the distance \( B \) travels is

\[ B : 2 \pi r_B \]

(Circumference of the circle)

\[ B \] travels farther than \( A \). Since it takes the same amount of time this means that \( B \) is going faster than \( A \)! If we chose a point \( C \) in between \( A \) and \( B \) than \( C \) is going faster than \( A \) but slower than \( B \). How many different speeds does a spinning object have? It has infinitely many linear speeds.

Why are the spokes in the center of the wheel more in focus? Because they are going slower near the center.
This is in complete contrast to the motion we discussed before. If I take the wheel and walk though the room, it has only one speed. In this case I can treat the wheel as a dot and I can assign one speed to it. However, the moment I am spinning the wheel, I have many infinitely many speeds, so I cannot describe a spinning wheel as a single dot. This is why we need two whole chapters to deal with rotational motion.

How can we discuss rotation?

**Angular Motion**

As a rigid body rotates, all the points will rotate through the same angle. So angle is important here. Rotational motion is sometimes also referred to as angular motion. So we will define an angular position, θ

While A and B travel different distances, they are always at the same angle.

All points rotate through the same angle Δθ. There are many different distances traveled by the points on the wheel but they are all rotating through the same angle.

We have to be careful here. The type of motion that we discussed before, we call linear motion. Distance traveled in meters which also includes the distance traveled on an arc.

When we use the angle as a measure for distance, we talk about angular motion.

We must distinguish linear motion = distance/time
from angular motion = angle/time

A rotating object has infinitely many linear speeds but only one angular speed.
Angle
In this chapter, we’ll find it necessary to use radians instead of degrees to measure angles. This is something all of you should know. The distance of part of a circle along its edge is called arc length $s$. Circumference is a special example of arc length for the entire circle. When we use radian then the relationship between arclength and the angle is very simple

$$s = r \theta \quad (\theta \text{ is in radians})$$

Conversion between radians and degrees:

$$2\pi \text{ rad} = 360^\circ = 1 \text{ rev} \quad (\text{revolution is also a unit of angle})$$

$$1 \text{ revolution} : \theta = \frac{s}{r} = \frac{2\pi r}{r} = 2\pi$$

Units: $\theta = \frac{s}{r} \Rightarrow \frac{m}{m} = 1 \leftarrow \textbf{No Unit!}$ Angles have no units, they are unitless.

“rad" is a way to specify an angular quantity. It is a unit we made up so that we can distinguish angle from just a number, like the $\frac{1}{2}$ when we calculate kinetic energy. In calculations we typically drop the units.

Now we have a measure for rotational distance and we can develop the idea corresponding to linear velocity, which corresponds to angular velocity for rotating rigid bodies.

Angular Velocity
If a point on the wheel starts at angle $\theta_1$ and it rotates to an angle $\theta_2$, the speed is – change in angle over change in time:

$$\text{Average angular velocity: } \omega_{\text{avg}} = \frac{\theta_2 - \theta_1}{t_2 - t_1} = \frac{\Delta \theta}{\Delta t} \left[ \begin{array}{c} \frac{\text{rad}}{s} \end{array} \right] \text{ or } \left[ \begin{array}{c} \frac{\text{rev}}{\text{min}} \end{array} \right] = \text{RPM}$$

The rate at which a particle circles is given by its angular velocity, $\omega$
Taking a derivative

(Instantaneous) angular velocity: \( \omega = \lim_{\Delta t \to 0} \frac{\Delta \theta}{\Delta t} = \frac{d\theta}{dt} \)

It tells me how is the angle changing in time.

**Example 1:**

A good baseball pitcher can throw a baseball toward home plate at 38 m/s with a spin of 1800 rev/min. How many revolutions does the baseball make on its way to home plate? For simplicity, assume that the 20 m path is a straight line.

\[
\omega = 1800 \text{ rev/min} = \frac{\Delta \theta}{\Delta t}
\]

Revolutions = \( \Delta \theta = \omega \Delta t \)

need to find \( \Delta t \) by using information from linear motion given:

\[
\begin{align*}
\nu &= 38.75 \text{ m/s} \\
\Delta x &= 20 \text{ m} \\
\Delta t &= \frac{\Delta x}{\nu} = \frac{20}{38} = 0.526 \text{ s} (\frac{1}{60} \text{ min}) = 0.0088 \text{ min} \\
\text{Revolutions} &= \Delta \theta = \omega \Delta t = (1800 \text{ rev/min})(0.0088 \text{ min}) = 15.8
\end{align*}
\]

The baseball makes approx 16 revolutions.
Thus far, I have talked about angular speed. Before we can talk about the vector quantity, angular velocity, we need to discuss the idea of rotational axis.

**Rotational Axis**

All rotation occurs about an axis. There is more than one way to spin a wheel. I can spin a wheel in its vertical position or in its horizontal position. What is the difference between these two rotations? The axis of rotation is different.

Axis of Rotation - The imaginary line passing through the point (or points) of zero linear velocity that is perpendicular to the motion. The center has zero linear velocity, when we spin the wheel about its center.

What vector tells you the direction of motion? The velocity.

Perpendicular to motion $\Rightarrow 90^\circ$ to all $\vec{v}$

What is the only direction which is perpendicular, at $90^\circ$, to all these velocity vectors? Into our out of the page - the rotational axis is along the z-axis (located at the center), i.e., into and out of the page.

Any rotation has an axis. The object does not have to be a wheel or circular object. For example the two black dots below perform a circular motion. So the axis is defined by the points on the object going around in a circle.
The Right-Hand-Rule
The angular velocity points along the axis of rotation. The axis goes in two directions. So how do I decide which in which direction the angular velocity vector points? This vector can only point into one direction. We use a right-hand-rule (RHR) to quickly determine which direction. This is a memory device.

RHR - Curl the fingers of your right hand in the “sense” of the rotation. Your extended thumb, points in direction of $\omega$

Sense = clockwise or counterclockwise

Angular Acceleration
Uniform angular motion is very unrealistic. We notice that a spinning wheel will eventually stop spinning. So when you when you have a change in angular velocity $\omega$, you have an angular acceleration, $\alpha$.

Either a change in magnitude or direction of $\omega$ involves an $\alpha$. The direction of $\omega$ can be changed by tilting the axis of rotation. Just like before, a change in magnitude or direction of velocity causes acceleration.

This term, we’ll deal only with changes in magnitude, change in angular speed.

Average angular acceleration: $\alpha_{av} = \frac{\omega_2 - \omega_1}{t_2 - t_1} = \frac{\Delta \omega}{\Delta t}$, [rad $\frac{rad}{s^2}$],

the rate at which angular speed is changing.

Taking a derivative
(Instantaneous) angular acceleration: $\alpha = \lim_{\Delta t \to 0} \frac{\Delta \omega}{\Delta t} = \frac{d \omega}{dt}$

Direction:
If $\omega$ (speed) is increasing: $\alpha$ in same direction as $\omega$
If $\omega$ (speed) is decreasing: $\alpha$ in opposite direction to $\omega$
Example 2:
The angular position of a point on a rotating wheel is given by \( \theta(t) = 2.0 + 4.0 \, t^2 + 2.0 \, t^3 \), where \( \theta \) is in radians and \( t \) is in seconds. At \( t = 0 \), what are
(a) the point’s angular position and
(b) its angular velocity?
(c) What is its angular velocity at \( t = 4.0 \) s?
(d) Calculate its angular acceleration at \( t = 2.0 \) s.
(e) Is its angular acceleration constant?

\[
\begin{align*}
\text{(a)} & \quad \theta(t) = 2 + 4t^2 + 2t^3 \\
& \quad \theta(0) = 2 \text{ rad} \\
\text{(b)} & \quad \omega = \frac{d\theta}{dt} = 0 + 8(4t) + 6(2)t^2 \\
& \quad \omega(0) = 0 \\
\text{(c)} & \quad \omega(t) = 8t + 6t^2 \\
& \quad \omega(4) = 8(4) + 6(4^2) = 128 \text{ rad/s} \\
\text{(d)} & \quad \omega(t) = 8t + 6t^2 \\
& \quad \alpha(t) = \frac{d\omega}{dt} = 8 + 12t \\
& \quad \alpha(2) = 8 + 12(2) = 32 \text{ rad/s}^2 \\
\text{(e)} & \quad \text{Angular acceleration in (d) depends on time. Therefore \( \alpha \) is not constant.}
\end{align*}
\]