Today:
- Projectile Motion (4.5/4.6)
- Quiz 1

**Projectile Motion:**
One type of problem: projectile motion – most common example of 2D motion with constant acceleration

**DEMO: Shoot drop**
Drop two identical objects simultaneously from the same height.
Drop one straight down while giving the other object an initial horizontal speed.

Observation: Both objects reach the ground at the same time

Inference:
1) horizontal motion does not affect vertical motion (different vector components do not affect each other)
2) after dropping what causes all subsequent motion is gravity. Gravity pulls straight down, so it causes acceleration in the y-direction only. Hence all objects fall with the same speed and acceleration. (We are ignoring air resistance here.)

These two facts mean that we can predict the trajectories of all kinds of projectiles.

**Projectile:**
Any object (moving through air) that is launched into motion and then acted on by gravity only.

\[ g = \text{is the magnitude of the acceleration caused by gravity (I am not supposed to be sloppy since I said previously that } g \text{ stands for gravity) } \]
**Equations of motion in 2D:**
Now we just need to modify our 1D equations of linear motion for constant acceleration from chapter 2 (lecture 3):
\[
\begin{align*}
\mathbf{v} &= \mathbf{v}_0 + \mathbf{a}t \\
x &= x_0 + v_0 t + \frac{1}{2} at^2 \\
\mathbf{v}^2 &= \mathbf{v}_0^2 + 2\mathbf{a} \cdot (x-x_0) \\
x &= x_0 + vt - \frac{1}{2} at^2 \\
x &= x_0 + \frac{1}{2} (v_0 + v) t
\end{align*}
\]

These equations can be applied in 2D and 3D as well. They are applied to each component separately (in 2D we will end up with twice as many equations, and in 3D we will end up with three times as many equation). Let's look at this for our projectile motion problem

A vector that points straight down has only a y component (no x component) acceleration vector due to gravity:
\[
a_x = 0; \\
a_y = - g = - 9.8 \text{ m/s}^2 \text{ (down is negative) (we have taken away any freedom of choice}
\]

1) **Projectile motion: x-component**
no acceleration in x, means uniform motion in x. When an object is in uniform motion the velocity is a constant:
\[
\mathbf{v}_x = \mathbf{v}_{0x}
\]

When we launched the metal ball it was going to the right. If I tell you it was going 5 m/s to the right at the launch how fast was it going to the right at the midpoint, 5 m/s (ignore air resistance). How fast was it going the instant it hits the ground to the right? 5 m/s

Derive an equation for distance:
\[
x = x_0 + v_{0x} t
\]
graph looks like a straight line - slope of this line equals the x component of velocity.

2) Projectile motion: y-component

\[ a_y = -g \]

\[ v_y = v_{0y} - gt \]  
(negative sign for gravity already in the equation – put in by hand)

\[ y = y_0 + v_{0y} t - \frac{1}{2}gt^2 \]  
(negative sign for gravity already in the equation – put in by hand)

In general it looks like this:

In both of these equations: \( g = +9.8 \text{ m/s}^2 \) (for earth, do you know the value on Venus? \( 8.8 \text{ m/s}^2 \))

**Range of flight:**
The range of flight is the horizontal distance traveled.

**Demo Example:**
Estimate the range of our metal ball which was given an initial speed of 1 m/s and was released at 1.5 m.

**x-component:**
\[ x_0 = 0 \text{ m} \]
\[ v_{0x} = 1 \text{ m/s} \]

**y-component**
\[ y_0 = 0 \text{ m} \]
\[ y = -1.5 \text{ m} \]
\[ v_{0y} = 0 \text{ m/s} \]

need: \( x \)
formula: \( x = x_0 + v_{0x}t \)

**don’t know** \( t \): compute from y-components
\[ y = y_0 + v_0 t - \frac{1}{2} gt^2 \]

\[ t = \sqrt{\frac{2(y-y_0)}{a}} = \sqrt{\frac{2(-1.5)}{-9.8}} = 0.553 \text{s} \]

now solve for x:

\[ x = x_0 + v_0 x t = 0 + (1)(0.553) = 0.533 \text{ m} = 5.53 \times 10^{-1} \text{ m} \]

The horizontal range seems to be consistent with what we observed in class.

**Example 2:**

At what launch speed would our metallic ball in the demo orbit the earth?

The answer emerges from a basic fact about the curvature of the earth. For every 8000 meters measured along the horizon of the earth, the earth's surface curves downward by approximately 5 meters.

It so happens that the vertical distance that a horizontally launched projectile would fall in its first second is approximately

\[ y = -\frac{1}{2} gt^2 = -\frac{1}{2}(9.8)(1)^2 = -4.9 \text{ m} \]

For this reason, a projectile launched horizontally with a speed of about 8000 m/s will be capable of orbiting the earth in a circular path. This assumes that it is launched above the surface of the earth and encounters negligible atmospheric drag. As the projectile travels tangentially a distance of 8000 meters in 1 second, it will drop approximately 5 meters towards the earth. Yet, the projectile will remain the same distance above the earth due to the fact that the earth curves at the same rate that the projectile falls. If shot with a speed greater than 8000 m/s, it would orbit the earth in an elliptical path.

An orbiting satellite is a projectile in the sense that the only force acting upon an orbiting satellite is the force of gravity. Most Earth-orbiting satellites are orbiting at a distance high above the Earth such that their motion is unaffected by forces of air resistance.

Why is motion of satellites around earth is considered an accelerated motion? We will talk about this at the end of this week (lecture 8).
Example 3:
A cart is rolling at constant velocity of 0.50 m/s on a flat track. It fires a ball straight up into the air at 1.0 m/s as it moves. After it is fired, will the ball fall right back into the cart?

Yes: since the vector component in the x direction (horizontal) is the same for both the cart and the fired ball.

Show Demo!

Example 4:
Now the cart is being pulled along a horizontal track by an external force (a weight attached to the cart which is hanging over the table edge) and accelerating at 9.8 m/s². It fires a ball straight up into the air at 1.0 m/s as it moves. After it is fired, will the ball fall right back into the cart?

In this problem, the acceleration of the cart is completely unrelated to the ball. In fact, the ball does not have any horizontal acceleration. Hence the ball will lag behind the accelerating cart once it is shot out of the cannon.

Oblique launch and launch Angle:

When an object is launched at an angle, the first step is to resolve the velocity vector into x and y vector components.

\[
\begin{align*}
\cos \theta &= \frac{v_{0x}}{v_0} \implies v_{0x} = v_0 \cos \theta \\
\sin \theta &= \frac{v_{0y}}{v_0} \implies v_{0y} = v_0 \sin \theta
\end{align*}
\]

\(v_{0x}\) and \(v_{0y}\) are the components of the initial velocity vector \(\vec{v}_0\). Usually, we are given the launch speed, \(v_0\) and angle, \(\theta\).
## Projectile Equations

<table>
<thead>
<tr>
<th>$a_x = 0$</th>
<th>$a_y = -g$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$v_x = v_{0x}$</td>
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Example 5:
A cannon Ball is launched from ground level at a speed of 25.0 m/s and at an angle of 40.0º above the horizontal.
a) What is the maximum height of the cannon ball?
b) What is the range of the cannon ball?

\[ \theta = 40.0^\circ \]
\[ v_0 = 25.0 \text{ m/s} \]
\[ y_0 = 0 \]
\[ x_0 = 0 \]

**a) Maximum height of the cannon ball:**

\[ v_y = 0 \text{ m/s} \]

\[ v_y^2 = v_{0y}^2 + 2a(y - y_0) \]
\[ \frac{v_y^2 - v_{0y}^2}{2a} = y - y_0 \]
\[ y = \frac{v_y^2 - v_{0y}^2}{2a} \]

\[ v_{0y} = v_0 \sin \theta = 25 \text{ m/s} \]
\[ v_{0y}^2 = 10.0 \text{ m/s} \]
\[ y = \frac{0 - (13.2)^2}{2(-1.8)} = 13.2 \text{ m} \]

**b) Range of the cannon ball:**

\[ x = x_0 + v_{0x} t \]
\[ y = y_0 + v_{0y} t - \frac{1}{2} gt^2 \]
\[ y = y_0 + v_{0y} t - \frac{1}{2} gt^2 \]
\[ 0 = v_{0y} t - \frac{1}{2} gt^2 \]
\[ 0 = v_{0y} - \frac{1}{2} gt \]
\[ t = \frac{2v_{0y}}{g} = \frac{2(16.0.\text{m/s})}{9.8 \text{ m/s}^2} = 3.28 \text{ s} \]

\[ x = x_0 + v_{0x} t = 0 + (19.2 \text{ m/s})(3.28 \text{ s}) = 62.8 \text{ m} \]

\[ v_{0x} = v_0 \cos \theta = 25 \cos 40^\circ = 19.2 \text{ m/s} \]
Example 6: Maximum range for launch on level ground.
Given a launch speed $v_0$, at what angle with the maximum range be achieved?

\[ y = y_0 = 0 = x_0 \]

Use:
\[ x = x_0 + v_{ox} t \]  \hspace{1cm} (1)

\[ v_{ox} = \cos(\theta_{max}) v_0 \]  \hspace{1cm} (2)

Need to find $t$:
\[ y = y_0 + v_{oy} t - \frac{1}{2} (9.8) t^2 \]

\[ 0 = 0 + \sin(\theta_{max}) v_0 t - 4.9 t^2 \]

\[ -\sin(\theta_{max}) v_0 = -4.9 t \]  \hspace{1cm} (3)

\[ t = \frac{+\sin(\theta_{max}) v_0}{4.9} \]

\[ x = x_0 + v_{ox} t = 0 + \left( \cos(\theta_{max}) v_0 \right) \left( \frac{\sin(\theta_{max}) v_0}{4.9} \right) \]

Using $2 \sin \theta \cos \theta = \sin(2\theta)$
\[ x = \frac{v_0^2 \cos \theta_{max} \sin \theta_{max}}{4.9} \]

$x$ will be max when $\theta_{max} = 45^\circ$ since $\sin(90) = 1$

You can also use calculus:
Set $\frac{dx}{d\theta} = 0$ and solve for $\theta$