

Math 482/582 Exam I, Name:
Show all your work and use back sheet if necessary.

1. (20 pts) A random sample size 4: X_1, X_2, X_3, X_4 is taken from the pdf

$$f(x|\theta) = \frac{\theta}{x^{\theta+1}}, \text{ for } \theta > 0 \text{ and } x > 1.$$

- (a) Find its likelihood function.
(b) Find the mle for θ .
(c) What is the mle if $X_1 = 12$, $X_2 = 11$, $X_3 = 10$, and $X_4 = 12.5$.

2. (20 pts) Given a random sample size 4: X_1, X_2, X_3, X_4 from a Poisson distribution with mean λ .
- (a) Show that both $\bar{\lambda}_1 = \bar{X}$ and $\bar{\lambda}_2 = X_1/5 + X_2/5 + 2X_3/5 + X_4/5$ are unbiased for the mean λ .
- (b) Show that $\bar{\lambda}_1 = \bar{X}$ is more efficient than $\bar{\lambda}_2$.

3. (20 pts) Use both the method of moments and the mle to estimate θ for the uniform density

$$f(x) = 1/\theta, 0 \leq x \leq \theta.$$

Compute your estimators (moment and mle) if $X_i = 17, 46, 92, 56$ for $i = 1, 2, 3, 4$.

4. (20 pts) Suppose that $H_0 : \mu = 95$ and $H_1 : \mu < 95$. Level $\alpha = 0.01$, $n = 25$, and $\sigma = 15$ are given.

(a) If \bar{X} is 92, do you want to reject H_0 ?

(b) If you assume that $\mu = 90$, find the power.

Math 482/582 Exam I (Take Home part).

If you take this course as 585, you should do it. If you take it as 485, you can do it as bonus (5 pts).

5. (20 pts) Let X_1, \dots, X_n be sample from the following density

$$f(x; \theta) = (\theta + 1)x^\theta \text{ if } 0 \leq x \leq 1 \text{ and } f(x; \theta) = 0 \text{ if otherwise.}$$

- (a) Find the mle of θ .
- (b) Find the moment estimator of θ .
- (c) Find the variance of the mle.
- (d) Find Cramer and Rao's lower bound for $f(x; \theta)$.