Error analysis

If $r = (x_2 - x_1)^3 + x_1 x_2$

$x_1 = 2 \pm .2$

$x_2 = 4 \pm .4$

how do you find error in r ?

Two methods.

1) Use rules from last lecture
   Apply sequentially from inside out

$x_2 - x_1 = 4 - 2 = 2$

$\delta(x_2 - x_1) = \delta x_2 + \delta x_1 = .2 + .4 = .6$ *if errors might be correlated*

$(x_2 - x_1)^3 = 2^3 = 8$

$\frac{\delta[(x_2 - x_1)^3]}{(x_2 - x_1)^3} = 3\frac{\delta(x_2 - x_1)}{x_2 - x_1} = \frac{\delta[(x_2 - x_1)^3]}{8} = 3\frac{.6}{2}$

$\delta[(x_2 - x_1)^3] = 7.2$

other term:

$x_1 x_2 = 8$

$\frac{\delta(x_1 x_2)}{x_1 x_2} = \frac{\delta x_1}{x_1} + \frac{\delta x_2}{x_2} = \frac{.2}{2} + \frac{.4}{4}$

$\delta(x_1 x_2) = 1.6$

$r = (x_2 - x_1)^3 - x_1 x_2 = 8 - 8 = 0$

$\delta r = \delta[(x_2 - x_1)^3] + \delta(x_1 x_2) = 7.2 + 1.6 = 8.8$

$r = 0 \pm 8.8$

2) If errors are uncorrelated
(expect smaller answer)

eqn 1.30 $f(x_1, x_2 \cdots x_n)$

$$s_f^2 = \sum_{i=1}^{n} \left(\frac{\partial f}{\partial x_i}\right)^2 s_{x_i}^2$$

$$s_r^2 = \left(\frac{\partial f}{\partial x_1}\right)^2 s_{x_1}^2 + \left(\frac{\partial f}{\partial x_2}\right)^2 s_{x_2}^2$$

$$\frac{\partial f}{\partial x_1} = -3(x_2 - x_1)^2 - x_2 = -3(4-2)^2 - 4 = -16$$

$$\frac{\partial f}{\partial x_2} = 3(x_2 - x_1)^2 - x_1 = 3(4-2)^2 - 2 = 10$$

$$s_r^2 = (-16)^2(.2)^2 + (10)^2(.4)^2$$
$$= 26.24$$

$$s_r = 5.12$$