Chapter 34
Wave-Particle Duality and Quantum Physics

Conceptual Problems

1  •  [SSM]  The quantized character of electromagnetic radiation is observed by (a) the Young double-slit experiment, (b) diffraction of light by a small aperture, (c) the photoelectric effect, (d) the J. J. Thomson cathode-ray experiment.

Determine the Concept The Young double-slit experiment and the diffraction of light by a small aperture demonstrated the wave nature of electromagnetic radiation. J. J. Thomson’s experiment showed that the rays of a cathode-ray tube were deflected by electric and magnetic fields and therefore must consist of electrically charged particles. Only the photoelectric effect requires an explanation based on the quantization of electromagnetic radiation. (c) is correct.

2  ••  Two monochromatic light sources, A and B, emit the same number of photons per second. The wavelength of A is \( \lambda_A = 400 \, \text{nm} \), and the wavelength of B is \( \lambda_B = 600 \, \text{nm} \). The power radiated by source B (a) is equal to the power of source A, (b) is less than the power of source A, (c) is greater than the power of source A, (d) cannot be compared to the power from source A using the available data.

Determine the Concept Since the power radiated by a source is the energy radiated per unit area and per unit time, it is directly proportional to the energy. The energy radiated varies inversely with the wavelength \( (E = \frac{hc}{\lambda}) \); i.e., the longer the wavelength, the less energy is associated with the electromagnetic radiation. (b) is correct.

3  •  [SSM]  The work function of a surface is \( \phi \). The threshold wavelength for emission of photoelectrons from the surface is equal to (a) \( \frac{hc}{\phi} \), (b) \( \frac{\phi}{hf} \), (c) \( \frac{hf}{\phi} \), (d) none of above.

Determine the Concept The work function is equal to the minimum energy required to remove an electron from the material. A photon that has that energy also has the threshold wavelength required for photoemission. Thus, \( hf = \phi \). In addition, \( c = f\lambda \). It follows that \( \frac{hc}{\lambda_t} = \phi \), so \( \lambda_t = \frac{hc}{\phi} \) and (a) is correct.
When light of wavelength $\lambda_1$ is incident on a certain photoelectric cathode, no electrons are emitted, no matter how intense the incident light is. Yet, when light of wavelength $\lambda_2 < \lambda_1$ is incident, electrons are emitted, even when the incident light has low intensity. Explain this observation.

**Determine the Concept** We can use Einstein’s photoelectric equation to explain this observation.

Einstein’s photoelectric equation is:

$$K_{\text{max}} = hf - \phi$$

where $\phi$, called the work function, is a characteristic of the particular metal.

Because $f = c/\lambda$:

$$K_{\text{max}} = \frac{hc}{\lambda} - \phi$$

We’re given that when light of wavelength $\lambda_1$ is incident on a certain photoelectric cathode; no electrons are emitted independently of the intensity of the incident light. Hence, we can conclude that:

$$\frac{hc}{\lambda_1} - \phi < 0 \Rightarrow \frac{hc}{\lambda_1} < \phi$$

When light of wavelength $\lambda_2 < \lambda_1$ is incident, electrons are emitted, electrons are emitted independently of the intensity of the incident light. Hence, we can conclude that:

$$\frac{hc}{\lambda_2} - \phi > 0 \Rightarrow \frac{hc}{\lambda_2} > \phi$$

Thus, photons with wavelengths greater than the threshold wavelength $\lambda_t = c/f_t$, where $f_t$ is the threshold frequency, do not have enough energy to eject an electron from the metal.

5 • True or false:

(a) The wavelength of an electron’s matter wave varies inversely with the momentum of the electron.

(b) Electrons can undergo diffraction.

(c) Neutrons can undergo diffraction.

(a) True. The de Broglie wavelength of an electron is given by $\lambda = h/p$.

(b) True

(c) True
6 • If the wavelength of an electron is equal to the wavelength of a proton, then (a) the speed of the proton is greater than the speed of the electron, (b) the speeds of the proton and the electron are equal, (c) the speed of the proton is less than the speed of the electron, (d) the energy of the proton is greater than the energy of the electron, (e) Both (a) and (d) are correct.

**Determine the Concept** If the de Broglie wavelengths of an electron and a proton are equal, their momenta must be equal. Because $m_p > m_e$, $v_p < v_e$. (c) is correct.

7 • A proton and an electron have equal kinetic energies. It follows that the wavelength of the proton is (a) greater than the wavelength of the electron, (b) equal to the wavelength of the electron, (c) less than the wavelength of the electron.

**Determine the Concept** The kinetic energy of a particle can be expressed, in terms of its momentum, as $K = p^2 / 2m$. We can use the equality of the kinetic energies and the fact that $m_e < m_p$ to determine the relative sizes of their de Broglie wavelengths.

Express the equality of the kinetic energies of the proton and electron in terms of their momenta and masses:

$$\frac{p_p^2}{2m_p} = \frac{p_e^2}{2m_e}$$

Use the de Broglie relation for the wavelength of matter waves to obtain:

$$\frac{h^2}{2m_p\lambda_p^2} = \frac{h^2}{2m_e\lambda_e^2} \Rightarrow m_p\lambda_p^2 = m_e\lambda_e^2$$

Because $m_e < m_p$:

$$\lambda_p^2 < \lambda_e^2$$ and $$\lambda_e > \lambda_p$$

and (c) is correct.

8 • The parameter $x$ is the position of a particle, $\langle x \rangle$ is the expectation value of $x$, and $P(x)$ is the probability density function. Can the expectation value of $x$ ever equal a value of $x$ for which $P(x)$ is zero? If yes, give a specific example. If no, explain why not.

**Determine the Concept** Yes. Consider a particle in a one-dimensional box of length $L$ that is on the $x$ axis on the interval $0 < x < L$. The wave function for a particle in the $n = 2$ state (the lowest state above the ground state) is given by $\psi_2(x) = \sqrt{\frac{2}{L}} \sin \left(\frac{2\pi x}{L}\right)$ (see Figure 31-13). The expectation value of $x$ is $L/2$ and $P(L/2) = 0$ (see Figure 31-14).
It was once believed that if two identical experiments are done on identical systems under the same conditions, the results must be identical. Explain how this statement can be modified so that it is consistent with quantum physics.

Determine the Concept According to quantum theory, the average value of many measurements of the same quantity will yield the expectation value of that quantity. However, any single measurement may differ from the expectation value.

A six-sided die has the numeral 1 painted on three sides and the numeral 2 painted on the other three sides. (a) What is the probability of a 1 coming up when the die is thrown? (b) What is the expectation value of the numeral that comes up when the die is thrown? (c) What is the expectation value of the cube of the numeral that comes up when the die is thrown?

Determine the Concept The probability of a particular event occurring is the number of ways that event can occur divided by the number of possible outcomes. The expectation value, on the other hand, is the average value of the experimental results.

(a) Find the probability of a 1 coming up when the die is thrown: 
\[ P(1) = \frac{3}{6} = \frac{1}{2} \]

(b) Find the average value of a large number of throws of the die: 
\[ \langle n \rangle = \frac{3 \times 1 + 3 \times 2}{6} = 1.5 \]

(c) The expectation value of the cube of the numeral that comes up when the die is thrown is: 
\[ \langle n^3 \rangle = \frac{3 \times 1^3 + 3 \times 2^3}{6} = 4.5 \]

Estimation and Approximation

During an advanced physics lab, students use X rays to measure the Compton wavelength, \( \lambda_C \). The students obtain the following wavelength shifts \( \lambda_2 - \lambda_1 \) as a function of scattering angle \( \theta \):

<table>
<thead>
<tr>
<th>( \theta )</th>
<th>45°</th>
<th>75°</th>
<th>90°</th>
<th>135°</th>
<th>180°</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \lambda_2 - \lambda_1 )</td>
<td>0.647 pm</td>
<td>1.67 pm</td>
<td>2.45 pm</td>
<td>3.98 pm</td>
<td>4.95 pm</td>
</tr>
</tbody>
</table>

Use their data to estimate the value for the Compton wavelength. Compare this number with the accepted value.
**Picture the Problem** From the Compton-scattering equation we have
\[ \lambda_2 - \lambda_1 = \lambda_c (1 - \cos \theta) \]
where \[ \lambda_c = \frac{h}{m_e c} \]
is the Compton wavelength. Note that this equation is of the form \[ y = mx + b \]
provided we let \[ y = \lambda_2 - \lambda_1 \]
and \[ x = 1 - \cos \theta \]. Thus, we can linearize the Compton equation by plotting \[ \Delta \lambda = \lambda_2 - \lambda_1 \]
as a function of \[ 1 - \cos \theta \]. The slope of the resulting graph will yield an experimental value for the Compton wavelength.

(a) The spreadsheet solution is shown below. The formulas used to calculate the quantities in the columns are as follows:

<table>
<thead>
<tr>
<th>Cell</th>
<th>Formula/Content</th>
<th>Algebraic Form</th>
</tr>
</thead>
<tbody>
<tr>
<td>A3</td>
<td>45</td>
<td>[ \theta \text{ (deg)} ]</td>
</tr>
<tr>
<td>B3</td>
<td>1 - cos(A3*PI()/180)</td>
<td>[ 1 - \cos \theta ]</td>
</tr>
<tr>
<td>C3</td>
<td>6.47E^-13</td>
<td>[ \Delta \lambda = \lambda_2 - \lambda_1 ]</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( \theta ) (deg)</th>
<th>( 1 - \cos \theta )</th>
<th>( \lambda_2 - \lambda_1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>45</td>
<td>0.293</td>
<td>6.47E-13</td>
</tr>
<tr>
<td>75</td>
<td>0.741</td>
<td>1.67E-12</td>
</tr>
<tr>
<td>90</td>
<td>1.000</td>
<td>2.45E-12</td>
</tr>
<tr>
<td>135</td>
<td>1.707</td>
<td>3.98E-12</td>
</tr>
<tr>
<td>180</td>
<td>2.000</td>
<td>4.95E-12</td>
</tr>
</tbody>
</table>

The following graph was plotted from the data shown in the above table. Excel’s "Add Trendline" was used to fit a linear function to the data and to determine the regression constants.
From the trend line we note that the experimental value for the Compton wavelength \( \lambda_{C,\text{exp}} \) is:

\[
\lambda_{C,\text{exp}} = 2.48 \, \text{pm}
\]

The Compton wavelength is given by:

\[
\lambda_C = \frac{h}{m_e c} = \frac{hc}{m_e c^2}
\]

Substitute numerical values and evaluate \( \lambda_C \):

\[
\lambda_C = \frac{1240 \, \text{eV} \cdot \text{nm}}{5.11 \times 10^5 \, \text{eV}} = 2.43 \, \text{pm}
\]

Express the percent difference between \( \lambda_C \) and \( \lambda_{C,\text{exp}} \):

\[
\% \text{diff} = \frac{\lambda_{C,\text{exp}} - \lambda_{\exp}}{\lambda_{\exp}} = \frac{\lambda_{C,\text{exp}}}{\lambda_{\exp}} - 1
\]

\[
= \frac{2.48 \, \text{pm}}{2.43 \, \text{pm}} - 1 \approx 2\%
\]

Students in a physics lab are trying to determine the value of Planck’s constant \( h \), using a photoelectric apparatus similar to the one shown in Figure 34-2. The students are using a helium–neon laser that has a tunable wavelength as the light source. The data that the students obtain for the maximum electron kinetic energies are

<table>
<thead>
<tr>
<th>( \lambda )</th>
<th>K(_{\text{max}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>544 nm</td>
<td>0.360 eV</td>
</tr>
<tr>
<td>594 nm</td>
<td>0.199 eV</td>
</tr>
<tr>
<td>604 nm</td>
<td>0.156 eV</td>
</tr>
<tr>
<td>612 nm</td>
<td>0.117 eV</td>
</tr>
<tr>
<td>633 nm</td>
<td>0.062 eV</td>
</tr>
</tbody>
</table>

(a) Using a spreadsheet program or graphing calculator, plot \( K_{\text{max}} \) versus light frequency. (b) Use the graph to estimate the value of Planck’s constant. (Note: You may wish to use a feature of your spreadsheet program or graphing calculator to obtain the best straight-line fit to the data.) (c) Compare your result with the accepted value for Planck’s constant.

**Picture the Problem** From Einstein’s photoelectric equation we have

\[ K_{\text{max}} = hf - \phi, \]

which is of the form \( y = mx + b \), where the slope is \( h \) and the \( K_{\text{max}} \)-intercept is the work function. Hence we should plot a graph of \( K_{\text{max}} \) versus \( f \) in order to obtain a straight line whose slope will be an experimental value for Planck’s constant.

(a) The spreadsheet solution is shown below. The formulas used to calculate the quantities in the columns are as follows:

<table>
<thead>
<tr>
<th>Cell</th>
<th>Formula/Content</th>
<th>Algebraic Form</th>
</tr>
</thead>
<tbody>
<tr>
<td>A3</td>
<td>544</td>
<td>( \lambda ) (nm)</td>
</tr>
<tr>
<td>B3</td>
<td>0.36</td>
<td>( K_{\text{max}} ) (eV)</td>
</tr>
</tbody>
</table>
The following graph was plotted from the data shown in the above table. Excel’s "Add Trendline" was used to fit a linear function to the data and to determine the regression constants.

\[ K_{\text{max}} = af + b \]

\[ a = 6.19 \times 10^{-34} \text{ J} \cdot \text{s} \]

\[ b = -2.83 \times 10^{-19} \text{ J} \]

(b) From the trend line we note that the experimental value for Planck’s constant is:

\[ h_{\text{exp}} = 6.19 \times 10^{-34} \text{ J} \cdot \text{s} \]

(c) Express the percent difference between \( h_{\text{exp}} \) and \( h \):

\[ \% \text{diff} = \frac{h - h_{\text{exp}}}{h} = 1 - \frac{h_{\text{exp}}}{h} \]

\[ = 1 - \frac{6.19 \times 10^{-34} \text{ J} \cdot \text{s}}{6.63 \times 10^{-34} \text{ J} \cdot \text{s}} \approx 6.6\% \]
The cathode that was used by the students in the experiment described in Problem 12 is constructed from one of the following metals:

<table>
<thead>
<tr>
<th>Metals</th>
<th>Tungsten</th>
<th>Silver</th>
<th>Potassium</th>
<th>Cesium</th>
</tr>
</thead>
<tbody>
<tr>
<td>Work function</td>
<td>4.58 eV</td>
<td>2.4 eV</td>
<td>2.1 eV</td>
<td>1.9 eV</td>
</tr>
</tbody>
</table>

Determine which metal composes the cathode by using the same data given in Problem 12. 

(a) Using a spreadsheet program or graphing calculator, plot $K_{\text{max}}$ versus frequency. 

(b) Use the graph to estimate the value of the work function based on the students’ data. (Note: You may wish to use a feature of your spreadsheet program or graphing calculator to obtain the best straight-line fit to the data.) 

(c) Which metal was most likely used for the cathode in their experiment?

**Picture the Problem** From Einstein’s photoelectric equation we have $K_{\text{max}} = hf - \phi$, which is of the form $y = mx + b$, where the slope is $h$ and the $K_{\text{max}}$-intercept is the work function. Hence we should plot a graph of $K_{\text{max}}$ versus $f$ in order to obtain a straight line whose intercept will be an experimental value for the work function.

(a) The spreadsheet solution is shown below. The formulas used to calculate the quantities in the columns are as follows:

<table>
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<tbody>
<tr>
<td>A3</td>
<td>544</td>
<td>$\lambda$ (nm)</td>
</tr>
<tr>
<td>B3</td>
<td>0.36</td>
<td>$K_{\text{max}}$ (eV)</td>
</tr>
<tr>
<td>C3</td>
<td>A3*10^-19</td>
<td>$\lambda$ (m)</td>
</tr>
<tr>
<td>D3</td>
<td>3*10^8/C3</td>
<td>$c/\lambda$</td>
</tr>
<tr>
<td>E3</td>
<td>B3<em>1.6</em>10^-19</td>
<td>$K_{\text{max}}$ (J)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$\lambda$ (nm)</th>
<th>$K_{\text{max}}$ (eV)</th>
<th>$\lambda$ (m)</th>
<th>$f = c/\lambda$ (Hz)</th>
<th>$K_{\text{max}}$ (J)</th>
</tr>
</thead>
<tbody>
<tr>
<td>544</td>
<td>0.36</td>
<td>5.44E-07</td>
<td>5.51E+14</td>
<td>5.76E-20</td>
</tr>
<tr>
<td>594</td>
<td>0.199</td>
<td>5.94E-07</td>
<td>5.05E+14</td>
<td>3.18E-20</td>
</tr>
<tr>
<td>604</td>
<td>0.156</td>
<td>6.04E-07</td>
<td>4.97E+14</td>
<td>2.50E-20</td>
</tr>
<tr>
<td>612</td>
<td>0.117</td>
<td>6.12E-07</td>
<td>4.90E+14</td>
<td>1.87E-20</td>
</tr>
<tr>
<td>633</td>
<td>0.062</td>
<td>6.33E-07</td>
<td>4.74E+14</td>
<td>9.92E-21</td>
</tr>
</tbody>
</table>
The following graph was plotted from the data shown in the above table. Excel’s "Add Trendline" was used to fit a linear function to the data and to determine the regression constants.

\[
K_{\text{max}} = af + b \\
a = 6.19 \times 10^{-34} \text{ J} \cdot \text{s} \\
b = -2.83 \times 10^{-19} \text{ J}
\]

\( \phi \) from the trend line we note that the experimental value for the work function \( \phi \) is:

\[
\phi_{\text{exp}} = 2.83 \times 10^{-19} \text{ J} \times \frac{1 \text{ eV}}{1.602 \times 10^{-19} \text{ J}} = 1.77 \text{ eV}
\]

The value of \( \phi_{\text{exp}} = 1.77 \text{ eV} \) is closest to the work function for cesium.

**The Particle Nature of Light: Photons**

14 • Find the photon energy in electron volts for light of wavelength (a) 450 nm, (b) 550 nm, and (c) 650 nm.

**Picture the Problem** We can use \( E = \frac{hc}{\lambda} \) to find the photon energy when we are given the wavelength of the radiation.

\( (a) \) Express the photon energy as a function of wavelength and evaluate \( E \) for \( \lambda = 450 \text{ nm} \):

\[
E = \frac{hc}{\lambda} = \frac{1240 \text{ eV} \cdot \text{nm}}{450 \text{ nm}} = 2.76 \text{ eV}
\]

\( (b) \) For \( \lambda = 550 \text{ nm} \):

\[
E = \frac{1240 \text{ eV} \cdot \text{nm}}{550 \text{ nm}} = 2.25 \text{ eV}
\]
(c) For $\lambda = 650$ nm:

\[
E = \frac{1240 \text{ eV} \cdot \text{nm}}{650 \text{ nm}} = 1.91 \text{ eV}
\]

15. Find the photon energy in electron volts for an electromagnetic wave of frequency (a) 100 MHz in the FM radio band and (b) 900 kHz in the AM radio band.

**Picture the Problem** We can find the photon energy for an electromagnetic wave of a given frequency $f$ from $E = hf$ where $h$ is Planck’s constant.

(a) For $f = 100$ MHz:

\[
E = hf = \left(6.626 \times 10^{-34} \text{ J} \cdot \text{s}\right) \left(100 \times 10^6 \text{ s}^{-1}\right)
\]

\[
= 6.626 \times 10^{-26} \text{ J} \times \frac{1 \text{ eV}}{1.602 \times 10^{-19} \text{ J}}
\]

\[
= 4.14 \times 10^{-7} \text{ eV}
\]

(b) For $f = 900$ kHz:

\[
E = hf = \left(6.626 \times 10^{-34} \text{ J} \cdot \text{s}\right) \left(900 \times 10^3 \text{ s}^{-1}\right)
\]

\[
= 5.963 \times 10^{-28} \text{ J} \times \frac{1 \text{ eV}}{1.602 \times 10^{-19} \text{ J}}
\]

\[
= 3.72 \times 10^{-9} \text{ eV}
\]

16. What are the frequencies of photons that have the following energies (a) 1.00 eV, (b) 1.00 keV, and (c) 1.00 MeV?

**Picture the Problem** The energy of a photon, in terms of its frequency, is given by $E = hf$.

(a) Express the frequency of a photon in terms of its energy and evaluate $f$ for $E = 1.00$ eV:

\[
f = \frac{E}{h} = \frac{1.00 \text{ eV}}{4.136 \times 10^{-15} \text{ eV} \cdot \text{s}}
\]

\[
= 2.42 \times 10^{14} \text{ Hz}
\]

(b) For $E = 1.00$ keV:

\[
f = \frac{1.00 \text{ keV}}{4.136 \times 10^{-15} \text{ eV} \cdot \text{s}}
\]

\[
= 2.42 \times 10^{17} \text{ Hz}
\]

(c) For $E = 1.00$ MeV:

\[
f = \frac{1.00 \text{ MeV}}{4.136 \times 10^{-15} \text{ eV} \cdot \text{s}}
\]

\[
= 2.42 \times 10^{20} \text{ Hz}
\]
Find the photon energy in electron volts if the wavelength is
(a) 0.100 nm (about 1 atomic diameter) and (b) 1.00 fm (1 fm = \(10^{-15}\) m, about 1
nuclear diameter).

**Picture the Problem** We can use \(E = \frac{hc}{\lambda}\) to find the photon energy when we are
given the wavelength of the radiation.

The energy of a photon as a function
of its wavelength is given by:
\[
E = \frac{hc}{\lambda}
\]

(a) Substitute numerical values and
evaluate \(E\) for \(\lambda = 0.100\) nm:
\[
E = \frac{1240\text{ eV} \cdot \text{nm}}{0.100\text{ nm}} = 12.4\text{ keV}
\]

(b) Substitute numerical values and
evaluate \(E\) for \(\lambda = 1.00\) fm = \(1.00 \times 10^{-6}\) nm:
\[
E = \frac{1240\text{ eV} \cdot \text{nm}}{1.00 \times 10^{-6}\text{ nm}} = 1.24\text{ GeV}
\]

The wavelength of red light emitted by a 3.00-mW helium–neon laser
is 633 nm. If the diameter of the laser beam is 1.00 mm, what is the density of
photons in the beam? Assume that the intensity is uniformly distributed across
the beam.

**Picture the Problem** We can express the density of photons in the beam as the
number of photons per unit volume. The number of photons per unit volume is, in
turn, the ratio of the power of the laser to the energy of the photons and the
volume occupied by the photons emitted in one second is the product of the cross-
sectional area of the beam and the speed at which the photons travel, i.e., the
speed of light.

Express the density of photons in the
beam as a function of the number of
photons emitted per second and the
volume occupied by those photons:
\[
\rho = \frac{N}{V}
\]

Relate the number of photons
emitted per second to the power of
the laser and the energy of the
photons:
\[
N = \frac{P}{E} = \frac{P\lambda}{hc}
\]

Express the volume containing the
photons emitted in one second as a
function of the cross sectional area of
the beam:
\[
V = Ac
\]
Substitute for $N$ and $V$ and simplify to obtain:

$$\rho = \frac{P\lambda}{hc^2 A} = \frac{P\lambda}{hc^2 \left(\frac{\pi}{4} d^2\right)} = \frac{4P\lambda}{\pi hc^2 d^2}$$

where $d$ is the diameter of the laser beam.

Substitute numerical values and evaluate $\rho$:

$$\rho = \frac{4(3.00 \text{ mW})(633 \text{ nm})}{\pi (6.626 \times 10^{-34} \text{ J} \cdot \text{s})(2.998 \times 10^8 \text{ m/s})^2 (1.00 \text{ mm})^2} = 4.06 \times 10^{13} \text{ m}^{-3}$$

19 • [SSM] Lasers used in a telecommunications network typically produce light that has a wavelength near 1.55 $\mu$m. How many photons per second are being transmitted if such a laser has an output power of 2.50 mW?

**Picture the Problem** The number of photons per second is the ratio of the power of the laser to the energy of the photons.

Relate the number of photons emitted per second to the power of the laser and the energy of the photons:

$$N = \frac{P}{E} = \frac{P}{hc\frac{\lambda}{\lambda}}$$

Substitute numerical values and evaluate $N$:

$$N = \frac{(2.50 \text{ mW})(1.55 \mu\text{m})}{(6.626 \times 10^{-34} \text{ J} \cdot \text{s})(2.998 \times 10^8 \text{ m/s})} = 1.95 \times 10^{16} \text{ s}^{-1}$$

**The Photoelectric Effect**

20 • The work function for tungsten is 4.58 eV. (a) Find the threshold frequency and wavelength for the photoelectric effect to occur when monochromatic electromagnetic radiation is incident on the surface of a sample of tungsten. Find the maximum kinetic energy of the electrons if the wavelength of the incident light is (b) 200 nm and (c) 250 nm.

**Picture the Problem** The threshold wavelength and frequency for emission of photoelectrons is related to the work function of a metal through $\phi = hf_\text{th} = \frac{hc}{\lambda_\text{th}}$.

We can use Einstein’s photoelectric equation $K_{\text{max}} = \frac{hc}{\lambda} - \phi$ to find the maximum kinetic energy of the electrons for the given wavelengths of the incident light.
(a) Express the threshold frequency in terms of the work function for tungsten and evaluate $f_t$:

$$f_t = \frac{\phi}{h} = \frac{4.58 \text{eV}}{4.136 \times 10^{-15} \text{eV} \cdot \text{s}} = 1.107 \times 10^{15} \text{Hz} = 1.11 \times 10^{15} \text{Hz}$$

Using $c = f\lambda$, express the threshold wavelength in terms of the threshold frequency and evaluate $\lambda_t$:

$$\lambda_t = \frac{c}{f_t} = \frac{2.998 \times 10^8 \text{m/s}}{1.107 \times 10^{15} \text{Hz}} = 271 \text{nm}$$

(b) Using Einstein’s photoelectric equation, relate the maximum kinetic energy of the electrons to their wavelengths and evaluate $K_{\text{max}}$:

$$K_{\text{max}} = E - \phi = hf - \phi = \frac{hc}{\lambda} - \phi$$

$$= \frac{1240 \text{eV} \cdot \text{nm}}{200 \text{nm}} - 4.58 \text{eV}$$

$$= 1.62 \text{eV}$$

(c) Evaluate $K_{\text{max}}$ for $\lambda = 250 \text{ nm}$:

$$K_{\text{max}} = \frac{1240 \text{eV} \cdot \text{nm}}{250 \text{nm}} - 4.58 \text{eV}$$

$$= 0.380 \text{eV}$$

21. When monochromatic ultraviolet light that has a wavelength equal to 300 nm is incident on a sample of potassium, the emitted electrons have maximum kinetic energy of 2.03 eV. (a) What is the energy of an incident photon? (b) What is the work function for potassium? (c) What would be the maximum kinetic energy of the electrons if the incident electromagnetic radiation had a wavelength of 430 nm? (d) What is the maximum wavelength of incident electromagnetic radiation that will result in the photoelectric emission of electrons by a sample of potassium?

**Picture the Problem** (a) We can use the Einstein equation for photon energy to find the energy of an incident photon. (b) and (c) We can use the photoelectric equation to relate the work function for potassium to the maximum energy of the photoelectrons. (d) The maximum wavelength of incident electromagnetic radiation that will result in the photoelectric emission of electrons can be found from $\lambda_i = \frac{hc}{\phi}$.

(a) Use the Einstein equation for photon energy to relate the energy of the incident photon to its wavelength:

$$E = \frac{hc}{\lambda} = \frac{1240 \text{eV} \cdot \text{nm}}{300 \text{nm}} = 4.13 \text{eV}$$
(b) Using Einstein’s photoelectric equation, relate the work function for potassium to the maximum kinetic energy of the photoelectrons:

\[ K_{\text{max}} = E - \phi \]

Solve for and evaluate \( \phi \):

\[ \phi = E - K_{\text{max}} = 4.13 \text{ eV} - 2.03 \text{ eV} = 2.10 \text{ eV} \]

(c) Proceeding as in (b) with \( E = \frac{hc}{\lambda} \) gives:

\[ K_{\text{max}} = \frac{hc}{\lambda} - \phi \]

\[ = \frac{1240 \text{ eV} \cdot \text{nm}}{430 \text{ nm}} - 2.10 \text{ eV} \]

\[ = 0.78 \text{ eV} \]

(d) Express the maximum wavelength as a function of potassium’s work function and evaluate \( \lambda_i \):

\[ \lambda_i = \frac{hc}{\phi} = \frac{1240 \text{ eV} \cdot \text{nm}}{2.10 \text{ eV}} = 590 \text{ nm} \]

22 • The maximum wavelength of electromagnetic radiation that will result in the photoelectric emission of electrons from a sample of silver is 262 nm.

(a) Find the work function for silver.

(b) Find the maximum kinetic energy of the electrons if the incident radiation has a wavelength of 175 nm.

**Picture the Problem**

(a) We can find the work function for silver using \( \phi = \frac{hc}{\lambda_i} \), and (b) the maximum kinetic energy of the electrons using Einstein’s photoelectric equation.

(a) Express the work function for silver as a function of the maximum wavelength of the electromagnetic radiation that will result in photoelectric emission of electrons:

\[ \phi = \frac{hc}{\lambda_i} \]

Substitute numerical values and evaluate \( \phi \):

\[ \phi = \frac{1240 \text{ eV} \cdot \text{nm}}{262 \text{ nm}} = 4.73 \text{ eV} \]
(b) Using Einstein’s photoelectric equation, relate the work function for silver to the maximum kinetic energy of the photoelectrons:

\[ K_{\text{max}} = E - \phi = \frac{hc}{\lambda} - \phi \]

Substitute numerical values and evaluate \( K_{\text{max}} \):

\[
K_{\text{max}} = \frac{1240 \text{ eV} \cdot \text{nm}}{175 \text{ nm}} - 4.73 \text{ eV} = 2.36 \text{ eV}
\]

23. The work function for cesium is 1.90 eV. (a) Find the minimum frequency and maximum wavelength of electromagnetic radiation that will result in the photoelectric emission of electrons from a sample of cesium. Find the maximum kinetic energy of the electrons if the wavelength of the incident radiation is (b) 250 nm and (c) 350 nm.

**Picture the Problem** We can find the minimum frequency and maximum wavelength for cesium using \( \phi = hf_i = hc/\lambda_i \) and the maximum kinetic energy of the electrons using Einstein’s photoelectric equation.

(a) Use the Einstein equation for photon energy to express the maximum wavelength for cesium:

\[ \lambda_i = \frac{hc}{\phi} \]

Substitute numerical values and evaluate \( \lambda_i \):

\[
\lambda_i = \frac{1240 \text{ eV} \cdot \text{nm}}{1.90 \text{ eV}} = 653 \text{ nm}
\]

Use \( c = f\lambda \) to find the maximum frequency:

\[
f_i = \frac{c}{\lambda_i} = \frac{2.998 \times 10^8 \text{ m/s}}{653 \text{ nm}}
= 4.59 \times 10^{14} \text{ Hz}
\]

(b) Using Einstein’s photoelectric equation, relate the maximum kinetic energy of the photoelectrons to the wavelength of the incident light:

\[ K_{\text{max}} = \frac{hc}{\lambda} - \phi \]

Evaluating \( K_{\text{max}} \) for \( \lambda = 250 \text{ nm} \) gives:

\[
K_{\text{max}} = \frac{1240 \text{ eV} \cdot \text{nm}}{250 \text{ nm}} - 1.90 \text{ eV}
= 3.06 \text{ eV}
\]
(c) Proceed as in Part (b) with $\lambda = 350$ nm to obtain:

$$K_{\text{max}} = \frac{1240 \text{ eV} \cdot \text{nm}}{350 \text{ nm}} - 1.90 \text{ eV}$$

$$= 1.64 \text{ eV}$$

24 ** When a surface is illuminated with electromagnetic radiation of wavelength 780 nm, the maximum kinetic energy of the emitted electrons is 0.37 eV. What is the maximum kinetic energy if the surface is illuminated using radiation of wavelength 410 nm?

**Picture the Problem** We can use Einstein’s photoelectric equation to find the work function of this surface and then apply it a second time to find the maximum kinetic energy of the photoelectrons when the surface is illuminated with light of wavelength 410 nm.

Use Einstein’s photoelectric equation to relate the maximum kinetic energy of the emitted electrons to their total energy and the work function of the surface:

$$K_{\text{max}} = \frac{hc}{\lambda} - \phi$$  \hspace{1cm} (1)

The work function of the surface is given by:

$$\phi = E - K_{\text{max}} = \frac{hc}{\lambda} - K_{\text{max}}$$

Substitute numerical values and evaluate $\phi$:

$$\phi = \frac{1240 \text{ eV} \cdot \text{nm}}{780 \text{ nm}} - 0.37 \text{ eV} = 1.22 \text{ eV}$$

Substitute for $\phi$ and $\lambda$ in equation (1) and evaluate $K_{\text{max}}$:

$$K_{\text{max}} = \frac{1240 \text{ eV} \cdot \text{nm}}{410 \text{ nm}} - 1.22 \text{ eV}$$

$$= 1.80 \text{ eV}$$

**Compton Scattering**

25 ** Find the shift in wavelength of photons scattered by free stationary electrons at $\theta = 60^\circ$. (Assume that the electrons are initially moving with negligible speed and are virtually free of (unattached to) any atoms or molecules.)

**Picture the Problem** We can calculate the shift in wavelength using the Compton relationship $\Delta \lambda = \frac{h}{m_e c} (1 - \cos \theta)$. 
The shift in wavelength is given by: 
\[ \Delta \lambda = \frac{h}{m_e c} (1 - \cos \theta) \]

Substitute numerical values and evaluate \( \Delta \lambda \):

\[
\Delta \lambda = \frac{6.626 \times 10^{-34} \text{ J} \cdot \text{s}}{(9.109 \times 10^{-31} \text{ kg})(2.998 \times 10^8 \text{ m/s})}(1 - \cos 60^\circ) = 1.2 \text{ pm}
\]

26  
- When photons are scattered by electrons in a carbon sample, the shift in wavelength is 0.33 pm. Find the scattering angle. (Assume that the electrons are initially moving with negligible speed and are virtually free of (unattached to) any atoms or molecules.)

**Picture the Problem** We can calculate the scattering angle using the Compton relationship \( \Delta \lambda = \frac{h}{m_e c} (1 - \cos \theta) \).

Using the Compton scattering equation, relate the shift in wavelength to the scattering angle:

\[
\Delta \lambda = \frac{h}{m_e c} (1 - \cos \theta)
\]

Solving for \( \theta \) yields:

\[
\theta = \cos^{-1} \left(1 - \frac{m_e c}{h} \frac{\Delta \lambda}{\lambda}\right)
\]

Substitute numerical values and evaluate \( \theta \):

\[
\theta = \cos^{-1} \left(1 - \frac{(9.109 \times 10^{-31} \text{ kg})(2.998 \times 10^8 \text{ m/s})}{6.626 \times 10^{-34} \text{ J} \cdot \text{s}} \frac{0.33 \text{ pm}}{\lambda}\right) = 30^\circ
\]

27  
- The photons in a monochromatic beam are scattered by electrons. The wavelength of the photons that are scattered at an angle of 135° with the direction of the incident photon beam is 2.3 percent less than the wavelength of the incident photons. What is the wavelength of the incident photons?

**Picture the Problem** We can calculate the shift in wavelength using the Compton relationship \( \Delta \lambda = \frac{h}{m_e c} (1 - \cos \theta) \).

Express the wavelength of the incident photons in terms of the fractional change in wavelength:

\[
\frac{\Delta \lambda}{\lambda} = 0.023 \Rightarrow \lambda = \frac{\Delta \lambda}{0.023}
\]
Using the Compton scattering equation, relate the shift in wavelength to the scattering angle:

\[ \Delta \lambda = \frac{h}{m_e c} (1 - \cos \theta) \]

Substituting for \( \Delta \lambda \) yields:

\[ \lambda = \frac{h}{0.023m_e c} (1 - \cos \theta) \]

Substitute numerical values and evaluate \( \lambda \):

\[ \lambda = \frac{6.626 \times 10^{-34} \text{ J} \cdot \text{s}}{(0.023)(9.109 \times 10^{-31} \text{ kg})(2.998 \times 10^8 \text{ m/s})} \left(1 - \cos 135^\circ\right) = 0.18 \text{ nm} \]

28. Compton used photons of wavelength 0.0711 nm. (a) What is the energy of one of these photons? (b) What is the wavelength of the photons scattered in the direction opposite to the direction of the incident photons? (c) What is the energy of the photon scattered in this direction?

**Picture the Problem** We can use the Einstein equation for photon energy to find the energy of both the incident and scattered photon and the Compton scattering equation to find the wavelength of the scattered photon.

(a) Use the Einstein equation for photon energy to obtain:

\[ E = \frac{hc}{\lambda_1} = \frac{1240 \text{ eV} \cdot \text{nm}}{0.0711 \text{ nm}} = 17.4 \text{ keV} \]

(b) Express the wavelength of the scattered photon in terms of its pre-scattering wavelength and the shift in its wavelength during scattering:

\[ \lambda_2 = \lambda_1 + \Delta \lambda = \lambda_1 + \frac{h}{m_e c} (1 - \cos \theta) \]

Substitute numerical values and evaluate \( \lambda_2 \):

\[ \lambda_2 = 0.0711 \text{ nm} + \frac{6.626 \times 10^{-34} \text{ J} \cdot \text{s}}{(9.109 \times 10^{-31} \text{ kg})(2.998 \times 10^8 \text{ m/s})} (1 - \cos 180^\circ) = 0.0760 \text{ nm} \]

(c) Use the Einstein equation for photon energy to obtain:

\[ E = \frac{hc}{\lambda_2} = \frac{1240 \text{ eV} \cdot \text{nm}}{0.0760 \text{ nm}} = 16.3 \text{ keV} \]

29. For the photons used by Compton (see Problem 28), find the momentum of the incident photon and the momentum of the photon scattered in the direction opposite to the direction of the incident photons. Use the
conservation of momentum to find the momentum of the recoil electron in this case.

**Picture the Problem** Compton used X rays of wavelength 71.1 pm. Let the direction the incident photon (and the recoiling electron) is moving be the positive direction. We can use \( p = \frac{h}{\lambda} \) to find the momentum of the incident photon and the conservation of momentum to find its momentum after colliding with the electron.

Use the expression for the momentum of a photon to find the momentum of Compton’s photons:

\[
p_1 = \frac{h}{\lambda_1} = \frac{6.626 \times 10^{-34} \text{ J} \cdot \text{s}}{71.1 \text{ pm}} = 9.32 \times 10^{-24} \text{ kg} \cdot \text{m/s}
\]

Using the Compton scattering equation, relate the shift in wavelength to the scattering angle:

\[
\lambda_2 = \lambda_1 + \lambda_c (1 - \cos \theta)
\]

Substitute numerical values and evaluate \( \lambda_2 \):

\[
\lambda_2 = 71.1 \text{ pm} + \left(2.43 \times 10^{-12} \text{ m}\right)(1 - \cos 180°) = 76.0 \text{ pm}
\]

Apply conservation of momentum \( p_1 = p_e - p_2 \Rightarrow p_e = p_1 - p_2 \) to obtain:

Substitute for \( p_1 \) and \( p_2 \) and evaluate \( p_e \):

\[
p_e = 9.32 \times 10^{-24} \text{ kg} \cdot \text{m/s} - \left(\frac{6.626 \times 10^{-34} \text{ J} \cdot \text{s}}{76.0 \text{ pm}}\right) = 1.80 \times 10^{-23} \text{ kg} \cdot \text{m/s}
\]

30 A beam of photons have a wavelength equal to 6.00 pm is scattered by electrons initially at rest. A photon in the beam is scattered in a direction perpendicular to the direction of the incident beam. (a) What is the change in wavelength of the photon? (b) What is the kinetic energy of the electron?

**Picture the Problem** We can calculate the shift in wavelength using the Compton relationship \( \Delta \lambda = \frac{h}{m_c} (1 - \cos \theta) = \lambda_c (1 - \cos \theta) \) and use conservation of energy to find the kinetic energy of the scattered electron.
(a) Use the Compton scattering equation to find the change in wavelength of the photon:

\[ \Delta \lambda = \lambda_c (1 - \cos \theta) \]

\[ = (2.43 \times 10^{-12} \text{ m})(1 - \cos 90^\circ) \]

\[ = 2.43 \text{ pm} \]

(b) Use conservation of energy to relate the change in the kinetic energy of the electron to the energies of the incident and scattered photon:

\[ \Delta E_e = \frac{hc}{\lambda_1} - \frac{hc}{\lambda_2} \]

Find the wavelength of the scattered photon:

\[ \lambda_2 = \lambda_1 + \Delta \lambda = 6.00 \text{ pm} + 2.43 \text{ pm} \]

\[ = 8.43 \text{ pm} \]

Substitute numerical values and evaluate the kinetic energy of the electron (equal to the change in its energy since it was stationary prior to the collision with the photon):

\[ \Delta E_e = 1240 \text{ eV} \cdot \text{nm} \left( \frac{1}{6.00 \text{ pm}} - \frac{1}{8.43 \text{ pm}} \right) \]

\[ = 60 \text{ keV} \]

Electrons and Matter Waves

31 • An electron is moving at $2.5 \times 10^5$ m/s. Find the electron’s de Broglie wavelength.

Picture the Problem We can use its definition to find the de Broglie wavelength of this electron.

Use its definition to express the de Broglie wavelength of the electron in terms of its momentum:

\[ \lambda = \frac{h}{p} = \frac{h}{m_e v} \]

Substitute numerical values and evaluate \( \lambda \):

\[ \lambda = \frac{6.626 \times 10^{-34} \text{ J} \cdot \text{s}}{9.109 \times 10^{-31} \text{ kg} \cdot (2.5 \times 10^5 \text{ m/s})} \]

\[ = 2.9 \text{ nm} \]

32 • An electron has a wavelength of 200 nm. Find (a) the magnitude of its momentum and (b) its kinetic energy.
**Picture the Problem** We can find the momentum of the electron from the de Broglie equation and its kinetic energy from \( \lambda = \frac{1.226}{\sqrt{K}} \) nm, where \( K \) is in eV.

(a) Use the de Broglie relation to express the momentum of the electron:

\[
p = \frac{h}{\lambda} = \frac{6.626 \times 10^{-34} \text{ J} \cdot \text{s}}{200 \text{ nm}}
\]

\[
= 3.31 \times 10^{-27} \text{ kg} \cdot \text{m/s}
\]

(b) Use the electron wavelength equation to relate the electron’s wavelength to its kinetic energy:

\[
\lambda = \frac{1.226}{\sqrt{K}} \text{ nm}
\]

Solving for \( K \) gives:

\[
K = \left( \frac{1.226 \text{ eV}^{1/2} \text{ nm}}{200 \text{ nm}} \right)^2
\]

\[
= 3.76 \times 10^{-5} \text{ eV}
\]

33 ** An electron, a proton, and an alpha particle each have a kinetic energy of 150 keV. Find (a) the magnitudes of their momenta and (b) their de Broglie wavelengths.

**Picture the Problem** The momenta of these particles can be found from their kinetic energies. The values for the electron, however, are only approximately correct because at the given energy the speed of the electron is a significant fraction of the speed of light. The solution presented is valid only in the non-relativistic limit \( \nu \ll c \). The de Broglie wavelengths of the particles are given by \( \lambda = h/p \).

(a) The momentum of a particle \( p \), in terms of its kinetic energy \( K \), is given by:

\[
p = \sqrt{2mK}
\]

Substitute numerical values and evaluate \( p_e \):

\[
p_e = \sqrt{2m_e K} = \sqrt{2(9.109 \times 10^{-31} \text{ kg})(150 \text{ keV} \times \frac{1.602 \times 10^{-19} \text{ C}}{\text{eV}})}
\]

\[
= 2.092 \times 10^{-22} \text{ N} \cdot \text{s} = 2.09 \times 10^{-22} \text{ N} \cdot \text{s}
\]
Substitute numerical values and evaluate \( p_p \):

\[
p_p = \sqrt{2m_p K} = \sqrt{2(1.673 \times 10^{-27} \text{ kg})(150 \text{ keV} \times \frac{1.602 \times 10^{-19} \text{ C}}{\text{eV}})}
\]

\[
= 8.967 \times 10^{-21} \text{ N} \cdot \text{s} = 8.97 \times 10^{-21} \text{ N} \cdot \text{s}
\]

Substitute numerical values and evaluate \( p_a \):

\[
p_a = \sqrt{2m_a K} = \sqrt{2\left(4\text{u} \times \frac{1.661 \times 10^{-27} \text{ kg}}{\text{u}}\right)(150 \text{ keV} \times \frac{1.602 \times 10^{-19} \text{ C}}{\text{eV}})}
\]

\[
= 1.787 \times 10^{-20} \text{ N} \cdot \text{s} = 1.79 \times 10^{-20} \text{ N} \cdot \text{s}
\]

(b) The de Broglie wavelengths of the particles are given by:

\[
\lambda = \frac{h}{p}
\]

Substitute numerical values in equation (1) and evaluate \( \lambda_e \):

\[
\lambda_e = \frac{h}{p_e} = \frac{6.626 \times 10^{-34} \text{ J} \cdot \text{s}}{2.092 \times 10^{-21} \text{ N} \cdot \text{s}}
\]

\[
= 3.17 \text{ pm}
\]

Substitute numerical values in equation (1) and evaluate \( \lambda_p \):

\[
\lambda_p = \frac{h}{p_p} = \frac{6.626 \times 10^{-34} \text{ J} \cdot \text{s}}{8.967 \times 10^{-21} \text{ N} \cdot \text{s}}
\]

\[
= 73.9 \text{ fm}
\]

Substitute numerical values in equation (1) and evaluate \( \lambda_a \):

\[
\lambda_a = \frac{h}{p_a} = \frac{6.63 \times 10^{-34} \text{ J} \cdot \text{s}}{1.79 \times 10^{-20} \text{ N} \cdot \text{s}}
\]

\[
= 37.0 \text{ fm}
\]

A neutron in a reactor has kinetic energy of approximately 0.020 eV. Calculate the wavelength of this neutron.

**Picture the Problem** The wavelength associated with a particle of mass \( m \) and kinetic energy \( K \) is given by \( \lambda = \frac{1240 \text{ eV} \cdot \text{nm}}{\sqrt{2mc^2K}} \).
Substituting numerical values yields:
\[ \lambda = \frac{1240 \text{eV} \cdot \text{nm}}{\sqrt{2(940 \text{MeV})(0.020 \text{eV})}} = 0.20 \text{nm} \]

35 • Find the wavelength of a proton that has a kinetic energy of 2.00 MeV.

**Picture the Problem** The wavelength associated with a particle of mass \( m \) and kinetic energy \( K \) is given by
\[ \lambda = \frac{1240 \text{eV} \cdot \text{nm}}{\sqrt{2mc^2K}}. \]

Substituting numerical values yields:
\[ \lambda = \frac{1240 \text{eV} \cdot \text{nm}}{\sqrt{2(938 \text{MeV})(2.00 \text{MeV})}} = 20.2 \text{fm} \]

36 • What is the kinetic energy of a proton whose wavelength is (a) 1.00 nm and (b) 1.00 fm?

**Picture the Problem** We can solve Equation 34-15 (\[ \lambda = \frac{1240 \text{eV} \cdot \text{nm}}{\sqrt{2mc^2K}} \]) for the kinetic energy of the proton and use the rest energy of a proton \( mc^2 = 938 \text{ MeV} \) to simplify our computation.

Solving Equation 34-15 for the kinetic energy of the proton gives:
\[ K = \frac{(1240 \text{eV} \cdot \text{nm})^2}{2mc^2\lambda^2} \]

(a) Substitute numerical values and evaluate \( K \) for \( \lambda = 1.00 \) nm:
\[ K = \frac{(1240 \text{eV} \cdot \text{nm})^2}{2(938 \text{MeV})(1.00 \text{nm})^2} = 0.820 \text{meV} \]

(b) Evaluate \( K \) for \( \lambda = 1.00 \) fm:
\[ K = \frac{(1240 \text{eV} \cdot \text{nm})^2}{2(938 \text{MeV})(1.00 \text{fm})^2} = 820 \text{MeV} \]

37 • The kinetic energy of the electrons in the electron beam in a run of Davisson and Germer’s experiment was 54 eV. Calculate the wavelength of the electrons in the beam.

**Picture the Problem** If \( K \) is in electron volts, the wavelength of a particle is given by
\[ \lambda = \frac{1.226}{\sqrt{K}} \text{ nm} \text{ provided } K \text{ is in eV.} \]
Evaluate $\lambda$ for $K = 54 \text{ eV}$:

$$\lambda = \frac{1.226}{\sqrt{K}} \text{ nm} = \frac{1.226}{\sqrt{54}} \text{ nm} = 0.17 \text{ nm}$$

38 • The distance between Li$^+$ and Cl$^-$ ions in a LiCl crystal is 0.257 nm. Find the energy of electrons that have a wavelength equal to this spacing.

**Picture the Problem** We can use $\lambda = \frac{1.226}{\sqrt{K}} \text{ nm}$, where $K$ is in eV, to find the energy of electrons whose wavelength is $\lambda$.

Relate the wavelength of the electrons to their kinetic energy:

$$\lambda = \frac{1.226}{\sqrt{K}} \text{ nm}$$

Solving for $K$ yields:

$$K = \left( \frac{1.226 \text{ nm} \cdot \text{eV}^{1/2}}{\lambda} \right)^2$$

Substituting numerical values and evaluating $K$ gives:

$$K = \left( \frac{1.226 \text{ nm} \cdot \text{eV}^{1/2}}{0.257 \text{ nm}} \right)^2 = 22.8 \text{ eV}$$

39 • [SSM] An electron microscope uses electrons that have energies equal to 70 keV. Find the wavelength of these electrons.

**Picture the Problem** We can approximate the wavelength of 70-keV electrons using $\lambda = \frac{1.226}{\sqrt{K}} \text{ nm}$, where $K$ is in eV. This solution is, however, only approximately correct because at the given energy the speed of the electron is a significant fraction of the speed of light. The solution presented is valid only in the non-relativistic limit $v \ll c$.

Relate the wavelength of the electrons to their kinetic energy:

$$\lambda = \frac{1.23}{\sqrt{K}} \text{ nm}$$

Substitute numerical values and evaluate $\lambda$:

$$\lambda = \frac{1.226}{\sqrt{70 \times 10^3 \text{ eV}}} \text{ nm} = 4.6 \text{ pm}$$

40 • What is the de Broglie wavelength of a neutron that has a speed of $1.00 \times 10^6 \text{ m/s}$?

**Picture the Problem** We can use its definition to calculate the de Broglie wavelength of a neutron with speed $10^6 \text{ m/s}$. 
Use its definition to express the de Broglie wavelength of the neutron: 

\[ \lambda_n = \frac{h}{p_n} = \frac{h}{m_n v_n} \]

Substitute numerical values and evaluate \( \lambda_n \):

\[ \lambda_n = \frac{6.626 \times 10^{-34} \text{ J} \cdot \text{s}}{\left(1.675 \times 10^{-27} \text{ kg}\right) \left(1.00 \times 10^6 \text{ m/s}\right)} = \boxed{0.396 \text{ pm}} \]

### A Particle in a Box

41 ** (a) Find the energy of the ground state \((n = 1)\) and the first two excited states of a neutron in a one-dimensional box of length \(L = 1.00 \times 10^{-15} \text{ m} = 1.00 \text{ fm}\) (about the diameter of an atomic nucleus). Make an energy-level diagram for this system. Calculate the wavelength of electromagnetic radiation emitted when the neutron makes a transition from (b) \(n = 2\) to \(n = 1\), (c) \(n = 3\) to \(n = 2\), and (d) \(n = 3\) to \(n = 1\).

**Picture the Problem** We can find the ground-state energy using 
\[ E_1 = \frac{h^2}{8m_n L^2} \] and the energies of the excited states using 
\[ E_n = n^2 E_1 \]. The wavelength of the electromagnetic radiation emitted when the neutron transitions from one state to another is given by the Einstein equation for photon energy 
\[ E = \frac{hc}{\lambda} \).

(a) Express the ground-state energy: 
\[ E_1 = \frac{h^2}{8m_n L^2} \]

Substitute numerical values and evaluate \( E_1 \):

\[ E_1 = \frac{\left(6.626 \times 10^{-34} \text{ J} \cdot \text{s}\right)^2}{8\left(1.6749 \times 10^{-27} \text{ kg}\right)\left(1.00 \times 10^{-15} \text{ m}\right)^2} \frac{1 \text{ eV}}{1.602 \times 10^{-19} \text{ J}} = 204.53 \text{ MeV} \]

\[ = \boxed{205 \text{ MeV}} \]

Find the energies of the first two excited states: 
\[ E_2 = 2^2 E_1 = 4(204.53 \text{ MeV}) = \boxed{818 \text{ MeV}} \]

and 
\[ E_3 = 3^2 E_1 = 9(204.53 \text{ MeV}) = \boxed{1.84 \text{ GeV}} \]
The energy-level diagram for this system is shown to the right:

\[ \begin{array}{c}
0 & 1 & 2 & 3 & 4 & 5 \\
\end{array} \]

(b) Relate the wavelength of the electromagnetic radiation emitted during a neutron transition to the energy released in the transition:

\[ \lambda = \frac{hc}{\Delta E} = \frac{1240 \text{ eV} \cdot \text{nm}}{\Delta E} \]

For the \( n = 2 \) to \( n = 1 \) transition:

\[ \Delta E = E_2 - E_1 = 4E_1 - E_1 = 3E_1 \]

Substitute numerical values and evaluate \( \lambda_{2 \rightarrow 1} \):

\[ \lambda_{2 \rightarrow 1} = \frac{1240 \text{ eV} \cdot \text{nm}}{3E_1} = \frac{1240 \text{ eV} \cdot \text{nm}}{3(204.53 \text{ MeV})} = 2.02 \text{ fm} \]

(c) For the \( n = 3 \) to \( n = 2 \) transition:

\[ \Delta E = E_3 - E_2 = 9E_1 - 4E_1 = 5E_1 \]

Substitute numerical values and evaluate \( \lambda_{3 \rightarrow 2} \):

\[ \lambda_{3 \rightarrow 2} = \frac{1240 \text{ eV} \cdot \text{nm}}{5E_1} = \frac{1240 \text{ eV} \cdot \text{nm}}{5(204.53 \text{ MeV})} = 1.21 \text{ fm} \]

(d) For the \( n = 3 \) to \( n = 1 \) transition:

\[ \Delta E = E_3 - E_1 = 9E_1 - E_1 = 8E_1 \]

Substitute numerical values and evaluate \( \lambda_{3 \rightarrow 1} \):

\[ \lambda_{3 \rightarrow 1} = \frac{1240 \text{ eV} \cdot \text{nm}}{8E_1} = \frac{1240 \text{ eV} \cdot \text{nm}}{8(204.53 \text{ MeV})} = 0.758 \text{ fm} \]

(a) Find the energy of the ground state \( n = 1 \) and the first two excited states of a neutron in a one-dimensional box of length 0.200 nm (about the diameter of a \( \text{H}_2 \) molecule). Calculate the wavelength of electromagnetic radiation emitted when the neutron makes a transition from (b) \( n = 2 \) to \( n = 1 \), (c) \( n = 3 \) to \( n = 2 \), and (d) \( n = 3 \) to \( n = 1 \).
**Picture the Problem** We can find the ground-state energy using \( E_1 = \frac{\hbar^2}{8mL^2} \) and the energies of the excited states using \( E_n = n^2 E_1 \). The wavelength of the electromagnetic radiation emitted when the neutron transitions from one state to another is given by the Einstein equation for photon energy \( E = \frac{hc}{\lambda} \).

\[(a)\] Express the ground-state energy:

\[
E_1 = \frac{\hbar^2}{8mL^2}.
\]

Substitute numerical values and evaluate \( E_1 \):

\[
\begin{align*}
E_1 &= \frac{(6.626 \times 10^{-34} \text{ J} \cdot \text{s})^2}{8\left(1.6749 \times 10^{-27} \text{ kg}\right)(0.200 \text{ nm})^2} \frac{1 \text{ eV}}{1.602 \times 10^{-19} \text{ J}} = 5.1133 \text{ meV} = \boxed{5.11 \text{ meV}}.
\end{align*}
\]

Find the energies of the first two excited states:

\[
E_2 = 2^2 E_1 = 4(5.1133 \text{ meV}) = \boxed{20.5 \text{ meV}}
\]

and

\[
E_3 = 3^2 E_1 = 9(5.1133 \text{ meV}) = \boxed{46.0 \text{ meV}}
\]

\[(b)\] Relate the wavelength of the electromagnetic radiation emitted during a neutron transition to the energy released in the transition:

\[
\lambda = \frac{hc}{\Delta E} = \frac{1240 \text{ eV} \cdot \text{nm}}{\Delta E}
\]

For the \( n = 2 \) to \( n = 1 \) transition:

\[
\Delta E = E_2 - E_1 = 4E_1 - E_1 = 3E_1
\]

Substitute numerical values and evaluate \( \lambda_{2\rightarrow1} \):

\[
\begin{align*}
\lambda_{2\rightarrow1} &= \frac{1240 \text{ eV} \cdot \text{nm}}{3E_1} = \frac{1240 \text{ eV} \cdot \text{nm}}{3(5.1133 \text{ meV})} = \boxed{80.8 \mu\text{m}}.
\end{align*}
\]

\[(c)\] For the \( n = 3 \) to \( n = 2 \) transition:

\[
\Delta E = E_3 - E_2 = 9E_1 - 4E_1 = 5E_1
\]

Substitute numerical values and evaluate \( \lambda_{3\rightarrow2} \):

\[
\begin{align*}
\lambda_{3\rightarrow2} &= \frac{1240 \text{ eV} \cdot \text{nm}}{5E_1} = \frac{1240 \text{ eV} \cdot \text{nm}}{5(5.1133 \text{ meV})} = \boxed{48.5 \mu\text{m}}.
\end{align*}
\]
\[(d)\] For the \(n = 3\) to \(n = 1\) transition: \[\Delta E = E_3 - E_1 = 9E_1 - E_1 = 8E_1\]

Substitute numerical values and evaluate \(\lambda_{3\to1}\):
\[
\lambda_{3\to1} = \frac{1240 \text{ eV} \cdot \text{nm}}{8E_1} = \frac{1240 \text{ eV} \cdot \text{nm}}{8(5.1133 \text{ meV})}
\]
\[= 30.3 \mu\text{m}\]

### Calculating Probabilities and Expectation Values

43 \(\star\star\) A particle is in the ground state of a one-dimensional box that has length \(L\). (The box has one end at the origin and the other end on the positive \(x\) axis.) Determine the probability of finding the particle in the interval of length \(\Delta x = 0.002L\) and centered at \((a)\) \(\frac{1}{4}L\), \((b)\) \(\frac{1}{2}L\), and \((c)\) \(\frac{3}{4}L\). (Because \(\Delta x\) is very small you need not do any integration.)

**Picture the Problem** The probability of finding the particle in some range \(\Delta x\) is \(\psi^2 dx\). The interval \(\Delta x = 0.002L\) is so small that we can neglect the variation in \(\psi(x)\) and just compute \(\psi^2 \Delta x\).

Express the probability of finding the particle in the interval \(\Delta x\):
\[P = P(x) \Delta x = \psi^2(x) \Delta x\]

Express the wave function for a particle in the ground state:
\[\psi_1(x) = \sqrt{\frac{2}{L}} \sin \frac{\pi x}{L}\]

Substitute for \(\psi_1(x)\) to obtain:
\[P = \frac{2}{L} \sin^2 \frac{\pi x}{L} \Delta x\]
\[= \frac{2}{L} \left( \sin^2 \frac{\pi x}{L} \right)(0.002L)\]
\[= 0.004 \sin^2 \left( \frac{\pi x}{L} \right)\]

\((a)\) Evaluate \(P\) at \(x = 4L\):
\[P(4L) = 0.004 \sin^2 \left( \frac{\pi (4L)}{2L} \right)\]
\[= 0.004 \sin^2 (2\pi) = 0\]

\((b)\) Evaluate \(P\) at \(x = L/2\):
\[P(L/2) = 0.004 \sin^2 \left( \frac{\pi L}{2L} \right)\]
\[= 0.004 \sin^2 \left( \frac{\pi}{2} \right) = 1\]
(c) Evaluate \( P \) at \( x = 3L/4 \):

\[
P(3L/4) = 0.004\sin^2\left(\frac{3\pi L}{4L}\right)
\]

\[
= 0.004\sin^2\left(\frac{3\pi}{4}\right) = 0.002
\]

44 ** A particle is in the second excited state \((n = 3)\) of a one-dimensional box that has length \(L\). (The box has one end at the origin and the other end on the positive \(x\) axis.) Determine the probability of finding the particle in the interval of length \(\Delta x = 0.002L\) and centered at \((a)\) \(x = \frac{1}{3}L\), \((b)\) \(x = \frac{1}{2}L\), and \((c)\) \(x = \frac{2}{3}L\). (Because \(\Delta x\) is very small you need not do any integration.)

**Picture the Problem** The probability of finding the particle in some range \(dx\) is \(\psi^2 dx\). The interval \(\Delta x = 0.002L\) is so small that we can neglect the variation in \(\psi(x)\) and just compute \(\psi^2 \Delta x\).

Express the probability of finding the particle in the interval \(\Delta x\):

\[
P = P(x)\Delta x = \psi^2(x)\Delta x
\]

Express the wave function for a particle in its second excited state:

\[
\psi_2(x) = \sqrt{\frac{2}{L}} \sin\left(\frac{2\pi x}{L}\right)
\]

Substitute for \(\psi_2(x)\) to obtain:

\[
P = \frac{2}{L} \sin^2\left(\frac{2\pi x}{L}\right) \Delta x
\]

\[
= \frac{2}{L} \left(\sin^2\left(\frac{2\pi x}{L}\right)\right)(0.002L)
\]

\[
= 0.004\sin^2\left(\frac{2\pi}{L}\right)
\]

(a) Evaluate \( P \) at \( x = L/3 \):

\[
P(L/3) = 0.004\sin^2\left(\frac{2\pi L}{3L}\right)
\]

\[
= 0.004\sin^2\left(\frac{2\pi}{3}\right) = 0.003
\]

(b) Evaluate \( P \) at \( x = L/2 \):

\[
P(L/2) = 0.004\sin^2\left(\frac{2\pi L}{2L}\right)
\]

\[
= 0.004\sin^2(\pi) = 0
\]
(c) Evaluate $P$ at $x = 2L/3$:

$$P(2L/3) = 0.004 \sin^2 \left( \frac{4\pi L}{3L} \right)$$

$$= 0.004 \sin^2 \left( \frac{4\pi}{3} \right) = 0.003$$

45 •• A particle is in the first excited ($n = 2$) state of a one-dimensional box that has length $L$. (The box has one end at the origin and the other end on the positive $x$ axis.) Find (a) $\langle x \rangle$ and (b) $\langle x^2 \rangle$.

**Picture the Problem** We’ll use $\langle f(x) \rangle = \int f(x) \psi^2(x) dx$ with $\psi_n(x) = \sqrt{\frac{2}{L}} \sin \frac{n\pi x}{L}$.

(a) Express $\psi(x)$ for the $n = 2$ state:

$$\psi_2(x) = \sqrt{\frac{2}{L}} \sin \frac{2\pi x}{L}$$

Express $\langle x \rangle$ using the $n = 2$ wave function:

$$\langle x \rangle = \int_0^L \frac{2x}{L} \sin^2 \frac{2\pi x}{L} dx$$

Change variables by letting $x = \frac{L}{2\pi} \theta$, $d\theta = \frac{2\pi}{L} dx$, $dx = \frac{L}{2\pi} d\theta$ and the limits on $\theta$ are 0 and $2\pi$.

Substitute to obtain:

$$\langle x \rangle = \frac{2}{L} \int_0^{2\pi} \left( \frac{L}{2\pi} \theta \right) \sin^2 \theta \left( \frac{L}{2\pi} d\theta \right)$$

$$= \frac{L}{2\pi^2} \int_0^{2\pi} \theta \sin^2 \theta d\theta$$

Use a table of integrals to evaluate the integral:

$$\langle x \rangle = \frac{L}{2\pi^2} \left[ \frac{\theta^2}{4} - \frac{\theta \sin 2\theta}{4} - \frac{\cos 2\theta}{8} \right]_0^{2\pi}$$

$$= \frac{L}{2\pi^2} \left[ \frac{\pi^2}{8} - \frac{1}{8} + \frac{1}{8} \right] = \frac{L}{2}$$

(b) Express $\langle x^2 \rangle$ using the $n = 2$ wave function:

$$\langle x^2 \rangle = \int_0^L \frac{2x^2}{L} \sin^2 \frac{2\pi x}{L} dx$$
Change variables as in (a) and substitute to obtain:

\[
\langle x^2 \rangle = \frac{2L}{L} \int_0^{\frac{2\pi}{L}} \left( \frac{L}{2\pi} \theta \right)^2 \sin^2 \left( \frac{L}{2\pi} d\theta \right)
\]

\[
= \frac{L^2}{4\pi} \int_0^{2\pi} \theta^2 \sin^2 \theta \, d\theta
\]

Use a table of integrals to evaluate the integral:

\[
\langle x^2 \rangle = \frac{L^2}{4\pi} \left[ \frac{\theta^3}{6} - \left( \frac{\theta^2}{4} - \frac{1}{8} \right) \sin 2\theta - \frac{\theta \cos 2\theta}{4} \right]_0^{2\pi} = \frac{L^2}{4\pi} \left[ \frac{4\pi^3}{3} - \frac{\pi}{2} \right]
\]

\[
= L^2 \left( \frac{1}{3} - \frac{1}{8\pi^2} \right) = 0.321L^2
\]

46 •• A particle in a one-dimensional box that has length \( L \) is in the first excited state \( n = 2 \). (The box has one end at the origin and the other end on the positive \( x \) axis.) (a) Sketch \( \psi^2(x) \) versus \( x \) for this state. (b) What is the expectation value \( \langle x \rangle \) for this state? (c) What is the probability of finding the particle in some small region \( dx \) centered at \( x = L/2 \)? (d) Are your answers for Part (b) and Part (c) contradictory? If not, explain why your answers are not contradictory.

**Picture the Problem** We’ll use \( \langle f(x) \rangle = \int f(x)\psi^2(x) \, dx \) with

\[
\psi_n(x) = \sqrt{\frac{2}{L}} \sin \frac{n\pi x}{L}.
\]

In Part (c) we’ll use \( P(x) = \psi^2_2(x) \) to determine the probability of finding the particle in some small region \( dx \) centered at \( x = \frac{1}{2}L \).

(a) Express the wave function for a particle in its first excited state:

\[
\psi_2(x) = \sqrt{\frac{2}{L}} \sin \frac{2\pi x}{L}
\]

Square both sides of the equation to obtain:

\[
\psi_2^2(x) = \frac{2}{L} \sin^2 \frac{2\pi x}{L}
\]

The graph of \( \psi^2(x) \) as a function of \( x \) is shown to the right:
(b) Express \( \langle x \rangle \) using the \( n = 2 \) wave function:

\[
\langle x \rangle = \int_{0}^{L} \frac{x}{L} \sin^2 \frac{2\pi x}{L} \, dx
\]

Change variables by letting

\[
x = \frac{L}{2\pi} \theta,
\]

\[
d\theta = \frac{2\pi}{L} \, dx, \text{ and}
\]

\[
dx = \frac{L}{2\pi} \, d\theta
\]

and the limits on \( \theta \) are \( 0 \) and \( 2\pi \).

Substitute in the expression for \( \langle x \rangle \) to obtain:

\[
\langle x \rangle = \frac{2}{L} \int_{0}^{\frac{L}{2\pi}} \left( \frac{L}{2\pi} \right) \sin^2 \left( \frac{L}{2\pi} \, d\theta \right)
\]

\[
= \frac{L}{2\pi^2} \int_{0}^{\frac{\pi}{2}} \theta \sin^2 \theta \, d\theta
\]

Using a table of integrals, evaluate the integral:

\[
\langle x \rangle = \frac{L}{2\pi^2} \left[ \frac{\theta^2}{4} - \frac{\theta \sin 2\theta}{4} - \frac{\cos 2\theta}{8} \right]_{0}^{\frac{\pi}{2}}
\]

\[
= \frac{L}{2\pi^2} \left[ \frac{\pi^2}{16} - \frac{1}{8} + \frac{1}{8} \right]
\]

\[
= \frac{L}{2}
\]

(c) Express \( P(x) \):

\[
P(x) = \psi_2^2(x) = \frac{2}{L} \sin^2 \frac{2\pi x}{L}
\]

Evaluate \( P(L/2) \):

\[
P \left( \frac{L}{2} \right) = \frac{2}{L} \sin^2 \frac{2\pi \cdot \frac{L}{2}}{L}
\]

\[
= \frac{2}{L} \sin^2 \pi = 0
\]

Because \( P(L/2) = 0 \):

\[
P \left( \frac{L}{2} \right) \, dx = 0
\]

(d) The answers to Parts (b) and (c) are not contradictory. Part (b) states that the average value of measurements of the position of the particle will yield \( L/2 \) even though the probability that any one measurement of position will yield this value is zero.
47 •• A particle of mass $m$ has a wave function given by $\psi(x) = Ae^{-|x/a|}$, where $A$ and $a$ are positive constants. (a) Find the normalization constant $A$. (b) Calculate the probability of finding the particle in the region $-a \leq x \leq a$.

**Picture the Problem** We can find the constant $A$ by applying the normalization condition $\int_{-\infty}^{\infty} \psi^2(x) dx = 1$ and finding the value for $A$ that satisfies this condition. As soon as we have found the normalization constant, we can calculate the probability of the finding the particle in the region $-a \leq x \leq a$ using

$$P = \int_{-a}^{a} \psi^2(x) dx .$$

(a) The normalization condition is:

$$\int_{-\infty}^{\infty} \psi^2(x) dx = 1$$

Substitute $\psi(x) = Ae^{-|x/a|}$:

$$\int_{-\infty}^{\infty} \left( Ae^{-|x/a|} \right)^2 dx = 2A^2 \int_{0}^{\infty} e^{-2x/a} dx$$

From integral tables:

$$\int_{0}^{\infty} e^{-ax} dx = \frac{1}{a}$$

Therefore:

$$2A^2 \int_{0}^{\infty} e^{-2x/a} dx = 2A^2 \left( \frac{a}{2} \right) = aA^2 = 1$$

Solving for $A$ yields:

$$A = \frac{1}{\sqrt{a}}$$

(b) Express the normalized wave function:

$$\psi(x) = \frac{1}{\sqrt{a}} e^{-|x/a|}$$

The probability of finding the particle in the region $-a \leq x \leq a$ is:

$$P = \int_{-a}^{a} \psi^2(x) dx = \int_{0}^{a} \frac{1}{a} e^{-2x/a} dx$$

$$= \frac{2}{a} \int_{0}^{a} e^{-2x/a} dx = 1 - e^{-2} = 0.865$$

48 •• A one-dimensional box is on the $x$-axis in the region of $0 \leq x \leq L$. A particle in this box is in its ground state. Calculate the probability that the particle will be found in the region (a) $0 < x < \frac{1}{3}L$ , (b) $0 < x < \frac{2}{3}L$ , and (c) $0 < x < \frac{3}{4}L$. 
**Picture the Problem** The probability density for the particle in its ground state is given by \( P(x) = \frac{2}{L} \sin^2 \frac{\pi}{L} x \). We’ll evaluate the integral of \( P(x) \) between the limits specified in \((a), (b), \text{and} (c)\).

Express \( P(x) \) for \( 0 < x < d \):

\[
P(x) = \frac{2}{L} \int_0^d \sin^2 \frac{\pi}{L} x \, dx
\]

Change variables by letting \( \theta = \frac{\pi}{L} x \). Then:

\[
x = \frac{L}{\pi} \theta, \, \, d\theta = \frac{L}{\pi} \, dx,
\]

Thus:

\[
dx = \frac{L}{\pi} \, d\theta
\]

Substitute to obtain:

\[
P(x) = \frac{2}{L} \int_0^\theta \sin^2 \left( \frac{L}{\pi} \theta \right) \, d\theta
\]

\[
= \frac{2}{\pi} \int_0^\theta \sin^2 \theta \, d\theta
\]

Use a table of integrals to evaluate

\[
\int_0^\theta \sin^2 \theta \, d\theta
\]

(a) Noting that the limits on \( \theta \) are 0 and \( \pi/2 \), evaluate equation (1) over the interval \( 0 < x < \frac{1}{2}L \):

\[
P(x) = \frac{2}{\pi} \left[ \frac{\theta}{2} - \frac{\sin 2\theta}{4} \right]_0^{\pi/2} = \frac{2}{\pi} \left[ \frac{\pi}{4} \right] = \boxed{0.500}
\]

(b) Noting that the limits on \( \theta \) are 0 and \( \pi/3 \), evaluate equation (1) over the interval \( 0 < x < L/3 \):

\[
P(x) = \frac{2}{\pi} \left[ \frac{\theta}{2} - \frac{\sin 2\theta}{4} \right]_0^{\pi/3} = \frac{2}{\pi} \left[ \frac{\pi}{6} - \frac{\sin 2\pi/3}{4} \right]
\]

\[
= \frac{1}{3} \left[ \frac{\sqrt{3}}{4\pi} \right] = \boxed{0.196}
\]
(c) Noting that the limits on $\theta$ are 0 and $3\pi/4$, Evaluate equation (1) over the interval $0 < x < 3L/4$:

\[
P(x) = \frac{2}{\pi} \left[ \frac{\theta}{2} - \frac{\sin 2\theta}{4} \right]^{3\pi/4}_0
\]

\[
= \frac{2}{\pi} \left[ \frac{3\pi}{8} - \frac{\sin 6\pi/4}{4} \right]
\]

\[
= \frac{3}{4} + \frac{1}{2\pi} = 0.909
\]

49  •• A one-dimensional box is on the $x$-axis in the region of $0 \leq x \leq L$. A particle in this box is in its first excited state. Calculate the probability that the particle will be found in the region (a) $0 < x < \frac{1}{2}L$, (b) $0 < x < \frac{1}{3}L$, and (c) $0 < x < \frac{3}{4}L$.

**Picture the Problem** The probability density for the particle in its first excited state is given by $P(x) = \frac{2}{L} \sin^2 \frac{2\pi}{L} x$. We’ll evaluate the integral of $P(x)$ between the limits specified in (a), (b), and (c).

Express $P(x)$ for $0 < x < d$:

\[
P(x) = \frac{2}{L} \int_0^x \sin^2 \frac{2\pi}{L} \, dx
\]

Change variables by letting $\theta = \frac{2\pi}{L} x$. Then:

\[
x = \frac{L}{2\pi} \theta, \, d\theta = \frac{2\pi}{L} \, dx, \text{ and}
\]

\[
dx = \frac{L}{2\pi} \, d\theta
\]

Substitute to obtain:

\[
P(x) = \frac{2}{L} \int_0^\theta \sin^2 \left( \frac{L}{2\pi} \theta \right) \, d\theta
\]

\[
= \frac{1}{\pi} \int_0^\theta \sin^2 \theta \, d\theta
\]

Use a table of integrals to evaluate

\[
\int_0^\theta \sin^2 \theta \, d\theta
\]

(a) Noting that the limits on $\theta$ are 0 and $\pi$, evaluate equation (1) over the interval $0 < x < \frac{1}{2}L$:

\[
P(x) = \frac{1}{\pi} \left[ \frac{\theta}{2} - \frac{\sin 2\theta}{4} \right]_0^\theta
\]

\[
= 0.500
\]
(b) Noting that the limits on $\theta$ are 0 and $2\pi/3$, evaluate equation (1) over the interval $0 < x < L/3$:

$$P(x) = \frac{1}{\pi} \left[ \frac{\theta}{2} - \frac{\sin 2\theta}{4} \right]_{0}^{2\pi/3} = \frac{1}{\pi} \left[ \frac{2\pi}{6} - \frac{\sin 4\pi/3}{4} \right] = \frac{1}{3} + \frac{\sqrt{3}/2}{4\pi} = 0.402$$

(c) Noting that the limits on $\theta$ are 0 and $3\pi/2$, evaluate equation (1) over the interval $0 < x < 3L/4$:

$$P(x) = \frac{1}{\pi} \left[ \frac{\theta}{2} - \frac{\sin 2\theta}{4} \right]_{0}^{3\pi/2} = \frac{1}{\pi} \left[ \frac{3\pi}{4} \right] = 0.750$$

50  

The classical probability distribution function for a particle in a one-dimensional box on the $x$ axis in the region $0 < x < L$ is given by $P(x) = 1/L$. Use this expression to show that $\langle x \rangle = \frac{1}{2}L$ and $\langle x^2 \rangle = \frac{1}{3}L^2$ for a classical particle in the box.

**Picture the Problem** Classically, $\langle x \rangle = \int_0^L xP(x)dx$ and $\langle x^2 \rangle = \int_0^L x^2P(x)dx$.

Evaluate $\langle x \rangle$ with $P(x) = 1/L$:

$$\langle x \rangle = \int_0^L x \frac{1}{L} dx = \left[ \frac{x^2}{2L} \right]_0^L = \frac{1}{2}L$$

Evaluate $\langle x^2 \rangle$ with $P(x) = 1/L$:

$$\langle x^2 \rangle = \int_0^L x^2 \frac{1}{L} dx = \left[ \frac{x^3}{3L} \right]_0^L = \frac{1}{3}L^2$$

51  

A one-dimensional box is on the $x$-axis in the region of $0 \leq x \leq L$.

(a) The wave functions for a particle in the box are given by

$$\psi_n(x) = \sqrt{\frac{2}{L}} \sin \frac{n\pi x}{L} \quad n = 1, 2, 3, \ldots$$

For a particle in the $n$th state, show that $\langle x \rangle = \frac{1}{2}L$ and $\langle x^2 \rangle = L^2/3 - L^2/(2n^2\pi^2)$.

(b) Compare these expressions for $\langle x \rangle$ and $\langle x^2 \rangle$, for $n >> 1$, with the expressions for $\langle x \rangle$ and $\langle x^2 \rangle$ for the classical distribution of Problem 50.

**Picture the Problem** We’ll use $\langle f(x) \rangle = \int_0^L f(x)\psi^2(x)dx$ with

$$\psi_n(x) = \sqrt{\frac{2}{L}} \sin \frac{n\pi x}{L}$$

to show that $\langle x \rangle = \frac{L}{2}$ and $\langle x^2 \rangle = \frac{L^2}{3} = \frac{L^2}{2n^2\pi^2}$.
(a) Express $\langle x \rangle$ for a particle in the $n$th state:

$$\langle x \rangle = \frac{L}{2} \int_0^L \frac{2x}{L} \sin^2 \frac{n\pi x}{L} \, dx$$

Change variables by letting $\theta = \frac{n\pi x}{L}$. Then:

$$x = \frac{L}{n\pi} \theta, \, d\theta = \frac{n\pi}{L} \, dx, \, dx = \frac{L}{n\pi} \, d\theta$$

and the limits on $\theta$ are $0$ and $n\pi$.

Substitute to obtain:

$$\langle x \rangle = \frac{2}{L} \int_0^{n\pi} \left( \frac{L}{n\pi} \theta \right) \sin^2 \theta \left( \frac{L}{n\pi} \, d\theta \right)$$

$$= \frac{2L}{n^2 \pi^2} \int_0^{n\pi} \theta \sin^2 \theta \, d\theta$$

Use a table of integrals to evaluate the integral:

$$\langle x \rangle = \frac{2L}{n^2 \pi^2} \left[ \frac{\theta^2}{4} - \frac{\theta \sin 2\theta}{4} - \frac{\cos 2\theta}{8} \right]_0^{n\pi}$$

$$= \frac{2L}{n^2 \pi^2} \left[ \frac{n^2 \pi^2}{4} - \frac{n\pi \sin 2n\pi}{4} - \frac{\cos 2n\pi}{8} + \frac{1}{8} \right] = \frac{2L}{n^2 \pi^2} \left[ \frac{n^2 \pi^2}{4} - \frac{1}{8} + \frac{1}{8} \right]$$

$$= \frac{L}{2}$$

Express $\langle x^2 \rangle$ for a particle in the $n$th state:

$$\langle x^2 \rangle = \frac{L}{2} \int_0^L \frac{x^2}{L} \sin^2 \frac{n\pi x}{L} \, dx$$

Change variables by letting $\theta = \frac{n\pi x}{L}$. Then:

$$x = \frac{L}{n\pi} \theta, \, d\theta = \frac{n\pi}{L} \, dx, \, dx = \frac{L}{n\pi} \, d\theta$$

and the limits on $\theta$ are $0$ and $n\pi$.

Substitute to obtain:

$$\langle x^2 \rangle = \frac{L}{2} \int_0^{n\pi} \left( \frac{L}{n\pi} \theta \right)^2 \sin^2 \theta \left( \frac{L}{n\pi} \, d\theta \right)$$

$$= \frac{2L^2}{n^3 \pi^3} \int_0^{n\pi} \theta^2 \sin^2 \theta \, d\theta$$
Use a table of integrals to evaluate the integral:

\[
\langle x^2 \rangle = \frac{2L^2}{n^3\pi^3} \left[ \frac{\theta^3}{6} - \left( \frac{\theta^2}{4} - \frac{1}{8} \right) \sin 2\theta - \frac{\theta \cos 2\theta}{4} \right]_0^n = \frac{2L^2}{n^3\pi^3} \left[ \frac{n^3\pi^3}{6} - \frac{n\pi}{4} \right] = \frac{L^2}{3} - \frac{L^2}{2n^2\pi^2}
\]

\(b\) For large values of \(n\) the result agrees with the classical value of \(L^2/3\) given in Problem 50.

52  **  (a) Use a spreadsheet program or graphing calculator to plot \(\langle x^2 \rangle\) as a function of the quantum number \(n\) for the particle in the box described in Problem 48 and for values of \(n\) from 1 to 100. Assume \(L = 1.00\) m for your graph. Refer to Problem 51. (b) Comment on the significance of any asymptotic limits that your graph shows.

Picture the Problem  (a) From Problem 51 we have \(\langle x \rangle = \frac{L}{2}\) and

\[
\langle x^2 \rangle = \frac{L^2}{3} - \frac{L^2}{2n^2\pi^2}
\]

A spreadsheet program was used to plot the following graph of \(\langle x^2 \rangle\) as a function of \(n\).
(b) As \( n \to \infty \), \( \langle x^2 \rangle \to \frac{L^2}{3} \)

53 \quad \bullet \quad \text{The wave functions for a particle of mass } m \text{ in a one-dimensional box of length } L \text{ centered at the origin (so that the ends are at } x = \pm \frac{1}{2} L) \text{ are given by}

\[
\psi(x) = \sqrt{\frac{2}{L}} \cos \frac{n \pi x}{L} \quad n = 1, 3, 5, 7, \ldots
\]
and

\[
\psi(x) = \sqrt{\frac{2}{L}} \sin \frac{n \pi x}{L} \quad n = 2, 4, 6, 8, \ldots
\]

Calculate \( \langle x \rangle \) and \( \langle x^2 \rangle \) for the ground state \( (n = 1) \).

**Picture the Problem** For the ground state, \( n = 1 \) and so we’ll evaluate

\[
\langle f(x) \rangle = \int f(x) \psi^2(x) \, dx \quad \text{using} \quad \psi_1(x) = \sqrt{\frac{2}{L}} \cos \frac{\pi x}{L}.
\]

Because \( \psi_1^2(x) \) is an even function of \( x \), \( x \psi_1^2(x) \) is an odd function of \( x \). It follows that the integral of \( x \psi_1^2(x) \) between \(-L/2\) and \( L/2 \) is zero. Thus:

Express \( \langle x^2 \rangle \):

\[
\langle x^2 \rangle = \frac{2}{L^2} \int_{-L/2}^{L/2} x^2 \cos^2 \frac{\pi x}{L} \, dx
\]

Change variables by letting \( \theta = \frac{\pi x}{L} \). \( x = \frac{L}{\pi} \theta, \, d\theta = \frac{\pi}{L} \, dx, \, dx = \frac{L}{\pi} \, d\theta \)

Then:

Substitute for \( x \) and \( dx \) to obtain:

\[
\langle x^2 \rangle = \frac{2}{L} \int_{-\pi/2}^{\pi/2} \left( \frac{L}{\pi} \theta \right)^2 \cos^2 \left( \frac{L}{\pi} \theta \right) \, d\theta
\]

Use a trigonometric identity to rewrite the integrand:

\[
\langle x^2 \rangle = \frac{2L^2}{\pi^3} \int_{-\pi/2}^{\pi/2} \theta^2 (1 - \sin^2 \theta) \, d\theta
\]

\[
= \frac{2L^2}{\pi^3} \left[ \frac{\pi^2}{8} - \frac{\pi^2}{2} \right]
\]
Evaluate the second integral by looking it up in the tables:

\[
\left\langle x^2 \right\rangle = \frac{2L^2}{\pi^3} \left[ \frac{\theta^3}{3} - \frac{\theta^3}{6} - \frac{\theta^2}{4} - \frac{1}{8} \right] \sin 2\theta - \frac{\theta \cos 2\theta}{4} \right\rangle_{-\pi/2}^{\pi/2}
\]

\[
= 2L^2 \frac{\pi^3}{\pi^3} \left[ \frac{\pi^3}{48} + \frac{\pi^3}{16} - \frac{3}{8} \right] \sin \pi + \frac{\pi \cos \pi}{8}
\]

\[
+ \frac{pi^3}{48} + \frac{\pi^3}{16} - \frac{3}{8} \right] \sin \pi + \frac{\pi \cos \pi}{8}
\]

\[
= 2L^2 \left[ \frac{\pi^3}{24} - \frac{\pi}{4} \right] = \frac{L^2}{12} - \frac{1}{2\pi^2}
\]

**Remarks:** The result differs from that of Example 34-8. Since we have shifted the origin by \( \Delta x = L/2 \), we could have arrived at the above result, without performing the integration, by subtracting \((\Delta x)^2 = L^2/4\) from \(\left\langle x^2 \right\rangle\).

**54** Calculate \(\left\langle x \right\rangle\) and \(\left\langle x^2 \right\rangle\) for the first excited state \((n = 2)\) of the box described in Problem 53.

**Picture the Problem** For the first excited state, \(n = 2\), and so we’ll evaluate

\[
\left\langle f(x) \right\rangle = \int f(x)\psi^2(x)dx \text{ using } \psi(x) = \sqrt{\frac{2}{L}} \sin \frac{2\pi x}{L}.
\]

Because \(\psi^2(x)\) is an even function of \(x\), \(x\psi^2(x)\) is an odd function of \(x\). It follows that the integral of \(x\psi^2(x)\) between \(-L/2\) and \(L/2\) is zero. Thus:

Express \(\left\langle x^2 \right\rangle\):

\[
\left\langle x^2 \right\rangle = \frac{2}{L} \int_{-L/2}^{L/2} x^2 \sin \frac{2\pi x}{L} dx
\]

Change variables by letting \(\theta = \frac{2\pi x}{L}\).

\[
x = \frac{L}{2\pi} \theta, \; d\theta = \frac{2\pi}{L} dx, \; dx = \frac{L}{2\pi} d\theta
\]

Then:

\[
\left\langle x^2 \right\rangle = \frac{2}{L} \int_{-\pi}^{\pi} \theta^2 \sin \theta d\theta
\]

and the limits on \(\theta\) are \(-\pi\) and \(\pi\).
Substitute to obtain:

\[
\langle x^2 \rangle = \frac{2}{L} \int_{-\pi}^{\pi} \left( \frac{L}{2\pi} \theta \right)^2 \sin^2 \theta \left( \frac{L}{2\pi} d\theta \right)
\]

\[
= \frac{L^2}{4\pi} \int_{-\pi}^{\pi} \theta^2 \sin^2 \theta \, d\theta
\]

Evaluate the integral by looking it up in the tables:

\[
\langle x^2 \rangle = \frac{L^2}{4\pi^3} \left[ \frac{\theta^3}{6} - \left( \frac{\theta^2}{4} - \frac{1}{8} \right) \sin 2\theta - \frac{\theta \cos 2\theta}{4} \right]_{-\pi}^{\pi}
\]

\[
= \frac{L^2}{4\pi^3} \left[ \frac{\pi^3}{6} - \left( \frac{\pi^2}{4} - \frac{1}{8} \right) \sin 2\pi - \frac{\pi \cos 2\pi}{4} \right]
\]

\[
+ \frac{\pi^3}{6} - \left( \frac{\pi^2}{4} - \frac{1}{8} \right) \sin 2\pi - \frac{\pi \cos 2\pi}{4} \right]
\]

\[
= \frac{L^2}{4\pi^3} \left[ \frac{\pi^3}{6} + \frac{\pi^3}{6} - \frac{\pi^3}{6} - \frac{\pi^3}{6} \right]
\]

\[
= \frac{L^2}{4\pi^3} \left[ \frac{\pi^3}{3} - \frac{\pi}{2} \right] = \left[ \frac{L^2}{12} - \frac{1}{8\pi^2} \right]
\]

Remarks: The result differs from that of Example 34-8. Since we have shifted the origin by \(\Delta x = L/2\), we could have arrived at the above result, without performing the integration, by subtracting \((\Delta x)^2 = L^2/4\) from \(\langle x^2 \rangle\).

General Problems

55 • [SSM] Photons in a uniform 4.00-cm-diameter light beam have wavelengths equal to 400 nm and the beam has an intensity of 100 W/m^2.

(a) What is the energy of each photon in the beam? (b) How much energy strikes an area of 1.00 cm^2 perpendicular to the beam in 1 s? (c) How many photons strike this area in 1.00 s?

Picture the Problem We can use the Einstein equation for photon energy to find the energy of each photon in the beam. The intensity of the energy incident on the surface is the ratio of the power delivered by the beam to its delivery time. Hence, we can express the energy incident on the surface in terms of the intensity of the beam.
(a) Use the Einstein equation for photon energy to express the energy of each photon in the beam:

\[ E_{\text{photon}} = hf = \frac{hc}{\lambda} \]

Substitute numerical values and evaluate \( E_{\text{photon}} \):

\[ E_{\text{photon}} = \frac{1240 \text{ eV} \cdot \text{nm}}{400 \text{ nm}} = 3.10 \text{ eV} \]

(b) Relate the energy incident on a surface of area \( A \) to the intensity of the beam:

\[ E = IA\Delta t \]

Substitute numerical values and evaluate \( E \):

\[ E = (100 \text{ W/m}^2)(1.00 \times 10^{-4} \text{ m}^2)(1.00 \text{ s}) \]
\[ = 0.0100 \text{ J} \times \frac{1 \text{ eV}}{1.602 \times 10^{-19} \text{ J}} \]
\[ = 6.242 \times 10^{16} \text{ eV} = 6.24 \times 10^{16} \text{ eV} \]

(c) Express the number of photons striking this area in 1.00 s as the ratio of the total energy incident on the surface to the energy delivered by each photon:

\[ N = \frac{E}{E_{\text{photon}}} = \frac{6.242 \times 10^{16} \text{ eV}}{3.10 \text{ eV}} \]
\[ = 2.08 \times 10^{16} \]

56 • A 1-\( \mu \)g particle is moving with a speed of approximately 1 mm/s in a one-dimensional box that has a length equal to 1 cm. Calculate the approximate value of the quantum number \( n \) of the state occupied by the particle.

**Picture the Problem** The particle’s \( n \)th-state energy is \( E_n = n^2 \frac{h^2}{8mL^2} \). We can find \( n \) by solving this equation for \( n \) and substituting the particle’s kinetic energy for \( E_n \).

Express the energy of the particle when it is in its \( n \)th state:

\[ E_n = n^2 \frac{h^2}{8mL^2} \Rightarrow n = \frac{L}{h} \sqrt{8mE_n} \]

Express the energy (kinetic) of the particle:

\[ E_n = \frac{1}{2}mv^2 \]

Substitute for \( E_n \) and simplify to obtain:

\[ n = \frac{2mvL}{h} \]
Substitute numerical values and evaluate \( n \):
\[
n = \frac{2 \left( 10^{-9} \text{ kg} \right) \left( 1 \times 10^{-3} \text{ m/s} \right) \left( 10^{-2} \text{ m} \right)}{6.626 \times 10^{-34} \text{ J} \cdot \text{s}}
\]
\[
\approx 3 \times 10^{19}
\]

57 • (a) For the particle and box of Problem 56, find \( \Delta x \) and \( \Delta p_x \), assuming that these uncertainties are given by \( \Delta x/L = 0.01 \) percent and \( \Delta p_{x}/p_x = 0.01 \) percent. (b) What is \( (\Delta x \Delta p_{x})/\hbar \)?

**Picture the Problem** We can use the fact that the uncertainties are given by \( \Delta x/L = 0.01 \) percent and \( \Delta p_{x}/p_x = 0.01 \) percent to find \( \Delta x \) and \( \Delta p \).

(a) Assuming that \( \Delta x/L = 0.01 \) percent, find \( \Delta x \):
\[
\Delta x = 10^{-4} (L) = 10^{-4} \left( 10^{-2} \text{ m} \right) = 1 \mu\text{m}
\]

Assuming that \( \Delta p_{x}/p_x = 0.01 \) percent, find \( \Delta p \):
\[
\Delta p_x = 10^{-4} mv = 10^{-4} \left( 10^{-9} \text{ kg} \right) \left( 10^{-3} \text{ m/s} \right)
\]
\[
= 10^{-16} \text{ kg} \cdot \text{m/s}
\]

(b) Evaluate \( (\Delta x \Delta p_{x})/\hbar \):
\[
\frac{\Delta x \Delta p_x}{\hbar} = \frac{1 \mu\text{m} \left( 10^{-16} \text{ kg} \cdot \text{m/s} \right)}{6.626 \times 10^{-34} \text{ J} \cdot \text{s}}
\]
\[
= 2 \times 10^{11}
\]

58 • In 1987, a laser at Los Alamos National Laboratory produced a flash that lasted \( 1 \times 10^{-12} \) s and had a power of \( 5 \times 10^{15} \) W. Estimate the number of emitted photons, assuming they all had wavelengths equal to 400 nm.

**Picture the Problem** We can estimate the number of emitted photons from the ratio of the total energy in the flash to the energy of a single photon.

Letting \( N \) be the number of emitted photons, express the ratio of the total energy in the flash to the energy of a single photon:
\[
N = \frac{E}{E_{\text{photon}}}
\]

Relate the energy in the flash to the power produced:
\[
E = P \Delta t
\]

Express the energy of a single photon as a function of its wavelength:
\[
E_{\text{photon}} = \frac{h c}{\lambda}
\]
Substituting for $E$ and $E_{\text{photon}}$ and simplifying gives:

$$N = \frac{P\Delta t\lambda}{hc}$$

Substitute numerical values and evaluate $N$:

$$N = \left(5 \times 10^{15} \text{ W}\right)\left(1 \times 10^{-12} \text{ s}\right)\left(400 \times 10^{-9} \text{ m}\right) \approx 1 \times 10^{-22}$$

59. You cannot "see" anything smaller than the wavelength of the wave used to make the observation. What is the minimum energy of an electron needed in an electron microscope to "see" an atom that has a diameter of about 0.1 nm?

**Picture the Problem** We can use the electron wavelength equation $\lambda = \frac{1.23}{\sqrt{K}} \text{ nm}$, where $K$ is in eV to find the minimum energy required to see an atom.

Relate the energy of the electron to the size of an atom (the wavelength of the electron):

$$\lambda = \frac{1.23}{\sqrt{K}} \text{ nm} \Rightarrow K = \left(\frac{1.23 \text{ eV}^{1/2} \cdot \text{ nm}}{\lambda}\right)^2$$

provided $K$ is in eV.

Substitute numerical values and evaluate $K$:

$$K = \frac{\left(1.23 \text{ eV}^{1/2} \cdot \text{ nm}\right)^2}{(0.1 \text{ nm})^2} \approx 0.2 \text{ keV}$$

60. A common flea that has a mass of 0.008 g can jump vertically as high as 20 cm. Estimate the de Broglie wavelength for the flea immediately after takeoff.

**Picture the Problem** The flea’s de Broglie wavelength is $\lambda = \frac{h}{p}$, where $p$ is the flea’s momentum immediately after takeoff. We can use a constant acceleration equation to find the flea’s speed and, hence, momentum immediately after takeoff.

Express the de Broglie wavelength of the flea immediately after takeoff:

$$\lambda = \frac{h}{p} = \frac{h}{mv_0}$$

Using a constant acceleration equation, express the height the flea can jump as a function of its takeoff speed:

$$v^2 = v_0^2 + 2a\Delta y$$

or, since $v = 0$ and $a = -g$,

$$v_0 = \sqrt{2g\Delta y}$$
Substitute to obtain:

\[ \lambda = \frac{h}{m\sqrt{2g\Delta y}} \]

Substitute numerical values and evaluate \( \lambda \):

\[
\lambda = \frac{6.626 \times 10^{-34} \text{ J} \cdot \text{s}}{(8 \times 10^{-6} \text{ kg})\sqrt{2(9.81 \text{ m/s}^2)(0.2 \text{ m})}}
\approx 4 \times 10^{-20} \text{ m}
\]

61  •  [SSM] A 100-W source radiates light of wavelength 600 nm uniformly in all directions. An eye that has been adapted to the dark has a 7-mm-diameter pupil and can detect the light if at least 20 photons per second enter the pupil. How far from the source can the light be detected under these rather extreme conditions?

**Picture the Problem** We can relate the fraction of the photons entering the eye to ratio of the area of the pupil to the area of a sphere of radius \( R \). We can find the number of photons emitted by the source from the rate at which it emits and the energy of each photon which we can find using the Einstein equation.

Letting \( r \) be the radius of the pupil, \( N_{\text{entering, eye}} \) the number of photons per second entering the eye, and \( N_{\text{emitted}} \) the number of photons emitted by the source per second, express the fraction of the light energy entering the eye at a distance \( R \) from the source:

\[
\frac{N_{\text{entering, eye}}}{N_{\text{emitted}}} = \frac{A_{\text{eye}}}{4\pi R^2} = \frac{\pi r^2}{4\pi R^2} = \frac{r^2}{4R^2}
\]

Solving for \( R \) yields:

\[
R = \frac{r}{2} \sqrt{\frac{N_{\text{emitted}}}{N_{\text{entering, eye}}}} \quad (1)
\]

Find the number of photons emitted by the source per second:

\[
N_{\text{emitted}} = \frac{P}{E_{\text{photon}}}
\]

Using the Einstein equation, express the energy of the photons:

\[
E_{\text{photon}} = \frac{hc}{\lambda}
\]

Substitute numerical values and evaluate \( E_{\text{photon}} \):

\[
E_{\text{photon}} = \frac{1240 \text{ eV} \cdot \text{nm}}{600 \text{ nm}} = 2.07 \text{ eV}
\]
Substitute and evaluate $N_{\text{emitted}}$:

$$N_{\text{emitted}} = \frac{100 \text{ W}}{(2.07 \text{ eV})(1.602 \times 10^{-19} \frac{\text{J}}{\text{eV}})} = 3.02 \times 10^{20} \text{ s}^{-1}$$

Substitute for $N_{\text{emitted}}$ in equation (1) and evaluate $R$:

$$R = \frac{3.5 \text{ mm}}{2} \sqrt{\frac{3.02 \times 10^{20} \text{ s}^{-1}}{20 \text{ s}^{-1}}} \approx 7 \times 10^3 \text{ km}$$

62 ** The diameter of the pupil of an eye under room-light conditions is approximately 5 mm. Find the intensity of light that has a wavelength equal to 600 nm so that 1 photon per second passes through the pupil.

**Picture the Problem** The intensity of the light such that one photon per second passes through the pupil is the ratio of the energy of one photon to the product of the area of the pupil and time interval during which the photon passes through the pupil. We’ll use the Einstein equation to express the energy of the photon.

Use its definition to relate the intensity of the light to the energy of a 600-nm photon:

$$I_{\text{1 photon}} = \frac{P}{A} = \frac{E_{\text{1 photon}}}{A\Delta t}$$

Using the Einstein equation, express the energy of a 600-nm photon:

$$E_{\text{1 photon}} = \frac{hc}{\lambda}$$

Substitute for $E_{\text{1 photon}}$ to obtain:

$$I_{\text{1 photon}} = \frac{hc}{\lambda A\Delta t}$$

Substitute numerical values and evaluate $I_{\text{1 photon}}$:

$$I_{\text{1 photon}} = \frac{(1240 \text{ eV} \cdot \text{nm})(1.602 \times 10^{-19} \frac{\text{J}}{\text{eV}})}{(600 \text{ nm})\left[\frac{\pi}{4}(5 \times 10^{-3} \text{ m})^2\right](1 \text{ s})} \approx 2 \times 10^{-14} \text{ W/m}^2$$

63 ** A 100-W incandescent light bulb radiates 2.6 W of visible light uniformly in all directions. (a) Find the intensity of the light from the bulb at a distance of 1.5 m. (b) If the average wavelength of the visible light is 650 nm, and counting only those photons in the visible spectrum, find the number of photons per second that strike a surface that has an area equal to 1.0 cm$^2$, is oriented so that the line to the bulb is perpendicular to the surface, and is a distance of 1.5 m from the bulb.
**Picture the Problem** We can find the intensity at a distance of 1.5 m directly from its definition. The number of photons striking the surface each second can be found from the ratio of the energy incident on the surface to the energy of a 650-nm photon.

(a) Use its definition to express the intensity of the light as a function of distance from the light bulb:

\[ I = \frac{P}{A} = \frac{P}{4\pi R^2} \]

Substitute numerical data to obtain:

\[ I = \frac{2.6 \text{ W}}{4\pi (1.5 \text{ m})^2} = 91.96 \text{ mW/m}^2 \]

\[ = 92 \text{ mW/m}^2 \]

(b) Express the number of photons per second that strike the surface as the ratio of the energy incident on the surface to the energy of a 650-nm photon:

\[ N = \frac{IA}{E_{\text{photon}}} \]

where \( A \) is the area of the surface.

Use the Einstein equation to express the energy of the 650-nm photons:

\[ E = \frac{hc}{\lambda} \]

Substituting for \( E \) yields:

\[ N = \frac{IA\lambda}{hc} \]

Substitute numerical values and evaluate \( N \):

\[ N = \frac{(91.96 \text{ mW/m}^2)(1.0 \times 10^{-4} \text{ m}^2)(650 \text{ nm})}{(1240 \text{ eV} \cdot \text{nm})(1.602 \times 10^{-19} \text{ J/eV})} = 3.0 \times 10^4 \]

64 ** When light of wavelength \( \lambda_1 \) is incident on the cathode of a photoelectric tube, the maximum kinetic energy of the emitted electrons is 1.8 eV. If the wavelength is reduced to \( \frac{1}{2} \lambda_1 \), the maximum kinetic energy of the emitted electrons is 5.5 eV. Find the work function \( \phi \) of the cathode material.

**Picture the Problem** The maximum kinetic energy of the photoelectrons is related to the frequency of the incident photons and the work function of the cathode material through the Einstein equation. We can apply this equation to the two sets of data and solve the resulting equations simultaneously for the work function.
Using the Einstein equation, relate the maximum kinetic energy of the emitted electrons to the frequency of the incident photons and the work function of the cathode material:

\[
K_{\text{max}} = hf - \phi = \frac{hc}{\lambda} - \phi
\]

Substitute numerical values for the light of wavelength \( \lambda_1 \):

\[
1.8 \text{ eV} = \frac{hc}{\lambda_1} - \phi \quad (1)
\]

Substitute numerical values for the light of wavelength \( \frac{1}{2} \lambda_1 \):

\[
5.5 \text{ eV} = \frac{hc}{\frac{1}{2} \lambda_1} - \phi = \frac{2hc}{\lambda_1} - \phi \quad (2)
\]

Solve equations (1) and (2) simultaneously for \( \phi \) to obtain:

\[
\phi = 1.9 \text{ eV}
\]

65  An incident photon of energy \( E_i \) undergoes Compton scattering at an angle of \( \theta \). Show that the energy \( E_s \) of the scattered photon is given by

\[
E_s = \frac{E_i}{1 + \left( \frac{E_i}{m_e c^2} \right) \left( 1 - \cos \theta \right)}
\]

**Picture the Problem** We can use the Einstein equation to express the energy of the scattered photon in terms of its wavelength and the Compton scattering equation to relate this wavelength to the scattering angle and the pre-scattering wavelength.

Express the energy of the scattered photon \( E_s \) as a function of their wavelength \( \lambda_s \):

\[
E_s = \frac{hc}{\lambda_s}
\]

Express the wavelength of the scattered photon as a function of the scattering angle \( \theta \):

\[
\lambda_s = \frac{h}{m_e c} (1 - \cos \theta) + \lambda
\]

where \( \lambda \) is the wavelength of the incident photon.
Substitute for $\lambda$ and simplify to obtain:

$$E_s = \frac{hc}{m_v c} (1 - \cos \theta) + \lambda$$

$$= \frac{hc}{\lambda}$$

$$= \frac{E_i}{1 + \left( \frac{E_i}{m_v c^2} \right) (1 - \cos \theta)}$$

66 ** A particle is confined to a one-dimensional box. While the particle makes a transition from the state $n$ to the state $n - 1$, radiation of 114.8 nm is emitted. While the particle makes the transition from the state $n - 1$ to the state $n - 2$, radiation of wavelength 147 nm is emitted. The ground-state energy of the particle is 1.2 eV. Determine $n$.

**Picture the Problem** While we can work with either of the transitions described in the problem statement, we’ll use the first transition in which radiation of wavelength 114.8 nm is emitted. We can express the energy released in the transition in terms of the difference between the energies in the two states and solve the resulting equation for $n$.

Express the energy of the emitted radiation as the particle goes from the $n$th to $n - 1$ state:

$$\Delta E = E_n - E_{n-1}$$

Express the energy of the particle in $n$th state:

$$E_n = n^2 E_i$$

Express the energy of the particle in the $n - 1$ state:

$$E_{n-1} = (n-1)^2 E_i$$

Substitute for $E_n$ and $E_{n-1}$ and simplify to obtain:

$$\Delta E = n^2 E_i - (n-1)^2 E_i$$

$$= (2n-1)E_i = \frac{hc}{\lambda}$$

Solving for $n$ yields:

$$n = \frac{hc}{2\lambda E_i} + \frac{1}{2}$$
Substitute numerical values and evaluate \( n \):
\[
n = \frac{1240 \text{ eV} \cdot \text{nm}}{2(114.8 \text{ nm})(1.2 \text{ eV})} + \frac{1}{2} = 5
\]

**67 [SSM]** The Pauli exclusion principle states that no more than one electron may occupy a particular quantum state at a time. Electrons intrinsically occupy two spin states. Therefore, if we wish to model an atom as a collection of electrons trapped in a one-dimensional box, no more than two electrons in the box can have the same value of the quantum number \( n \). Calculate the energy that the most energetic electron(s) would have for the uranium atom that has an atomic number 92. Assume the box has a length of 0.050 nm and the electrons are in the lowest possible energy states. How does this energy compare to the rest energy of the electron?

**Picture the Problem** We can use the expression for the energy of a particle in a well to find the energy of the most energetic electron in the uranium atom.

Relate the energy of an electron in the uranium atom to its quantum number \( n \):
\[
E_n = n^2 \left( \frac{\hbar^2}{8m_e L^2} \right)
\]

Substitute numerical values and evaluate \( E_{92} \):
\[
E_{92} = (92)^2 \left[ \frac{(6.626 \times 10^{-34} \text{ J} \cdot \text{s})^2}{8(9.109 \times 10^{-31} \text{ kg})(0.050 \text{ nm})^2} \times \frac{1 \text{ eV}}{1.602 \times 10^{-19} \text{ J}} \right] = 1.273 \text{ MeV}
\]
\[
= 1.3 \text{ MeV}
\]

The rest energy of an electron is:
\[
m_e c^2 = (9.109 \times 10^{-31} \text{ kg})(2.998 \times 10^8 \text{ m/s})^2 \left( \frac{1 \text{ eV}}{1.602 \times 10^{-19} \text{ J}} \right) = 0.512 \text{ MeV}
\]

Express the ratio of \( E_{92} \) to \( m_e c^2 \):
\[
\frac{E_{92}}{m_e c^2} = \frac{1.273 \text{ MeV}}{0.512 \text{ MeV}} \approx 2.5
\]

The energy of the most energetic electron is approximately 2.5 times the rest-energy of an electron.

**68** A beam of electrons that each have the same kinetic energy illuminates a pair of slits separated by a distance 54 nm. The beam forms bright and dark fringes on a screen located a distance 1.5 m beyond the two slits. The arrangement is otherwise identical to that used in the optical two-slit interference
experiment described in Chapter 33 and in Figure 33-7 and the fringes have the appearance shown in Figure 34-18d. The bright fringes are found to be separated by a distance of 0.68 mm. What is the kinetic energy of the electrons in the beam?

**Picture the Problem** We can express the kinetic energy of an electron in the beam in terms of its momentum. We can use the de Broglie relationship to relate the electron’s momentum to its wavelength and use the condition for constructive interference to find $\lambda$.

Express the kinetic energy of an electron in terms of its momentum:  
$$K = \frac{p^2}{2m} \quad (1)$$  

Using the de Broglie relationship, relate the momentum of an electron to its wavelength:  
$$p = \frac{h}{\lambda}$$  

Substitute for $p$ in equation (1) to obtain:  
$$K = \frac{h^2}{2m\lambda^2} \quad (2)$$

The condition for constructive interference is:  
$$d \sin \theta = n\lambda \Rightarrow \lambda = \frac{d \sin \theta}{n}$$

where $d$ is the slit separation and $n = 0, 1, 2, \ldots$

For $\theta << 1$, $\sin \theta$ is also given by:  
$$\sin \theta \approx \frac{\Delta y}{L}$$  

Substitute for $\sin \theta$ to obtain:  
$$\lambda = \frac{d\Delta y}{nL}$$

Substitute for $\lambda$ in equation (2) to obtain:  
$$K = \frac{n^2 L^2 h^2}{2md^2(\Delta y)^2}$$
Substitute numerical values \((n = 1)\) and evaluate \(K\):

\[
K = \frac{(1)^2(1.5\, \text{m})^2(6.626 \times 10^{-34}\, \text{J} \cdot \text{s})^2}{2(9.109 \times 10^{-31}\, \text{kg})(54\, \text{nm})^2(0.68\, \text{mm})^2} \times \frac{1\, \text{eV}}{1.602 \times 10^{-19}\, \text{J}} = 2.5\, \text{keV}
\]

**69**  When a surface is illuminated by light of wavelength \(\lambda\), the maximum kinetic energy of the emitted electrons is 1.20 eV. If the wavelength \(\lambda' = 0.800\lambda\) is used, the maximum kinetic energy increases to 1.76 eV. For wavelength \(\lambda' = 0.600\lambda\), the maximum kinetic energy of the emitted electrons is 2.676 eV. Determine the work function of the surface and the wavelength \(\lambda\).

**Picture the Problem** The maximum kinetic energy of the photoelectrons is related to the frequency of the incident photons and the work function of the illuminated surface through the Einstein equation. We can apply this equation to either set of data and solve the resulting equations simultaneously for the work function of the surface and the wavelength of the incident photons.

Using the Einstein equation, relate the maximum kinetic energy of the emitted electrons to the frequency of the incident photons and the work function of the cathode material:

\[
K_{\text{max}} = hf - \phi = \frac{hc}{\lambda} - \phi
\]

Substitute numerical values for the light of wavelength \(\lambda\):

\[
1.20\, \text{eV} = \frac{hc}{\lambda} - \phi
\]

Substitute numerical values for the light of wavelength \(\lambda'\):

\[
1.76\, \text{eV} = \frac{hc}{\lambda'} - \phi = \frac{hc}{0.800\lambda} - \phi
\]

Solve these equations simultaneously for \(\phi\) to obtain:

\[
\phi = 1.04\, \text{eV}
\]

Substitute in either of the equations and solve for \(\lambda\):

\[
\lambda = 554\, \text{nm}
\]

**70**  A simple pendulum has a length equal to 1.0 m and has a bob that has a mass equal to 0.30 kg. The energy of this oscillator is quantized, and the allowed values of the energy are given by \(E_n = \left(n + \frac{1}{2}\right)hf_0\), where \(n\) is an integer and \(f_0\) is the frequency of the pendulum. (a) Find \(n\) if the angular amplitude is 1.0°. (b) Find \(n\) such that \(E_{n+1}\) exceeds \(E_n\) by 0.010 percent.
Picture the Problem The diagram shows the pendulum when its angular displacement is \( \theta \). The energy of the oscillator is equal to its initial potential energy \( mg \theta \). We can find \( n \) by equating this initial energy to \( E_n = \left( n + \frac{1}{2} \right) h f_0 \) and solving for \( n \).

\[(a) \text{ Express the nth-state energy as a function of the frequency of the pendulum:} \]

\[ E_n = \left( n + \frac{1}{2} \right) h f_0 \]

Because the small-amplitude frequency of the pendulum is given by \( f_0 = \frac{1}{2\pi} \sqrt{\frac{g}{L}} \):

\[ E_n = \left( n + \frac{1}{2} \right) \frac{h}{2\pi} \sqrt{\frac{g}{L}} \]

The energy of the pendulum is also equal to its initial gravitational potential energy:

\[ E_n = mgL(1 - \cos \theta) \]

Equating these expressions gives:

\[ mgL(1 - \cos \theta) = \left( n + \frac{1}{2} \right) \frac{h}{2\pi} \sqrt{\frac{g}{L}} \]

Solving for \( n \) gives:

\[ n = \frac{2\pi m \sqrt{gL^{3/2}(1 - \cos \theta)}}{h} - \frac{1}{2} \]

Substitute numerical values and evaluate \( n \):

\[ n = \frac{2\pi (0.30 \text{ kg}) \sqrt{9.81 \text{ m/s}^2 (1.0 \text{ m})^{3/2}(1 - \cos 1.0^\circ)}}{6.626 \times 10^{-34} \text{ J} \cdot \text{s}} - \frac{1}{2} \]

\[ = 1.357 \times 10^{30} = 1.4 \times 10^{30} \]

\[(b) \text{ Because } E_{n+1} \text{ exceeds } E_n \text{ by 0.010 percent:} \]

\[ \frac{E_{n+1} - E_n}{E_n} = 1.0 \times 10^{-4} \]
Substituting for $E_{n+1}$ and $E_n$ gives:

$$\frac{(n+1 + \frac{i}{2})hf_0 - (n + \frac{i}{2})hf_0}{(n + \frac{i}{2})hf_0} = 1.0 \times 10^{-4}$$

or

$$(n+1) + \frac{i}{2} - (n + \frac{i}{2}) = 1.0 \times 10^{-4}(n + \frac{i}{2})$$

Solving for $n$ yields:

$$n = 1.0 \times 10^4 - \frac{i}{2} = 1.0 \times 10^4$$

71  **  [SSM]  (a) Show that for large $n$, the fractional difference in energy between state $n$ and state $n + 1$ for a particle in a one-dimensional box is given approximately by $(E_{n+1} - E_n)/E_n \approx 2/n$  (b) What is the approximate percentage energy difference between the states $n_1 = 1000$ and $n_2 = 1001$?  (c) Comment on how this result is related to Bohr’s correspondence principle.

**Picture the Problem** We can use the fact that the energy of the $n$th state is related to the energy of the ground state according to $E_n = n^2E_1$, to express the fractional change in energy in terms of $n$ and then examine this ratio as $n$ grows without bound.

(a) Express the ratio $(E_{n+1} - E_n)/E_n$:

$$\frac{E_{n+1} - E_n}{E_n} = \frac{(n+1)^2 - n^2}{n^2} = \frac{2n+1}{n^2}$$

$$= \frac{2}{n} + \frac{1}{n^2} \approx \frac{2}{n}$$

for $n \gg 1$.

(b) Evaluate $\frac{E_{1001} - E_{1000}}{E_{1000}}$:

$$\frac{E_{1001} - E_{1000}}{E_{1000}} \approx \frac{2}{1000} = 0.2\%$$

(c) Classically, the energy is continuous. For very large values of $n$, the energy difference between adjacent levels is infinitesimal.

72  **  A mode-locked, titanium–sapphire laser has a wavelength of 850 nm and produces 100 million pulses of light each second. Each pulse has a duration of 125 femtoseconds ($1$ fs = $10^{-15}$ s) and consists of $5 \times 10^9$ photons. What is the average power produced by the laser?

**Picture the Problem** We can apply the definition of power in conjunction with the de Broglie equation for the energy of a photon to derive an expression for the average power produced by the laser.
The average power produced by the laser is:

\[ P = \frac{\Delta E}{\Delta t} \]

Use the de Broglie equation to express the energy of the emitted photons:

\[ \Delta E = Nhf = \frac{Nhc}{\lambda} \]

where \( N \) is number of photons in each pulse.

Substitute for \( \Delta E \) to obtain:

\[ P = \frac{Nhc}{\lambda \Delta t} \]

Substitute numerical values and evaluate \( P \):

\[ P = \frac{(5 \times 10^9) (6.626 \times 10^{-34} \text{ J} \cdot \text{s}) (2.998 \times 10^8 \text{ m/s})}{(850 \text{ nm})(10^{-8} \text{ s})} \approx 0.1 \text{ W} \]

**Remarks:** Note that the pulse length has no bearing on the solution.

73  ** This problem estimates the time lag in the photoelectric effect that is expected classically but not observed. Let the intensity of the incident radiation falling on an atom be 0.010 W/m². (a) If the area presented by the atom is 0.010 nm², find the energy per second falling on an atom. (b) If the work function is 2.0 eV, how long would it take for this much energy to fall on the atom if the radiation energy was distributed uniformly rather than in compact packets (photons)?

**Picture the Problem** We can find the rate at which energy is delivered to the atom using the definitions of power and intensity. We can also use the definition of power to determine how much time is required for an amount of energy equal to the work function to fall on one atom.

(a) Relate the energy per second (power) falling on an atom to the intensity of the incident radiation:

\[ P = \frac{\Delta E}{\Delta t} = IA \]

Substitute numerical values and evaluate \( P \):

\[ P = (0.010 \text{ W/m}^2)(0.010 \times 10^{-18} \text{ m}^2) \]

\[ = 10^{-22} \text{ J/s} \times \frac{1 \text{ eV}}{1.602 \times 10^{-19} \text{ J}} \]

\[ = 6.242 \times 10^{-4} \text{ eV/s} \]

\[ = \left[ 6.2 \times 10^{-4} \text{ eV/s} \right] \]
(b) Classically:

\[ \Delta t = \frac{\Delta E}{P} = \frac{\phi}{P} \]

Substitute numerical values and evaluate \( \Delta t \):

\[ \Delta t = \frac{2.0 \text{ eV}}{6.242 \times 10^{-4} \text{ eV/s}} = 3204 \text{ s} \]

\[ \approx 53 \text{ min} \]