Chapter 13
Fluids

Conceptual Problems

1. If the gauge pressure is doubled, the absolute pressure will be (a) halved, (b) doubled, (c) unchanged, (d) increased by a factor greater than 2, (e) increased by a factor less than 2.

Determine the Concept The absolute pressure is related to the gauge pressure according to $P = P_{\text{gauge}} + P_{\text{at}}$.

Prior to doubling the gauge pressure, the absolute pressure is given by:

$$P = P_{\text{gauge}} + P_{\text{at}} \Rightarrow P_{\text{gauge}} = P - P_{\text{at}}$$

Doubling the gauge pressure results in an absolute pressure given by:

$$P' = 2P_{\text{gauge}} + P_{\text{at}}$$

Substituting for $P_{\text{gauge}}$ yields:

$$P' = 2(P - P_{\text{at}}) + P_{\text{at}} = 2P - P_{\text{at}}$$

Because $P' = 2P - P_{\text{at}}$, doubling the gauge pressure results in an increase in the absolute pressure by a factor less than 2. [e] is correct.

2. Two spherical objects differ in size and mass. Object A has a mass that is eight times the mass of object B. The radius of object A is twice the radius of object B. How do their densities compare? (a) $\rho_A > \rho_B$, (b) $\rho_A < \rho_B$, (c) $\rho_A = \rho_B$, (d) not enough information is given to compare their densities.

Determine the Concept The density of an object is its mass per unit volume. The pictorial representation shown to the right summarizes the information concerning the radii and masses of the two spheres. We can determine the relationship between the densities of A and B by examining their ratio.

Express the densities of the two spheres:

$$\rho_A = \frac{m_A}{V_A} = \frac{m_A}{\frac{4}{3} \pi r_A^3}$$

and

$$\rho_B = \frac{m_B}{V_B} = \frac{m_B}{\frac{4}{3} \pi r_B^3}$$
Divide the first of these equations by the second and simplify to obtain:

\[ \rho_A = \frac{\frac{m_A}{\frac{4}{3} \pi r_A^3}}{\frac{m_B}{\frac{4}{3} \pi r_B^3}} = \frac{m_A r_B^3}{m_B r_A^3} \]

Substituting for the masses and radii and simplifying yields:

\[ \frac{8m_B}{m_B (2r)} = 1 \Rightarrow \rho_A = \rho_B \]

and (c) is correct.

Two objects differ in density and mass. Object A has a mass that is eight times the mass of object B. The density of object A is four times the density of object B. How do their volumes compare? (a) \( V_A = \frac{1}{4} V_B \), (b) \( V_A = V_B \), (c) \( V_A = 2 V_B \), (d) not enough information is given to compare their volumes.

**Determine the Concept** The density of an object is its mass per unit volume. We can determine the relationship between the volumes of A and B by examining their ratio.

Express the volumes of the two objects:

\[ V_A = \frac{m_A}{\rho_A} \text{ and } V_B = \frac{m_B}{\rho_B} \]

Divide the first of these equations by the second and simplify to obtain:

\[ \frac{V_A}{V_B} = \frac{\frac{m_A}{\rho_A}}{\frac{m_B}{\rho_B}} = \frac{\rho_B}{\rho_A} \frac{m_A}{m_B} \]

Substituting for the masses and densities and simplifying yields:

\[ \frac{V_A}{V_B} = \frac{\rho_B}{\rho_B} \frac{8m_B}{m_B} = 2 \Rightarrow V_A = 2V_B \]

and (c) is correct.

A sphere is constructed by gluing together two hemispheres of different density materials. The density of each hemisphere is uniform, but the density of one is greater than the density of the other. True or false: The average density of the sphere is the numerical average of the two different densities. Clearly explain your reasoning.

**Determine the Concept** True. This is a special case. Because the volumes are equal, the average density is the numerical average of the two densities. This is not a general result.

In several jungle adventure movies, the hero and heroine escape the bad guys by hiding underwater for extended periods of time. To do this, they
breathe through long vertical hollow reeds. Imagine that in one movie, the water is so clear that to be safely hidden the two are at a depth of 15 m. As a science consultant to the movie producers, you tell them that this is not a realistic depth and the knowledgeable viewer will laugh during this scene. Explain why this is so.

**Determine the Concept** Pressure increases approximately 1 atm every 10 m of depth. To breathe requires creating a pressure of less than 1 atm in your lungs. At the surface you can do this easily, but not at a depth of 10 m.

6. Two objects are balanced as in Figure 13-28. The objects have identical volumes but different masses. Assume all the objects in the figure are denser than water and thus none will float. Will the equilibrium be disturbed if the entire system is completely immersed in water? Explain your reasoning.

**Determine the Concept** Yes. Because the volumes of the two objects are equal, the downward force on each side is reduced by the same amount (the buoyant force acting on them) when they are submerged. The buoyant force is independent of their masses. That is, if \( m_1L_1 = m_2L_2 \) and \( L_1 \neq L_2 \), then \( (m_1 - c)L_1 \neq (m_2 - c)L_2 \).

7. A solid 200-g block of lead and a solid 200-g block of copper are completely submerged in an aquarium filled with water. Each block is suspended just above the bottom of the aquarium by a thread. Which of the following is true?
   (a) The buoyant force on the lead block is greater than the buoyant force on the copper block.
   (b) The buoyant force on the copper block is greater than the buoyant force on the lead block.
   (c) The buoyant force is the same on both blocks.
   (d) More information is needed to choose the correct answer.

**Determine the Concept** The buoyant forces acting on these submerged objects are equal to the weight of the water each displaces. The weight of the displaced water, in turn, is directly proportional to the volume of the submerged object. Because \( \rho_{\text{Pb}} > \rho_{\text{Cu}} \), the volume of the copper must be greater than that of the lead and, hence, the buoyant force on the copper is greater than that on the lead. 

\[ (b) \] is correct.

8. A 20-cm\(^3\) block of lead and a 20-cm\(^3\) block of copper are completely in an aquarium filled with water. Each is suspended just above the bottom of the aquarium by a thread. Which of the following is true?
   (a) The buoyant force on the lead block is greater than the buoyant force on the copper block.
The buoyant force on the copper block is greater than the buoyant force on the lead block.

The buoyant force is the same on both blocks.

More information is needed to choose the correct answer.

Determine the Concept The buoyant forces acting on these submerged objects are equal to the weight of the water each displaces. The weight of the displaced water, in turn, is directly proportional to the volume of the submerged object. Because their volumes are the same, the buoyant forces on them must be the same. (c) is correct.

Two bricks are completely submerged in water. Brick 1 is made of lead and has rectangular dimensions of 2” × 4” × 8”. Brick 2 is made of wood and has rectangular dimensions of 1” × 8” × 8”. True or false: The buoyant force on brick 2 is larger than the buoyant force on brick 1.

Determine the Concept False. The buoyant force on a submerged object depends on the weight of the displaced fluid which, in turn, depends on the volume of the displaced fluid. Because the bricks have the same volume, they will displace the same volume of water and the buoyant force will be the same on both of them.

Figure 13-29 shows an object called a “Cartesian diver.” The diver consists of a small tube, open at the bottom, with an air bubble at the top, inside a closed plastic soda bottle that is partly filled with water. The diver normally floats, but sinks when the bottle is squeezed hard. (a) Explain why this happens. (b) Explain the physics behind how a submarine can “silently” sink vertically simply by allowing water to flow into empty tanks near its keel. (c) Explain why a floating person will oscillate up and down on the water surface as he or she breathes in and out.

Determine the Concept

(a) When the bottle is squeezed, the force is transmitted equally through the fluid, leading to a pressure increase on the air bubble in the diver. The air bubble shrinks, and the loss in buoyancy is enough to sink the diver.

(b) As water enters its tanks, the weight of the submarine increases. When the submarine is completely submerged, the volume of the displaced water and, hence, the buoyant force acting on the submarine become constant. Because the weight of the submarine is now greater than the buoyant force acting on it, the submarine will start to sink.

(c) Breathing in lowers one’s average density and breathing out increases your average density. Because denser objects float lower on the surface than do less dense objects, a floating person will oscillate up and down on the water surface as he or she breathes in and out.
11. A certain object has a density just slightly less than that of water so that it floats almost completely submerged. However, the object is more compressible than water. What happens if the floating object is given a slight downward push? Explain.

**Determine the Concept** Because the pressure increases with depth, the object will be compressed and its density will increase as its volume decreases. Thus, the object will sink to the bottom.

12. In Example 13.11 the fluid is accelerated to a greater speed as it enters the narrow part of the pipe. Identify the forces that act on the fluid at the entrance to the narrow region to produce this acceleration.

**Determine the Concept** The acceleration-producing force acting on the fluid is the product of the difference in pressure between the wide and narrow parts of the pipe and the area of the narrow part of the pipe.

13. An upright glass of water is accelerating to the right along a flat, horizontal surface. What is the origin of the force that produces the acceleration on a small element of water in the middle of the glass? Explain by using a diagram. *Hint: The water surface will not remain level as long as the glass of water is accelerating. Draw a free body diagram of the small element of water.*

**Determine the Concept** The pictorial representation shows the glass and an element of water in the middle of the glass. As is readily established by a simple demonstration, the surface of the water is not level while the glass is accelerated, showing that there is a pressure gradient (a difference in pressure) due to the differing depths \((h_1 > h_2)\) of water on the two sides of the element of water. This pressure gradient results in a net force on the element as shown in the figure. The upward buoyant force is equal in magnitude to the downward gravitational force.
14. You are sitting in a boat floating on a very small pond. You take the anchor out of the boat and drop it into the water. Does the water level in the pond rise, fall, or remain the same? Explain your answer.

**Determine the Concept** The water level in the pond will fall slightly. When the anchor is in the boat, the boat displaces enough water so that the buoyant force on it equals the sum of the weight of the boat, your weight, and the weight of the anchor. When you drop the anchor into the water, it displaces just its volume of water (rather than its weight as it did while in the boat). The total weight of the boat becomes less and the boat displaces less water as a consequence.

15. A horizontal pipe narrows from a diameter of 10 cm at location A to 5.0 cm at location B. For a nonviscous incompressible fluid flowing without turbulence from location A to location B, how do the flow speeds \( v \) (in m/s) at the two locations compare? (a) \( v_A = v_B \), (b) \( v_A = \frac{1}{2} v_B \), (c) \( v_A = \frac{1}{4} v_B \), (d) \( v_A = 2 v_B \), (e) \( v_A = 4 v_B \)

**Determine the Concept** We can use the equation of continuity to compare the flow rates at the two locations.

Apply the equation of continuity at locations A and B to obtain:

\[ A_A v_A = A_B v_B \Rightarrow v_A = \frac{A_B}{A_A} v_B \]

Substitute for \( A_B \) and \( A_A \) and simplify to obtain:

\[ v_A = \frac{\frac{1}{4} \pi d_B^2}{\frac{1}{4} \pi d_A^2} v_B = \left( \frac{d_B}{d_A} \right)^2 v_B \]

Substitute numerical values and evaluate \( v_A \):

\[ v_A = \left( \frac{5 \text{ cm}}{10 \text{ cm}} \right)^2 v_B = \frac{1}{4} v_B \]

and (c) is correct.

16. A horizontal pipe narrows from a diameter of 10 cm at location A to 5.0 cm at location B. For a nonviscous incompressible fluid flowing without turbulence from location A to location B, how do the pressures \( P \) (in N/m\(^2\)) at the two locations compare? (a) \( P_A = P_B \), (b) \( P_A = \frac{1}{2} P_B \), (c) \( P_A = \frac{1}{4} P_B \), (d) \( P_A = 2 P_B \), (e) \( P_A = 4 P_B \), (f) There is not enough information to compare the pressures quantitatively.

**Determine the Concept** We can use the equation of continuity to compare the flow rates at the two locations.

Apply Bernoulli’s equation for constant elevations at locations A and B to obtain:

\[ P_A + \frac{1}{2} \rho v_A^2 = P_B + \frac{1}{2} \rho v_B^2 \quad (1) \]
Apply the equation of continuity at locations A and B to obtain:

\[ A_A v_A = A_B v_B \Rightarrow v_A = \frac{A_B}{A_A} v_B \]

Substitute for \( A_B \) and \( A_A \) and simplify to obtain:

\[ v_A = \frac{1}{2} \frac{m_B^2}{m_A^2} v_B = \left( \frac{d_B}{d_A} \right)^2 v_B \]

Substituting for \( v_A \) in equation (1) yields:

\[ P_A + \frac{1}{2} \rho \left( \frac{d_B}{d_A} \right)^2 v_B^2 = P_B + \frac{1}{2} \rho v_B^2 \]

While the values of \( d_B, d_A, \) and \( \rho \) are known to us, we need a value for \( v_B \) (or \( v_A \)) in order to compare \( P_A \) and \( P_B \). Hence (f) is correct.

17 [SSM] Figure 13-30 is a diagram of a prairie dog tunnel. The geometry of the two entrances are such that entrance 1 is surrounded by a mound and entrance 2 is surrounded by flat ground. Explain how the tunnel remains ventilated, and indicate in which direction air will flow through the tunnel.

**Determine the Concept** The mounding around entrance 1 will cause the streamlines to curve concave downward over the entrance. An upward pressure gradient produces the downward centripetal force. This means there is a lowering of the pressure at entrance 1. No such lowering occurs over entrance 2, so the pressure there is higher than the pressure at entrance 1. The air circulates in entrance 2 and out entrance 1. It has been demonstrated that enough air will circulate inside the tunnel even with the slightest breeze outside.

**Estimation and Approximation**

18 Your undergraduate research project involves atmospheric sampling. The sampling device has a mass of 25.0 kg. Estimate the diameter of a helium-filled balloon required to lift the device off the ground. Neglect the mass of the balloon "skin" and the small buoyancy force on the device itself.

**Picture the Problem** We can use Archimedes’ principle and the condition for vertical equilibrium to estimate the diameter of the helium-filled balloon that would just lift the sampling device.

Express the equilibrium condition that must be satisfied if the balloon-payload is to "just lift off":

\[ F_{\text{bouyant}} = B = F_{\text{gravitational}} = mg \quad (1) \]

Using Archimedes’ principle, express the buoyant force \( B \):

\[ B = \rho_{\text{air}} V_{\text{balloons}} g \]
Substituting for $B$ in equation (1) yields:

$$\rho_{\text{air}} V_{\text{balloon}} g = mg$$

or

$$\rho_{\text{air}} V_{\text{balloon}} = m \quad (2)$$

The total mass $m$ to be lifted is the sum of the mass of the payload $m_p$ and the mass of the helium $m_{\text{He}}$:

$$m = m_p + m_{\text{He}} = m_p + \rho_{\text{He}} V_{\text{balloon}}$$

Substitute for $m$ in equation (2) to obtain:

$$\rho_{\text{air}} V_{\text{balloon}} = m_p + \rho_{\text{He}} V_{\text{balloon}}$$

Solving for $V_{\text{balloon}}$ yields:

$$V_{\text{balloon}} = \frac{m_p}{\rho_{\text{air}} - \rho_{\text{He}}}$$

The volume of the balloon is given by:

$$V_{\text{balloon}} = \frac{4}{3} \pi r^3 = \frac{1}{6} \pi d^3$$

Substituting for $V_{\text{balloon}}$ yields:

$$\frac{1}{6} \pi d^3 = \frac{m_p}{\rho_{\text{air}} - \rho_{\text{He}}} \Rightarrow d = 3 \sqrt{\frac{6m_p}{\pi(\rho_{\text{air}} - \rho_{\text{He}})}}$$

Substitute numerical values and evaluate $d$:

$$d = 3 \sqrt{\frac{6(25.0 \text{ kg})}{\pi(1.293 \text{ kg/m}^3 - 0.179 \text{ kg/m}^3)}} = 3.50 \text{ m}$$

19 Your friend wants to start a business giving hot-air balloon rides. To the empty balloon, the basket and the occupants have a total maximum mass of 1000 kg. If the balloon itself has a diameter of 22.0 m when fully inflated with hot air, estimate the required density of the hot air. Neglect the buoyancy force on the basket and people.

**Picture the Problem** We can use Archimedes’ principle and the condition for vertical equilibrium to estimate the density of the hot air that would enable the balloon and its payload to lift off.

Express the equilibrium condition that must be satisfied if the balloon-payload is to "just lift off":

$$F_{\text{bouyant}} = B = F_{\text{gravitational}} = mg \quad (1)$$

Using Archimedes’ principle, express the buoyant force $B$:

$$B = w_{\text{displaced fluid}} = m_{\text{displaced fluid}} g = \rho_{\text{air}} V_{\text{balloon}} g$$

Substituting for $B$ in equation (1)

$$\rho_{\text{air}} V_{\text{balloon}} g = mg \Rightarrow \rho_{\text{air}} V_{\text{balloon}} = m \quad (2)$$
The total mass \( m \) to be lifted is the sum of the mass of the payload \( m_p \) and the mass of the hot air:

\[
m = m_p + m_{\text{hot air}} = m_p + \rho_{\text{hot air}} V_{\text{balloon}}
\]

Substitute for \( m \) in equation (2) to obtain:

\[
\rho_{\text{air}} V_{\text{balloon}} = m_p + \rho_{\text{hot air}} V_{\text{balloon}}
\]

Solving for \( \rho_{\text{hot air}} \) and simplifying yields:

\[
\rho_{\text{hot air}} = \frac{\rho_{\text{air}} V_{\text{balloon}} - m_p}{V_{\text{balloon}}} = \rho_{\text{air}} \frac{m_p}{V_{\text{balloon}}}
\]

The volume of the balloon is given by:

\[
V_{\text{balloon}} = \frac{4}{3} \pi r^3 = \frac{1}{5} \pi d^3
\]

Substituting for \( V_{\text{balloon}} \) yields:

\[
\rho_{\text{hot air}} = \rho_{\text{air}} \frac{m_p}{\frac{4}{3} \pi d^3} = \rho_{\text{air}} \frac{6 m_p}{\pi d^3}
\]

Substitute numerical values and evaluate \( \rho_{\text{hot air}} \):

\[
\rho_{\text{hot air}} = 1.293 \text{ kg/m}^3 - \frac{6(1000 \text{ kg})}{\pi (22.0 \text{ m})^3} = 1.11 \text{ kg/m}^3
\]

Remarks: As expected, the density of the hot air is considerably less than the density of the surrounding cooler air.

Density

20 • Find the mass of a solid lead sphere with a radius equal to 2.00 cm.

Picture the Problem The mass of the sphere is the product of its density and volume. The density of lead can be found in Figure 13-1.

Using the definition of density, express the mass of the sphere:

\[
m = \rho V = \rho \left( \frac{4}{3} \pi R^3 \right)
\]

Substitute numerical values and evaluate \( m \):

\[
m = \frac{4}{3} \pi (11.3 \times 10^3 \text{ kg/m}^3)(2.00 \times 10^{-2} \text{ m})^3 = 0.379 \text{ kg}
\]

21 • [SSM] Consider a room measuring 4.0 m \( \times \) 5.0 m \( \times \) 4.0 m. Under normal atmospheric conditions at Earth’s surface, what would be the mass of the air in the room?

Picture the Problem The mass of the air in the room is the product of its density...
and volume. The density of air can be found in Figure 13-1.

Use the definition of density to express the mass of the air in the room:

\[ m = \rho V = \rho LWH \]

Substitute numerical values and evaluate \( m \):

\[ m = (1.293 \text{ kg/m}^3)(4.0 \text{ m})(5.0 \text{ m})(4.0 \text{ m}) = 1.0 \times 10^2 \text{ kg} \]

22  ●  An average neutron star has approximately the same mass as the Sun, but is compressed into a sphere of radius roughly 10 km. What would be the approximate mass of a teaspoonful of matter that dense?

**Picture the Problem** We can use the definition of density to find the approximate mass of a teaspoonful of matter from a neutron star. Assume that the volume of a teaspoon is about 5 mL.

Use the definition of density to express the mass of a teaspoonful of matter whose density is \( \rho \):

\[ m = \rho V_{\text{teaspoon}} \quad (1) \]

The density of the neutron star is given by:

\[ \rho_{\text{NS}} = \frac{m_{\text{NS}}}{V_{\text{NS}}} \]

Substituting for \( m_{\text{NS}} \) and \( V_{\text{NS}} \) yields:

\[ \rho_{\text{NS}} = \frac{m_{\text{Sun}}}{\frac{4}{3} \pi r_{\text{NS}}^3} \]

Let \( \rho = \rho_{\text{NS}} \) in equation (1) to obtain:

\[ m = \frac{3m_{\text{Sun}}V_{\text{teaspoon}}}{4\pi r_{\text{NS}}^3} \]

Substitute numerical values and evaluate \( m \):

\[ m = \frac{3(1.99 \times 10^{30} \text{ kg})(5 \times 10^{-6} \text{ m}^3)}{4\pi (10 \times 10^3 \text{ m}^3)} \approx 2 \text{ Tg} \]

23  ●  A 50.0-g ball consists of a plastic spherical shell and a water-filled core. The shell has an outside diameter equal to 50.0 mm and an inside diameter equal to 20.0 mm. What is the density of the plastic?

**Picture the Problem** We can use the definition of density to find the density of the plastic of which the spherical shell is constructed.
The density of the plastic is given by:

\[ \rho_{\text{plastic}} = \frac{m_{\text{plastic}}}{V_{\text{plastic}}} \]  \hspace{1cm} (1)

The mass of the plastic is the difference between the mass of the ball and the mass of its water-filled core:

\[ m_{\text{plastic}} = m_{\text{ball}} - m_{\text{water}} \]

Use the definition of density to express the mass of the water:

\[ m_{\text{water}} = \rho_{\text{water}} V_{\text{water}} \]

Substituting for \( m_{\text{water}} \) yields:

\[ m_{\text{plastic}} = m_{\text{ball}} - \rho_{\text{water}} V_{\text{water}} \]

Substitute for \( m_{\text{plastic}} \) in equation (1) to obtain:

\[ \rho_{\text{plastic}} = \frac{m_{\text{ball}} - \rho_{\text{water}} V_{\text{water}}}{V_{\text{plastic}}} \]

Because \( V_{\text{water}} = \frac{4}{3} \pi R_{\text{inside}}^3 \) and \( V_{\text{ball}} = \frac{4}{3} \pi (R_{\text{outside}}^3 - R_{\text{inside}}^3) \):

\[ \rho_{\text{plastic}} = \frac{m_{\text{ball}} - \rho_{\text{water}} \left(\frac{4}{3} \pi R_{\text{inside}}^3\right)}{\frac{4}{3} \pi (R_{\text{outside}}^3 - R_{\text{inside}}^3)} \]

Substitute numerical values and evaluate \( \rho_{\text{plastic}} \):

\[ \rho_{\text{plastic}} = \frac{50.0 \text{ g} - \left(1.00 \text{ g/cm}^3\right)\left(\frac{4}{3} \pi (20.0 \text{ mm})^3\right)}{\frac{4}{3} \pi (50.0 \text{ mm})^3 - (20.0 \text{ mm})^3} = 33.6 \text{ kg/m}^3 \]

24 A 60.0-mL flask is filled with mercury at 0°C (Figure 13-31). When the temperature rises to 80°C, 1.47 g of mercury spills out of the flask. Assuming that the volume of the flask stays constant, find the change in density of mercury at 80°C if its density at 0°C is 13 645 kg/m³.

**Picture the Problem** We can use the definition of density to relate the change in the density of the mercury to the amount spilled during the heating process.

The change in the density of the mercury as it is warmed is given by:

\[ \Delta \rho = \rho_0 - \rho \]  \hspace{1cm} (1)

where \( \rho_0 \) is the density of the mercury before it is warmed.

The density of the mercury before it is warmed is the ratio of its mass to the volume it occupies:

\[ \rho_0 = \frac{m_0}{V_0} \]  \hspace{1cm} (2)
The volume of the mercury that spills is the ratio of its mass to its density at the higher temperature:

\[ V_{\text{spilled}} = \frac{m_{\text{spilled}}}{\rho} \]

The density \( \rho \) of the mercury at the higher temperature is given by:

\[ \rho = \frac{m_0}{V_0 + V_{\text{spilled}}} = \frac{m_0}{V_0 + \frac{m_{\text{spilled}}}{\rho}} \]

Solving for \( \rho \) yields:

\[ \rho = \frac{m_0 - m_{\text{spilled}}}{V_0} = \rho_0 - \frac{m_{\text{spilled}}}{V_0} \quad (3) \]

Substituting equations (2) and (3) in equation (1) yields:

\[ \Delta \rho = \rho_0 - \left( \rho_0 - \frac{m_{\text{spilled}}}{V_0} \right) = \frac{m_{\text{spilled}}}{V_0} \]

Substitute numerical values and evaluate \( \Delta \rho \):

\[ \Delta \rho = \frac{1.47 \times 10^{-3} \text{ kg}}{60.0 \times 10^{-6} \text{ m}^3} = 24.5 \text{ kg/m}^3 \]

25 ** One sphere is made of gold and has a radius \( r_{\text{Au}} \) and another sphere is made of copper and has a radius \( r_{\text{Cu}} \). If the spheres have equal mass, what is the ratio of the radii, \( r_{\text{Au}}/r_{\text{Cu}} \)?

**Picture the Problem** We can use the definition of density to find the ratio of the radii of the two spheres. See Table 13-1 for the densities of gold and copper.

Use the definition of density to express the mass of the gold sphere:

\[ m_{\text{Au}} = \rho_{\text{Au}} V_{\text{Au}} = \frac{4}{3} \pi \rho_{\text{Au}} r_{\text{Au}}^3 \]

The mass of the copper sphere is given by:

\[ m_{\text{Cu}} = \rho_{\text{Cu}} V_{\text{Cu}} = \frac{4}{3} \pi \rho_{\text{Cu}} r_{\text{Cu}}^3 \]

Dividing the first of these equations by the second and simplifying yields:

\[ \frac{m_{\text{Au}}}{m_{\text{Cu}}} = \frac{4}{3} \pi \rho_{\text{Au}} \frac{r_{\text{Au}}^3}{\rho_{\text{Cu}} \rho_{\text{Cu}}} = \frac{\rho_{\text{Au}}}{\rho_{\text{Cu}}} \left( \frac{r_{\text{Au}}}{r_{\text{Cu}}} \right)^3 \]

Solve for \( r_{\text{Au}}/r_{\text{Cu}} \) to obtain:

\[ r_{\text{Au}} = \sqrt[3]{\frac{m_{\text{Au}} \rho_{\text{Cu}}}{m_{\text{Cu}} \rho_{\text{Au}}}} \]

Because the spheres have the same mass:

\[ r_{\text{Au}} = \sqrt[3]{\frac{\rho_{\text{Cu}}}{\rho_{\text{Au}}}} \]
Substitute numerical values and evaluate \( r_{Au}/r_{Cu} \):

\[
\frac{r_{Au}}{r_{Cu}} = \frac{8.93 \text{ kg/m}^3}{19.3 \text{ kg/m}^3} = 0.773
\]

26 Since 1983, the US Mint has made pennies out of zinc with a copper cladding. The mass of these pennies is 2.50 g. Model the penny as a uniform cylinder of height 1.23 mm and radius 9.50 mm. Assume the copper cladding is uniformly thick on all surfaces. If the density of zinc is 7140 kg/m\(^3\) and that of copper is 8930 kg/m\(^3\), what is the thickness of the copper cladding?

**Picture the Problem** The pictorial representation shows a zinc penny with its copper cladding. We can use the definition of density to relate the difference between the mass of an all-copper penny and the mass of a copper-zinc penny to the thickness \( d \) of the copper cladding.

Express the difference in mass between an all-copper penny and an all-zinc penny:

\[ \Delta m = m_{Cu \ penny} - m_{Zn \ penny} \]

In terms of the densities of copper and zinc, our equation becomes:

\[ \Delta m = \rho_{Cu \ penny} V_{Cu \ penny} - \rho_{Zn \ penny} V_{Zn \ penny} \]

Assuming that the thickness \( d \) of cladding is very small compared to the height \( h \) and radius \( r \) of the pennies:

\[ \Delta m \approx (\rho_{Cu \ penny} - \rho_{Zn \ penny}) V_{Cu \ penny} \]

Substituting for \( V_{Cu \ penny} \) yields:

\[ \Delta m \approx (\rho_{Cu \ penny} - \rho_{Zn \ penny}) d (2\pi r^2 + 2\pi rh) \]

Solve for \( d \) and simplify to obtain:

\[
\begin{align*}
d &= \frac{m_{Cu \ penny} - m_{Zn \ penny}}{2\pi (\rho_{Cu \ penny} - \rho_{Zn \ penny}) (r + h)} \\
&= \frac{m_{Cu \ penny} - \rho_{Zn \ penny} V_{Zn \ penny}}{2\pi (\rho_{Cu \ penny} - \rho_{Zn \ penny}) (r + h)} \\
&= \frac{m_{Cu \ penny} - \rho_{Zn \ penny} \pi r^2 h}{2\pi (\rho_{Cu \ penny} - \rho_{Zn \ penny}) (r + h)}
\end{align*}
\]
Substitute numerical values and evaluate $d$:

$$
d = \frac{2.50 \times 10^{-3} \text{ kg} - \pi (7140 \text{ kg/m}^3)(9.50 \text{ mm})^2 (1.23 \text{ mm})}{2\pi (9.50 \text{ mm})(8930 \text{ kg/m}^3 - 7140 \text{ kg/m}^3)(9.50 \text{ mm} + 1.23 \text{ mm})} \approx 9 \mu\text{m}
$$

**Remarks:** This small value for $d$ (approximately $9 \mu\text{m}$) justifies our assumptions that $d$ is much smaller than both $r$ and $h$.

**Pressure**

27 • Barometer readings are commonly given in inches of mercury (inHg). Find the pressure in inches of mercury equal to 101 kPa.

**Picture the Problem** The pressure due to a column of height $h$ of a liquid of density $\rho$ is given by $P = \rho gh$.

Letting $h$ represent the height of the column of mercury, express the pressure at its base:

$$\rho_{\text{Hg}} gh = 101 \text{ kPa} \Rightarrow h = \frac{101 \text{ kPa}}{\rho_{\text{Hg}} g}$$

Substitute numerical values and evaluate $h$:

$$h = \frac{101 \text{ kPa}}{(13.6 \times 10^3 \text{ kg/m}^3)(9.81 \text{ m/s}^2)}
= 0.7570 \text{ m} \times \frac{1 \text{ in}}{2.54 \times 10^{-2} \text{ m}}
= 29.8 \text{ inHg}$$

28 • The pressure on the surface of a lake is $P_{at} = 101 \text{ kPa}$. (a) At what depth is the pressure twice atmospheric pressure? (b) If the pressure at the top of a deep pool of mercury is $P_{at}$, at what depth is the pressure $2P_{at}$?

**Picture the Problem** The pressure due to a column of height $h$ of a liquid of density $\rho$ is given by $P = \rho gh$. The pressure at any depth in a liquid is the sum of the pressure at the surface of the liquid and the pressure due to the liquid at a given depth in the liquid.

(a) Express the pressure as a function of depth in the lake:

$$P = P_{at} + \rho_{\text{water}} gh \Rightarrow h = \frac{P - P_{at}}{\rho_{\text{water}} g}$$

Substitute for $P$ and simplify to obtain:

$$h = \frac{2P_{at} - P_{at}}{\rho_{\text{water}} g} = \frac{P_{at}}{\rho_{\text{water}} g}$$
Substitute numerical values and evaluate $h$:

$$h = \frac{101\text{kPa}}{\left(1.00 \times 10^3 \text{ kg/m}^3\right)\left(9.81 \text{ m/s}^2\right)} = \boxed{10.3 \text{ m}}$$

(b) Proceed as in (a) with $\rho_{\text{water}}$ replaced by $\rho_{\text{Hg}}$ to obtain:

$$h = \frac{2P_\text{at} - P_\text{at}}{\rho_{\text{Hg}}g} = \frac{P_\text{at}}{\rho_{\text{Hg}}g}$$

Substitute numerical values and evaluate $h$:

$$h = \frac{101\text{kPa}}{\left(13.6 \times 10^3 \text{ kg/m}^3\right)\left(9.81 \text{ m/s}^2\right)} = \boxed{75.7 \text{ cm}}$$

When at cruising altitude, a typical airplane cabin will have an air pressure equivalent to an altitude of about 2400 m. During the flight, ears often equilibrate, so that the air pressure inside the inner ear equalizes with the air pressure outside the plane. The Eustachian tubes allow for this equalization, but can become clogged. If an Eustachian tube is clogged, pressure equalization may not occur on descent and the air pressure inside an inner ear may remain equal to the pressure at 2400 m. In that case, by the time the plane lands and the cabin is repressurized to sea-level air pressure, what is the net force on one ear drum, due to this pressure difference, assuming the ear drum has an area of 0.50 cm$^2$?

**Picture the Problem**
Assuming the density of the air to be constant, we can use the definition of pressure and the expression for the variation of pressure with depth in a fluid to find the net force on one's ear drums.

The net force acting on one ear drum is given by:

$$F = \Delta PA$$

where $A$ is the area of an ear drum.

Relate the pressure difference to the pressure at 2400 m and the pressure at sea level:

$$\Delta P = P_{\text{sea level}} - P_{2400 \text{ m}} = \rho g \Delta h$$

Substituting for $\Delta P$ in the expression for $F$ yields:

$$F = \rho g \Delta h A$$

Substitute numerical values and evaluate $F$:

$$F = \left(1.293 \text{ kg/m}^3\right)\left(9.81 \text{ m/s}^2\right)(2400 \text{ m})(0.50 \text{ cm}^2) = 1.5 \text{ N}$$

The axis of a cylindrical container is vertical. The container is filled with equal masses of water and oil. The oil floats on top of the water, and the open surface of the oil is at a height $h$ above the bottom of the container. What is
the height $h$, if the pressure at the bottom of the water is 10 kPa greater than the pressure at the top of the oil? Assume the oil density is 875 kg/m$^3$.

**Picture the Problem** The pictorial representation shows oil floating on water in a cylindrical container that is open to the atmosphere. We can express the height of the cylinder as the sum of the heights of the cylinders of oil and water and then use the fact that the masses of oil and water are equal to obtain a relationship between $h_{\text{oil}}$ and $h_{\text{water}}$. This relationship, together with the equation for the pressure as function of depth in a liquid will lead us to expressions for $h_{\text{oil}}$ and $h_{\text{water}}$.

Express $h$ as the sum of the heights of the cylinders of oil and water:

$$h = h_{\text{oil}} + h_{\text{water}} \quad (1)$$

The difference in pressure between the bottom of the water and the top of oil is:

$$\Delta P = P_{\text{bottom of water}} - P_{\text{at}}$$

$$= P_{\text{bottom of oil}} + \rho_{\text{water}} gh_{\text{water}} - P_{\text{at}}$$

Because $P_{\text{bottom of oil}} = P_{\text{at}} + \rho_{\text{oil}} gh_{\text{oil}}$:

$$\Delta P = P_{\text{at}} + \rho_{\text{oil}} gh_{\text{oil}} + \rho_{\text{water}} gh_{\text{water}} - P_{\text{at}}$$

Express the volumes of oil and water in the cylinder:

$$V_{\text{oil}} = \frac{m_{\text{oil}}}{\rho_{\text{oil}}} = \pi r^2 h_{\text{oil}}$$

and

$$V_{\text{water}} = \frac{m_{\text{water}}}{\rho_{\text{water}}} = \pi r^2 h_{\text{water}}$$

Dividing the first of these equations by the second yields:

$$\frac{m_{\text{oil}}}{\rho_{\text{oil}}} = \frac{\pi r^2 h_{\text{oil}}}{\rho_{\text{oil}}} = \frac{h_{\text{oil}}}{h_{\text{water}}}$$

Because $m_{\text{oil}} = m_{\text{water}}$:

$$\frac{h_{\text{oil}}}{h_{\text{water}}} = \frac{\rho_{\text{water}}}{\rho_{\text{oil}}} \Rightarrow h_{\text{oil}} = \frac{\rho_{\text{water}}}{\rho_{\text{oil}}} h_{\text{water}} \quad (2)$$
Substitute for \( h_{\text{oil}} \) in the expression for \( \Delta P \) to obtain:

\[
\Delta P = \rho_{\text{oil}} g \left( \frac{\rho_{\text{water}}}{\rho_{\text{oil}}} h_{\text{water}} \right) + \rho_{\text{water}} gh_{\text{water}}
\]

\[
= \rho_{\text{water}} gh_{\text{water}} + \rho_{\text{water}} gh_{\text{water}}
\]

\[
= 2 \rho_{\text{water}} gh_{\text{water}}
\]

Solving for \( h_{\text{water}} \) yields:

\[
h_{\text{water}} = \frac{\Delta P}{2 \rho_{\text{water}} g}
\]

Solve equation (2) for \( h_{\text{water}} \) to obtain:

\[
h_{\text{water}} = \frac{\rho_{\text{oil}}}{\rho_{\text{water}}} h_{\text{oil}}
\]

Substitute for \( h_{\text{water}} \) in the expression for \( \Delta P \) to obtain:

\[
\Delta P = \rho_{\text{oil}} gh_{\text{oil}} + \rho_{\text{water}} g \left( \frac{\rho_{\text{oil}}}{\rho_{\text{water}}} h_{\text{oil}} \right)
\]

\[
= 2 \rho_{\text{oil}} gh_{\text{oil}}
\]

Solving for \( h_{\text{oil}} \) yields:

\[
h_{\text{oil}} = \frac{\Delta P}{2 \rho_{\text{oil}} g}
\]

Substituting for \( h_{\text{oil}} \) and \( h_{\text{water}} \) in equation (1) yields:

\[
h = \frac{\Delta P}{2 \rho_{\text{oil}} g} + \frac{\Delta P}{2 \rho_{\text{water}} g}
\]

\[
= \left( \frac{1}{\rho_{\text{oil}}} + \frac{1}{\rho_{\text{water}}} \right) \frac{\Delta P}{2g}
\]

Substitute numerical values and evaluate \( h \):

\[
h = \left( \frac{1}{875 \ \text{kg/m}^3} + \frac{1}{1.00 \times 10^3 \ \text{kg/m}^3} \right) \frac{10 \ \text{kPa}}{2(9.81 \ \text{m/s}^2)} = 1.1 \ \text{m}
\]

**31 \cdot [SSM]** A hydraulic lift is used to raise an automobile of mass 1500 kg. The radius of the shaft of the lift is 8.00 cm and that of the piston is 1.00 cm. How much force must be applied to the compressor’s piston to raise the automobile? **Hint:** The shaft of the lift is the other piston.

**Picture the Problem** The pressure applied to an enclosed liquid is transmitted undiminished to every point in the fluid and to the walls of the container. Hence we can equate the pressure produced by the force applied to the piston to the pressure due to the weight of the automobile and solve for \( F \).
Express the pressure the weight of the automobile exerts on the shaft of the lift:

\[ P_{\text{auto}} = \frac{W_{\text{auto}}}{A_{\text{shaft}}} \]

Express the pressure the force applied to the piston produces:

\[ P = \frac{F}{A_{\text{piston}}} \]

Because the pressures are the same, we can equate them to obtain:

\[ \frac{W_{\text{auto}}}{A_{\text{shaft}}} = \frac{F}{A_{\text{piston}}} \]

Solving for \( F \) yields:

\[ F = \frac{W_{\text{auto}} A_{\text{piston}}}{A_{\text{shaft}}} = m_{\text{auto}} g \frac{A_{\text{piston}}}{A_{\text{shaft}}} \]

Substitute numerical values and evaluate \( F \):

\[ F = \left(1500 \text{ kg}\right) \left(9.81 \text{ m/s}^2\right) \left(\frac{1.00 \text{ cm}}{8.00 \text{ cm}}\right)^2 = \left[230 \text{ N}\right] \]

32. A 1500-kg car rests on four tires, each of which is inflated to a gauge pressure of 200 kPa. If the four tires support the car’s weight equally, what is the area of contact of each tire with the road?

**Picture the Problem** The area of contact of each tire with the road is related to the weight on each tire and the pressure in the tire through the definition of pressure.

Using the definition of gauge pressure, relate the area of contact to the pressure and the weight of the car:

\[ A = \frac{\frac{1}{4} w}{P_{\text{gauge}}} = \frac{mg}{4P_{\text{gauge}}} \]

Substitute numerical values and evaluate \( A \):

\[ A = \frac{(1500 \text{ kg})(9.81 \text{ m/s}^2)}{4(200 \text{ kPa})} = \left[184 \text{ cm}^2\right] \]

33. What pressure increase is required to compress the volume of 1.00 kg of water from 1.00 L to 0.99 L? Could this compression occur in our oceans where the maximum depth is about 11 km? Explain.
**Picture the Problem** The required pressure $\Delta P$ is related to the change in volume $\Delta V$ and the initial volume $V$ through the definition of the bulk modulus $B$:

$$B = -\frac{\Delta P}{\Delta V/V}.$$ 

Using the definition of the bulk modulus, relate the change in volume to the initial volume and the required pressure:

$$B = -\frac{\Delta P}{\Delta V/V} \Rightarrow \Delta P = -B \frac{\Delta V}{V}.$$ 

Substitute numerical values and evaluate $\Delta P$:

$$\Delta P = -2.00 \times 10^6 \text{ Pa} \times \left(\frac{-0.01 \text{ L}}{1.00 \text{ L}}\right)$$

$$= 2.00 \times 10^7 \text{ Pa} \times \frac{1 \text{ atm}}{101.325 \text{ kPa}}$$

$$= 197.4 \text{ atm} = 197 \text{ atm}$$

Express the pressure at a depth $h$ in the ocean:

$$P(h) = P_{\text{at}} + \rho_{\text{seawater}} gh$$

or

$$\Delta P = \rho_{\text{seawater}} gh \Rightarrow h = \frac{\Delta P}{\rho_{\text{seawater}} g}.$$ 

Substitute numerical values and evaluate $h$:

$$h = \frac{197.4 \text{ atm} \times 101.325 \text{ kPa}}{1025 \text{ kg/m}^3 \times 9.81 \text{ m/s}^2} \approx 2.0 \text{ km}$$

Because the depth of 2 km required to produce the given compression is considerably less than 11 km, this compression occurs in our oceans.

34 When a woman in high-heeled shoes takes a step, she momentarily places her entire weight on one heel of her shoe. If her mass is 56.0 kg and if the area of the heel is 1.00 cm$^2$, what is the pressure exerted on the floor by the heel? Compare your answer to the pressure exerted by the one foot of an elephant on a flat floor. Assume the elephant’s mass is 5000 kg, that he has all four feet equally distributed on the floor, and that each foot has an area of 400 cm$^2$.

**Picture the Problem** The pressure exerted by the woman’s heel on the floor is her weight divided by the area of her heel.

Using its definition, express the pressure exerted on the floor by the woman’s heel:

$$P = \frac{F}{A} = \frac{w}{A} = \frac{mg}{A}.$$
Substitute numerical values and evaluate $P_{\text{woman}}$:

$$P_{\text{woman}} = \left( \frac{56.0 \text{ kg}}{} \right) \left( 9.81 \text{ m/s}^2 \right) \left( 1.00 \times 10^{-4} \text{ m}^2 \right)$$

$$= 5.49 \times 10^6 \text{ N/m}^2 \times \frac{1 \text{ atm}}{101.325 \text{ kPa}}$$

$$= 54.2 \text{ atm}$$

Express the pressure exerted by one of the elephant’s feet:

$$P_{\text{elephant}} = \frac{1}{4} \left( \frac{5000 \text{ kg}}{} \right) \left( 9.81 \text{ m/s}^2 \right) \left( 400 \times 10^{-4} \text{ m}^2 \right)$$

$$= 3.066 \times 10^3 \text{ N/m}^2 \times \frac{1 \text{ atm}}{101.325 \text{ kPa}}$$

$$= 3.03 \text{ atm}$$

Express the ratio of the pressure exerted by the woman’s heel to the pressure exerted by the elephant’s foot:

$$\frac{P_{\text{woman}}}{P_{\text{elephant}}} = \frac{54.2 \text{ atm}}{3.03 \text{ atm}} \approx 18$$

Thus $P_{\text{woman}} \approx 18P_{\text{elephant}}$.

35 In the seventeenth century, Blaise Pascal performed the experiment shown in Figure 13-32. A wine barrel filled with water was coupled to a long tube. Water was added to the tube until the barrel burst. The radius of the lid was 20 cm and the height of the water in the tube was 12 m. (a) Calculate the force exerted on the lid due to the pressure increase. (b) If the tube had an inner radius of 3.0 mm, what mass of water in the tube caused the pressure that burst the barrel?

**Picture the Problem** The force on the lid is related to pressure exerted by the water and the cross-sectional area of the column of water through the definition of density. We can find the mass of the water from the product of its density and volume.

(a) Using the definition of pressure, express the force exerted on the lid:

$$F = PA$$

Express the pressure due to a column of water of height $h$:

$$P = \rho_{\text{water}} gh$$

Substitute for $P$ and $A$ to obtain:

$$F = \rho_{\text{water}} gh \pi r^2$$

Substitute numerical values and evaluate $F$:

$$F = \left( 1.00 \times 10^3 \text{ kg/m}^3 \right) \left( 9.81 \text{ m/s}^2 \right) \times (12 \text{ m}) \pi (0.20 \text{ m})^2$$

$$= 15 \text{ kN}$$
(b) Relate the mass of the water to its density and volume:

\[ m = \rho_{\text{water}} V = \rho_{\text{water}} \pi r^2 \]

Substitute numerical values and evaluate \( m \):

\[ m = (1.00 \times 10^3 \text{ kg/m}^3)(12 \text{ m})\pi (3.0 \times 10^{-3} \text{ m})^2 = 0.34 \text{ kg} \]

36. Blood plasma flows from a bag through a tube into a patient’s vein, where the blood pressure is 12 mm-Hg. The specific gravity of blood plasma at 37°C is 1.03. What is the minimum elevation the bag must have so the plasma flows into the vein?

**Picture the Problem** The minimum elevation of the bag \( h \) that will produce a pressure of at least 12 mm-Hg is related to this pressure and the density of the blood plasma through \( P = \rho_{\text{blood}} gh \).

Using the definition of the pressure due to a column of liquid, relate the pressure at its base to its height:

\[ P = \rho_{\text{blood}} gh \Rightarrow h = \frac{P}{\rho_{\text{blood}} g} \]

Substitute numerical values and evaluate \( h \):

\[ h = \frac{12 \text{ mm-Hg} \times 133.32 \text{ Pa}}{(1.03 \times 10^3 \text{ kg/m}^3)(9.81 \text{ m/s}^2)} \]
\[ = 16 \text{ cm} \]

37. Many people have imagined that if they were to float the top of a flexible snorkel tube out of the water, they would be able to breathe through it while walking underwater (Figure 13-33). However, they generally do not take into account just how much water pressure opposes the expansion of the chest and the inflation of the lungs. Suppose you can just breathe while lying on the floor with a 400-N (90-lb) weight on your chest. How far below the surface of the water could your chest be for you still to be able to breathe, assuming your chest has a frontal area of 0.090 m²?

**Picture the Problem** The depth \( h \) below the surface at which you would be able to breathe is related to the pressure at that depth and the density of water \( \rho_w \) through \( P = \rho_w gh \).

Express the pressure due to a column of water of height \( h \):

\[ P = \rho_w gh \Rightarrow h = \frac{P}{\rho_w g} \]
Express the pressure at depth \( h \) in terms of the weight on your chest:

\[
P = \frac{W_{\text{on your chest}}}{A_{\text{of your chest}}}
\]

Substituting for \( P \) yields:

\[
h = \frac{W_{\text{on your chest}}}{A_{\text{of your chest}} \rho w g}
\]

Substitute numerical values and evaluate \( h \):

\[
h = \frac{400 \text{ N}}{0.090 \text{ m}^2 (1.00 \times 10^3 \text{ kg/m}^3)(9.81 \text{ m/s}^2)} = 45 \text{ cm}
\]

38  In Example 13-3 a 150-N force is applied to a small piston to lift a car that weighs 15 000 N. Demonstrate that this does not violate the law of conservation of energy by showing that, when the car is lifted some distance \( h \), the work done by the 150-N force acting on the small piston equals the work done on the car by the large piston.

**Picture the Problem** Let \( A_1 \) and \( A_2 \) represent the cross-sectional areas of the large piston and the small piston, and \( F_1 \) and \( F_2 \) the forces exerted by the large and on the small piston, respectively. The work done by the large piston is \( W_1 = F_1 h_1 \) and that done on the small piston is \( W_2 = F_2 h_2 \). We’ll use Pascal’s principle and the equality of the volume of the displaced liquid in both pistons to show that \( W_1 \) and \( W_2 \) are equal.

Express the work done in lifting the car a distance \( h \):

\[
W_1 = F_1 h_1
\]

where \( F_1 \) is the weight of the car.

Using the definition of pressure, relate the forces \( F_1 (= w) \) and \( F_2 \) to the areas \( A_1 \) and \( A_2 \):

\[
\frac{F_1}{A_1} = \frac{F_2}{A_2} \Rightarrow F_1 = F_2 \frac{A_1}{A_2}
\]

Equate the volumes of the displaced fluid in the two pistons:

\[
h_1 A_1 = h_2 A_2 \Rightarrow h_1 = h_2 \frac{A_2}{A_1}
\]

Substitute in the expression for \( W_1 \) and simplify to obtain:

\[
W_1 = F_2 A_2 h_2 \frac{A_2}{A_1} = F_2 h_2 = [W_2]
\]

39  A 5.00-kg lead sinker is accidentally dropped overboard by fishermen in a boat directly above the deepest portion of the Marianas trench, near the Philippines. By what percentage does the volume of the sinker change by the time it settles to the trench bottom, which is 10.9 km below the surface?
**Picture the Problem** This problem is an application of the definition of the bulk modulus. The change in volume of the sinker is related to the pressure change and the bulk modulus by

\[ B = -\frac{\Delta P}{\Delta V/V}. \]

From the definition of bulk modulus:

\[ \frac{\Delta V}{V} = -\frac{\Delta P}{B}. \]  \hspace{1cm} (1)

Express the pressure at a depth \( h \) in the ocean:

\[ P(h) = P_{atm} + \rho gh. \]

The difference in pressure between the pressure at depth \( h \) and the surface of the water is:

\[ \Delta P = P(h) - P_{atm} = \rho gh. \]

Substitute for \( \Delta P \) in equation (1) to obtain:

\[ \frac{\Delta V}{V} = -\frac{\rho gh}{B}. \]

Substitute numerical values and evaluate the fractional change in volume of the sinker:

\[ \Delta V = -\frac{(1025 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(10.9 \text{ km})}{7.7 \text{ GPa}} = \boxed{-1.4\%}. \]

40  \hspace{0.5cm} \text{The volume of a cone of height} \ h \ \text{and base radius} \ r \ \text{is} \ V = \pi r^2 h/3. \ \text{A jar in the shape of a cone of height} \ 25 \ \text{cm} \ \text{has a base with a radius equal to} \ 15 \ \text{cm}. \ \text{The jar is filled with water. Then its lid (the base of the cone) is screwed on and the jar is turned over so its lid is horizontal.} (a) \ \text{Find the volume and weight of the water in the jar.} (b) \ \text{Assuming the pressure inside the jar at the top of the cone is equal to} \ 1 \ \text{atm, find the excess force exerted by the water on the base of the jar.} \ \text{Explain how this force can be greater than the weight of the water in the jar.}

**Picture the Problem** The weight of the water in the jar is the product of its mass and the gravitational field. Its mass, in turn, is related to its volume through the definition of density. The force the water exerts on the base of the jar can be determined from the product of the pressure it creates and the area of the base.

\( (a) \) The volume of the water is:

\[ V = \frac{1}{3} \pi \left(15 \times 10^{-2} \text{ m}\right)^2 \left(25 \times 10^{-2} \text{ m}\right) = 5.890 \times 10^{-3} \text{ m}^3 = \boxed{5.9 \text{ L}} \]
Using the definition of density, relate the weight of the water to the volume it occupies:

\[ w = \left(1.00 \times 10^3 \text{ kg/m}^3\right) \left(5.890 \times 10^{-3} \text{ m}^3\right) \left(9.81 \text{ m/s}^2\right) = 57.79 \text{ N} = \boxed{58 \text{ N}} \]

(b) Using the definition of pressure, relate the force exerted by the water on the base of the jar to the pressure it exerts and the area of the base:

Substitute numerical values and evaluate \( F \):

\[ F = \left(1.00 \times 10^3 \text{ kg/m}^3\right) \left(9.81 \text{ m/s}^2\right) \left(25 \times 10^{-2} \text{ m}\right) \pi \left(15 \times 10^{-2} \text{ m}\right)^2 = 0.17 \text{ kN} \]

This occurs in the same way that the force on Pascal’s barrel is much greater than the weight of the water in the tube. The downward force on the base is also the result of the downward component of the force exerted by the slanting walls of the cone on the water.

**Buoyancy**

41 • A 500-g piece of copper (specific gravity 8.96) is suspended from a spring scale and is submerged in water (Figure 13-34). What force does the spring scale read?

**Picture the Problem** The scale’s reading will be the difference between the weight of the piece of copper in air and the buoyant force acting on it.

Express the apparent weight \( w' \) of the piece of copper:

\[ w' = w - B \]

Using the definition of density and Archimedes’ principle, substitute for \( w \) and \( B \) to obtain:

\[ w' = \rho_{Cu} Vg - \rho_w Vg = \left(\rho_{Cu} - \rho_w\right) Vg \]

Express \( w \) in terms of \( \rho_{Cu} \) and \( V \) and solve for \( Vg \):

\[ w = \rho_{Cu} Vg \Rightarrow Vg = \frac{w}{\rho_{Cu}} \]
Substitute for \( V_g \) in the expression for \( w' \) to obtain:

\[
w' = \left( \frac{\rho_{Cu} - \rho_w}{\rho_{Cu}} \right) \frac{w}{\rho_{Cu}} = \left( 1 - \frac{\rho_w}{\rho_{Cu}} \right) w
\]

Substitute numerical values and evaluate \( w' \):

\[
w' = \left( 1 - \frac{1}{8.96} \right) (0.500 \text{ kg}) (9.81 \text{ m/s}^2)
\]

\[
= 4.36 \text{ N}
\]

42 When a certain rock is suspended from a spring scale the scale-display reads 60 N. However, when the suspended rock is submerged in water, the display reads 40 N. What is the density of the rock?

**Picture the Problem** We can use the definition of density and Archimedes’ principle to find the density of the stone. The difference between the weight of the stone in air and in water is the buoyant force acting on the stone.

Using its definition, express the density of the stone:

\[
\rho_{\text{stone}} = \frac{m_{\text{stone}}}{V_{\text{stone}}}
\]

(1)

Apply Archimedes’ principle to obtain:

\[
B = w_{\text{displaced fluid}} = \frac{m_{\text{displaced fluid}} g}{\rho_{\text{fluid}}}
\]

\[= \frac{\rho_{\text{displaced fluid}}}{\rho_{\text{fluid}}} V_{\text{displaced fluid}} g
\]

Solve for \( V_{\text{displaced fluid}} \):

\[
V_{\text{displaced fluid}} = \frac{B}{\rho_{\text{displaced fluid}} g}
\]

Because \( V_{\text{displaced fluid}} = V_{\text{stone}} \) and \( \rho_{\text{displaced fluid}} = \rho_{\text{water}} \):

\[
V_{\text{stone}} = \frac{B}{\rho_{\text{water}} g}
\]

Substituting in equation (1) and simplifying yields:

\[
\rho_{\text{stone}} = \frac{m_{\text{stone}} g}{B} \rho_{\text{water}} = \frac{w_{\text{stone}}}{B} \rho_{\text{water}}
\]

Substitute numerical values and evaluate \( \rho_{\text{stone}} \):

\[
\rho_{\text{stone}} = \frac{60 \text{ N}}{60 \text{ N} - 40 \text{ N}} \left( 1.00 \times 10^3 \text{ kg/m}^3 \right)
\]

\[
= 3.0 \times 10^3 \text{ kg/m}^3
\]

43 [SSM] A block of an unknown material weighs 5.00 N in air and 4.55 N when submerged in water. (a) What is the density of the material? (b) From what material is the block likely to have been made?
**Picture the Problem** We can use the definition of density and Archimedes’ principle to find the density of the unknown object. The difference between the weight of the object in air and in water is the buoyant force acting on the object.

(a) Using its definition, express the density of the object:

\[
\rho_{\text{object}} = \frac{m_{\text{object}}}{V_{\text{object}}} \quad \text{(1)}
\]

Apply Archimedes’ principle to obtain:

\[
B = m_{\text{displaced fluid}} g = \rho_{\text{displaced fluid}} V_{\text{displaced fluid}} g
\]

Solve for \(V_{\text{displaced fluid}}\):

\[
V_{\text{displaced fluid}} = \frac{B}{\rho_{\text{displaced fluid}} g}
\]

Because \(V_{\text{displaced fluid}} = V_{\text{object}}\) and \(\rho_{\text{displaced fluid}} = \rho_{\text{water}}\),

\[
V_{\text{object}} = \frac{B}{\rho_{\text{water}} g}
\]

Substitute in equation (1) and simplify to obtain:

\[
\rho_{\text{object}} = \frac{m_{\text{object}} g}{B} \rho_{\text{water}} = \frac{w_{\text{object}}}{B} \rho_{\text{water}}
\]

Substitute numerical values and evaluate \(\rho_{\text{object}}\):

\[
\rho_{\text{object}} = \frac{5.00 \text{ N}}{5.00 \text{ N} - 4.55 \text{ N}} \left(1.00 \times 10^3 \text{ kg/m}^3\right) = 11 \times 10^3 \text{ kg/m}^3
\]

(b) From Table 13-1, we see that the density of the unknown material is close to that of lead.

44 • A solid piece of metal weighs 90.0 N in air and 56.6 N when submerged in water. Determine the density of this metal.

**Picture the Problem** We can use the definition of density and Archimedes’ principle to find the density of the unknown object. The difference between the weight of the object in air and in water is the buoyant force acting on it.

Using its definition, express the density of the metal:

\[
\rho_{\text{metal}} = \frac{m_{\text{metal}}}{V_{\text{metal}}} \quad \text{(1)}
\]
Apply Archimedes’ principle to obtain:

\[ B = w_{\text{displaced}} = m_{\text{displaced}}g = \rho_{\text{displaced}} V_{\text{displaced}}g \]

Solve for \( V_{\text{displaced}} \):

\[ V_{\text{displaced}} = \frac{B}{\rho_{\text{displaced}}g} \]

Because \( V_{\text{displaced}} = V_{\text{metal}} \) and \( \rho_{\text{displaced}} = \rho_{\text{water}} \):

\[ V_{\text{metal}} = \frac{B}{\rho_{\text{water}}g} \]

Substitute in equation (1) and simplify to obtain:

\[ \rho_{\text{metal}} = \frac{m_{\text{metal}}g}{B} \rho_{\text{water}} = \frac{W_{\text{metal}}}{B} \rho_{\text{water}} \]

Substitute numerical values and evaluate \( \rho_{\text{metal}} \):

\[ \rho_{\text{metal}} = \frac{90.0N}{90.0N - 56.6N} \left(1.00\times10^3 \text{ kg/m}^3 \right) = 2.69\times10^3 \text{ kg/m}^3 \]

A homogeneous solid object floats on water with 80.0 percent of its volume below the surface. The same object when placed in a second liquid floats on that liquid with 72.0 percent of its volume below the surface. Determine the density of the object and the specific gravity of the liquid.

**Picture the Problem** Let \( V \) be the volume of the object and \( V' \) be the volume that is submerged when it floats. The weight of the object is \( \rho Vg \) and the buoyant force due to the water is \( \rho_w V'g \). Because the floating object is in translational equilibrium, we can use \( \sum F_y = 0 \) to relate the buoyant forces acting on the object in the two liquids to its weight.

Apply \( \sum F_y = 0 \) to the object floating in water:

\[ \rho_w V'g - mg = \rho_w V'g - \rho Vg = 0 \] (1)

Solving for \( \rho \) yields:

\[ \rho = \rho_w \frac{V'}{V} \]

Substitute numerical values and evaluate \( \rho '\):

\[ \rho = \left(1.00\times10^3 \text{ kg/m}^3 \right) \frac{0.800V'}{V} = 800 \text{ kg/m}^3 \]
Apply \( \sum F_y = 0 \) to the object floating in the second liquid and solve for \( mg \):

\[
mg = 0.720 \rho_L g
\]

Solve equation (1) for \( mg \):

\[
mg = 0.800 \rho_w V g
\]

Equate these two expressions to obtain:

\[
0.720 \rho_L = 0.800 \rho_w
\]

Substitute in the definition of specific gravity to obtain:

\[
\text{specific gravity} = \frac{\rho_L}{\rho_w} = \frac{0.800}{0.720} = 1.11
\]

46 •• A 5.00-kg iron block is suspended from a spring scale and is submerged in a fluid of unknown density. The spring scale reads 6.16 N. What is the density of the fluid?

**Picture the Problem** We can use Archimedes’ principle to find the density of the unknown object. The difference between the weight of the block in air and in the fluid is the buoyant force acting on the block.

Apply Archimedes’ principle to obtain:

\[
B = w_{\text{displaced}} = m_{\text{displaced}} g = \rho_{\text{displaced}} V_{\text{displaced}} g
\]

Solve for \( \rho_{\text{displaced}} \):

\[
\rho_{\text{displaced}} = \frac{B}{V_{\text{displaced}} g}
\]

Because \( V_{\text{displaced}} = V_{\text{Fe block}} \):

\[
\rho_I = \frac{B}{V_{\text{Fe block}} g} = \frac{B}{m_{\text{Fe block}} g} \rho_{Fe}
\]

Substitute numerical values and evaluate \( \rho_I \):

\[
\rho_I = \frac{(5.00 \text{ kg}) (9.81 \text{ m/s}^2)}{5.00 \text{ kg}} \rho_{Fe} = \frac{(9.81 \times 10^3 \text{ kg/m}^3) - 6.16 \text{ N}}{(9.81 \times 10^3 \text{ kg/m}^3)} = 7.0 \times 10^3 \text{ kg/m}^3
\]

47 •• A large piece of cork weighs 0.285 N in air. When held submerged underwater by a spring scale as shown in Figure 13-35, the spring scale reads 0.855 N. Find the density of the cork.

**Picture the Problem** The forces acting on the cork are \( B \), the upward force due to the displacement of water, \( mg \), the weight of the piece of cork, and \( F_s \), the force
exerted by the spring. The piece of cork is in equilibrium under the influence of these forces.

Apply $\sum F_y = 0$ to the piece of cork:

$$B - w - F_s = 0 \quad (1)$$

or

$$B - \rho_{\text{cork}} Vg - F_s = 0 \quad (2)$$

Express the buoyant force as a function of the density of water:

$$B = w_{\text{displaced fluid}} = \rho_w Vg \Rightarrow Vg = \frac{B}{\rho_w}$$

Substitute for $Vg$ in equation (2) to obtain:

$$B - \rho_{\text{cork}} \frac{B}{\rho_w} - F_s = 0 \quad (3)$$

Solve equation (1) for $B$:

$$B = w + F_s$$

Substitute in equation (3) to obtain:

$$w + F_s - \rho_{\text{cork}} \frac{w + F_s}{\rho_w} - F_s = 0$$

or

$$w - \rho_{\text{cork}} \frac{w + F_s}{\rho_w} = 0$$

Solving for $\rho_{\text{cork}}$ yields:

$$\rho_{\text{cork}} = \rho_w \frac{w}{w + F_s}$$

Substitute numerical values and evaluate $\rho_{\text{cork}}$:

$$\rho_{\text{cork}} = \left(1.00 \times 10^3 \text{ kg/m}^3 \right) \frac{0.285 \text{ N}}{0.285 \text{ N} + 0.855 \text{ N}} = 250 \text{ kg/m}^3$$

A helium balloon lifts a basket and cargo of total weight 2000 N under standard conditions, at which the density of air is 1.29 kg/m$^3$ and the density of helium is 0.178 kg/m$^3$. What is the minimum volume of the balloon?

**Picture the Problem** Under minimum-volume conditions, the balloon will be in equilibrium. Let $B$ represent the buoyant force acting on the balloon, $w_{\text{tot}}$ represent its total weight, and $V$ its volume. The total weight is the sum of the weights of its basket, cargo, and helium in its balloon.

Apply $\sum F_y = 0$ to the balloon:

$$B - w_{\text{tot}} = 0$$
Express the total weight of the balloon: 

\[ w_{\text{tot}} = 2000 \text{ N} + \rho_{\text{He}} V g \]

Express the buoyant force due to the displaced air: 

\[ B = w_{\text{displaced}} = \rho_{\text{air}} V g \]

Substitute for \( B \) and \( w_{\text{tot}} \) to obtain: 

\[ \rho_{\text{air}} V g - 2000 \text{ N} = \rho_{\text{He}} V g = 0 \]

Solving for \( V \) yields: 

\[ V = \frac{2000 \text{ N}}{(\rho_{\text{air}} - \rho_{\text{He}}) g} \]

Substitute numerical values and evaluate \( V \):

\[
V = \frac{2000 \text{ N}}{(1.29 \text{ kg/m}^3 - 0.178 \text{ kg/m}^3)(9.81 \text{ m/s}^2)} = 183 \text{ m}^3
\]

49 [SSM] An object has “neutral buoyancy” when its density equals that of the liquid in which it is submerged, which means that it neither floats nor sinks. If the average density of an 85-kg diver is 0.96 kg/L, what mass of lead should, as dive master, suggest be added to give him neutral buoyancy?

**Picture the Problem** Let \( V \) = volume of diver, \( \rho_D \) the density of the diver, \( V_{\text{Pb}} \) the volume of added lead, and \( m_{\text{Pb}} \) the mass of lead. The diver is in equilibrium under the influence of his weight, the weight of the lead, and the buoyant force of the water.

Apply \( \sum F_y = 0 \) to the diver: 

\[ B - w_D - w_{\text{Pb}} = 0 \]

Substitute to obtain: 

\[ \rho_w V_{\text{D+Pb}} g - \rho_D V_{\text{D}} g - m_{\text{Pb}} g = 0 \]

or

\[ \rho_w V_{\text{D}} + \rho_w V_{\text{Pb}} - \rho_D V_{\text{D}} - m_{\text{Pb}} = 0 \]

Rewrite this expression in terms of masses and densities:

\[ \rho_w \frac{m_D}{\rho_D} + \rho_w \frac{m_{\text{Pb}}}{\rho_{\text{Pb}}} - \rho_D m_D - m_{\text{Pb}} = 0 \]

Solving for \( m_{\text{Pb}} \) yields:

\[ m_{\text{Pb}} = \rho_{\text{Pb}} \left( \frac{\rho_w - \rho_D}{\rho_D (\rho_{\text{Pb}} - \rho_w)} \right) m_D \]
Substitute numerical values and evaluate \(m_{\text{Pb}}\):

\[
m_{\text{Pb}} = \frac{(11.3 \times 10^3 \text{ kg/m}^3)(1.00 \times 10^3 \text{ kg/m}^3 - 0.96 \times 10^3 \text{ kg/m}^3)(85 \text{ kg})}{(0.96 \times 10^3 \text{ kg/m}^3)(11.3 \times 10^3 \text{ kg/m}^3 - 1.00 \times 10^3 \text{ kg/m}^3)} = 3.9 \text{ kg}
\]

50  A 1.00-kg beaker containing 2.00 kg of water rests on a scale. A 2.00-kg block of aluminum (density \(2.70 \times 10^3 \text{ kg/m}^3\)) suspended from a spring scale is submerged in the water, as in Figure 13-36. Find the readings of both scales.

**Picture the Problem** The hanging scale’s reading \(w'\) is the difference between the weight of the aluminum block in air \(w\) and the buoyant force acting on it. The buoyant force is equal to the weight of the displaced fluid, which, in turn, is the product of its density and mass. We can apply a condition for equilibrium to relate the reading of the bottom scale to the weight of the beaker and its contents and the buoyant force acting on the block.

Express the apparent weight \(w'\) of the aluminum block. This apparent weight is the reading on the hanging scale and is equal to the tension in the string:

\[
w' = w - B \quad (1)
\]

Letting \(F\) be the reading of the bottom scale and choosing upward to be the positive \(y\) direction, apply \(\sum F_y = 0\) to the system consisting of the beaker, water, and block to obtain:

\[
F + w' - M_{\text{tot}}g = 0 \quad (2)
\]

Using Archimedes’ principle, substitute for \(B\) in equation (1) to obtain:

\[
w' = w - \rho_w Vg \quad (3)
\]

Express \(w\) in terms of \(\rho_{\text{Al}}\) and \(V\) and solve for \(Vg\):

\[
w = \rho_{\text{Al}} Vg \Rightarrow Vg = \frac{w}{\rho_{\text{Al}}}
\]

Substitute for \(Vg\) in equation (3) and simplify to obtain:

\[
w' = w - \rho_w \frac{w}{\rho_{\text{Al}}} \left(1 - \frac{\rho_w}{\rho_{\text{Al}}}\right)w
\]
Substitute numerical values and evaluate $w'$:

$$w' = \left(1 - \frac{1.00 \times 10^3 \text{ kg/m}^3}{2.70 \times 10^3 \text{ kg/m}^3}\right) \left(2.00 \text{ kg}\right) \left(9.81 \text{ m/s}^2\right) = 12.4 \text{ N}$$

Solve equation (2) for $F$ to obtain:

$$F = M_{tot}g - w'$$

Substitute numerical values and evaluate the reading of the bottom scale:

$$F = (5.00 \text{ kg}) \left(9.81 \text{ m/s}^2\right) - 12.4 \text{ N} = 36.7 \text{ N}$$

51. When cracks form at the base of a dam, the water seeping into the cracks exerts a buoyant force that tends to lift the dam. As a result, the dam can topple. Estimate the buoyant force exerted on a 2.0-m thick by 5.0-m wide dam wall by water seeping into cracks at its base. The water level in the lake is 5.0 m above the cracks.

**Picture the Problem** We can apply Archimedes’ principle to estimate the buoyant force exerted by the water on the dam. We can determine whether the buoyant force is likely to lift the dam by comparing the buoyant and gravitational forces acting on the dam.

The buoyant force acting on the dam is given by Archimedes’ principle:

$$B = w_{\text{displaced}} = m_{\text{displaced}}g = \rho_{\text{water}} V_{\text{displaced}}g$$

The volume of the displaced water is the same as the volume of the dam:

$$B = \rho_{\text{water}} V_{\text{dam}}g = \rho_{\text{water}}LWHg$$

Where $L$, $W$, and $H$ are the length and width of the dam, and $H$ is the height of the water above the cracks.

Substitute numerical values and evaluate $B$:

$$B = \left(1.00 \times 10^3 \text{ kg/m}^3\right)(2.0 \text{ m})(4.0 \text{ m})(5.0 \text{ m})(9.81 \text{ m/s}^2) = 3.9 \times 10^5 \text{ N}$$

52. Your team is in charge of launching a large helium weather balloon that is spherical in shape, and whose radius is 2.5 m and total mass is 15 kg (balloon plus helium plus equipment). (a) What is the initial upward acceleration of the balloon when it is released from sea level? (b) If the drag force on the balloon is given by $F_D = \frac{1}{2} \pi r^2 \rho \nu^2$, where $r$ is the balloon radius, $\rho$ is the density of
air, and \( v \) the balloon’s ascension speed, calculate the terminal speed of the ascending balloon.

**Picture the Problem** The forces acting on the balloon are the buoyant force \( B \), its weight \( mg \), and a drag force \( F_D \). We can find the initial upward acceleration of the balloon by applying Newton’s 2\(^{nd} \) law at the instant it is released. We can find the terminal speed of the balloon by recognizing that when \( a_y = 0 \), the net force acting on the balloon will be zero.

\[(a) \text{ Apply } \sum F_y = ma_y \text{ to the balloon at the instant of its release to obtain:} \]

\[
B - m_{\text{balloon}}g = m_{\text{balloon}}a_y \quad (1)
\]

Using Archimedes principle, express the buoyant force \( B \) acting on the balloon:

\[
B = w_{\text{displaced fluid}} = m_{\text{displaced fluid}}g = \rho_{\text{displaced fluid}}V_{\text{displaced fluid}}g = \rho_{\text{air}}V_{\text{balloon}}g = \frac{4}{3} \pi \rho_{\text{air}} r^3 g
\]

Substitute in equation (1) to obtain:

\[
\frac{4}{3} \pi \rho_{\text{air}} r^3 g - m_{\text{balloon}}g = m_{\text{balloon}}a_y
\]

Solving for \( a_y \) yields:

\[
a_y = \left( \frac{4}{3} \pi \rho_{\text{air}} r^3 - 1 \right) \frac{g}{m_{\text{balloon}}}
\]

Substitute numerical values and evaluate \( a_y \):

\[
a_y = \left[ \frac{4}{3} \pi \left(1.29 \text{ kg/m}^3\right) \left(2.5 \text{ m}^3\right) \right] \left(9.81 \text{ m/s}^2\right) = 45 \text{ m/s}^2
\]

\[(b) \text{ Apply } \sum F_y = ma_y \text{ to the balloon under terminal-speed conditions to obtain:} \]

\[
B - mg - \frac{1}{2} \pi r^2 \rho v_i^2 = 0
\]

Substitute for \( B \):

\[
\frac{4}{3} \pi \rho_{\text{air}} r^3 g - mg - \frac{1}{2} \pi r^2 \rho v_i^2 = 0
\]

Solving for \( v_i \) yields:

\[
v_i = \sqrt{\frac{2 \left( \frac{4}{3} \pi \rho_{\text{air}} r^3 - m \right) g}{\pi r^2 \rho}}
\]
Substitute numerical values and evaluate $v$:

$$v_t = \sqrt{\frac{2\pi \frac{4}{3} \pi (1.29 \text{ kg/m}^3)(2.5 \text{ m})^3 - 15 \text{ kg}(9.81 \text{ m/s}^2)}{\pi (2.5 \text{ m})^2 (1.29 \text{ kg/m}^3)}} = 7.33 \text{ m/s} = 7.3 \text{ m/s}$$

53 [SSM] A ship sails from seawater (specific gravity 1.025) into freshwater, and therefore sinks slightly. When its 600,000-kg load is removed, it returns to its original level. Assuming that the sides of the ship are vertical at the water line, find the mass of the ship before it was unloaded.

**Picture the Problem** Let $V$ = displacement of ship in the two cases, $m$ be the mass of ship without load, and $\Delta m$ be the load. The ship is in equilibrium under the influence of the buoyant force exerted by the water and its weight. We’ll apply the condition for floating in the two cases and solve the equations simultaneously to determine the loaded mass of the ship.

Apply $\sum F_y = 0$ to the ship in fresh water:

$$\rho_w V g - mg = 0 \quad (1)$$

Apply $\sum F_y = 0$ to the ship in salt water:

$$\rho_{sw} V g - (m + \Delta m)g = 0 \quad (2)$$

Solve equation (1) for $V g$:

$$V g = \frac{mg}{\rho_w}$$

Substitute in equation (2) to obtain:

$$\rho_{sw} \frac{mg}{\rho_w} - (m + \Delta m)g = 0$$

Solving for $m$ yields:

$$m = \frac{\rho_{sw} \Delta m}{\rho_{sw} - \rho_w}$$

Add $\Delta m$ to both sides of the equation and simplify to obtain:

$$m + \Delta m = \frac{\rho_w \Delta m}{\rho_{sw} - \rho_w} + \Delta m = \Delta m \left( \frac{\rho_w}{\rho_{sw} - \rho_w} + 1 \right) = \frac{\Delta m \rho_{sw}}{\rho_{sw} - \rho_w}$$
Substitute numerical values and evaluate \( m + \Delta m \):

\[
m + \Delta m = \frac{(6.00 \times 10^5 \text{ kg})(1.025 \rho_w)}{1.025 \rho_w - \rho_w} = \frac{(6.00 \times 10^5 \text{ kg})(1.025)}{1.025 - 1} = 2.5 \times 10^7 \text{ kg}
\]

**Continuity and Bernoulli's Equation**

54  •  Water flows at 0.65 m/s through a 3.0-cm-diameter hose that terminates in a 0.30-cm-diameter nozzle. Assume laminar nonviscous steady-state flow. (a) At what speed does the water pass through the nozzle? (b) If the pump at one end of the hose and the nozzle at the other end are at the same height, and if the pressure at the nozzle is 1.0 atm, what is the pressure at the pump outlet?

**Picture the Problem** Let \( A_1 \) represent the cross-sectional area of the hose, \( A_2 \) the cross-sectional area of the nozzle, \( v_1 \) the speed of the water in the hose, and \( v_2 \) the speed of the water as it passes through the nozzle. We can use the continuity equation to find \( v_2 \) and Bernoulli's equation for constant elevation to find the pressure at the pump outlet.

(a) Using the continuity equation, relate the speeds of the water to the diameter of the hose and the diameter of the nozzle:

\[
A_1 v_1 = A_2 v_2
\]

or

\[
\frac{\pi d_1^2}{4} v_1 = \frac{\pi d_2^2}{4} v_2 \Rightarrow v_2 = \frac{d_1^2}{d_2^2} v_1
\]

Substitute numerical values and evaluate \( v_2 \):

\[
v_2 = \left( \frac{3.0 \text{ cm}}{0.30 \text{ cm}} \right)^2 \left( 0.65 \text{ m/s} \right) = 65.0 \text{ m/s}
\]

(b) Using Bernoulli's equation for constant elevation, relate the pressure at the pump \( P_p \) to the atmospheric pressure and the velocities of the water in the hose and the nozzle:

\[
P_p + \frac{1}{2} \rho v_1^2 = P_{at} + \frac{1}{2} \rho v_2^2
\]

Solve for the pressure at the pump:

\[
P_p = P_{at} + \frac{1}{2} \rho \left( v_2^2 - v_1^2 \right)
\]
Substitute numerical values and evaluate $P_\rho$:

$$
P_\rho = 101.325 \text{kPa} + \frac{1}{2} \left(1.00 \times 10^3 \text{ kg/m}^3\right) \left((65.0 \text{ m/s})^2 - (0.65 \text{ m/s})^2\right)
$$

$$
= 2.21 \times 10^6 \text{Pa} \times \frac{1 \text{ atm}}{101.325 \text{kPa}} = 22 \text{ atm}
$$

55  •  [SSM] Water is flowing at 3.00 m/s in a horizontal pipe under a pressure of 200 kPa. The pipe narrows to half its original diameter. (a) What is the speed of flow in the narrow section? (b) What is the pressure in the narrow section? (c) How do the volume flow rates in the two sections compare?

**Picture the Problem** Let $A_1$ represent the cross-sectional area of the larger-diameter pipe, $A_2$ the cross-sectional area of the smaller-diameter pipe, $v_1$ the speed of the water in the larger-diameter pipe, and $v_2$ the velocity of the water in the smaller-diameter pipe. We can use the continuity equation to find $v_2$ and Bernoulli’s equation for constant elevation to find the pressure in the smaller-diameter pipe.

(a) Using the continuity equation, relate the velocities of the water to the diameters of the pipe:

$$
A_1 v_1 = A_2 v_2
$$

or

$$
\frac{\pi d_1^2}{4} v_1 = \frac{\pi d_2^2}{4} v_2 \Leftrightarrow v_2 = \frac{d_1^2}{d_2^2} v_1
$$

Substitute numerical values and evaluate $v_2$:

$$
v_2 = \left(\frac{d_1}{\frac{1}{2} d_1}\right)^2 (3.00 \text{ m/s}) = 12.0 \text{ m/s}
$$

(b) Using Bernoulli’s equation for constant elevation, relate the pressures in the two segments of the pipe to the velocities of the water in these segments:

$$
P_1 + \frac{1}{2} \rho_w v_1^2 = P_2 + \frac{1}{2} \rho_w v_2^2
$$

Solving for $P_2$ yields:

$$
P_2 = P_1 + \frac{1}{2} \rho_w v_1^2 - \frac{1}{2} \rho_w v_2^2
$$

$$
= P_1 + \frac{1}{2} \rho_w (v_1^2 - v_2^2)
$$

Substitute numerical values and evaluate $P_2$:

$$
P_2 = 200 \text{kPa} + \frac{1}{2} \left(1.00 \times 10^3 \text{ kg/m}^3\right) \left((3.00 \text{ m/s})^2 - (12.0 \text{ m/s})^2\right) = 133 \text{kPa}
$$
(c) Using the continuity equation, evaluate $I_{V1}$:

$$I_{V1} = A_1 v_1 = \frac{\pi d_1^2}{4} v_1 = \frac{\pi d_1^2}{4}(3.00 \text{ m/s})$$

Using the continuity equation, express $I_{V2}$:

$$I_{V2} = A_2 v_2 = \frac{\pi d_2^2}{4} v_2$$

Substitute numerical values and evaluate $I_{V2}$:

$$I_{V2} = \frac{\pi}{4} \left( \frac{d_1}{2} \right)^2 (12.0 \text{ m/s})$$

$$= \frac{\pi d_1^2}{4}(3.00 \text{ m/s})$$

Thus, as we expected would be the case:

$$I_{V1} = I_{V2}$$

56  •  The pressure in a section of horizontal pipe with a diameter of 2.00 cm is 142 kPa. Water flows through the pipe at 2.80 L/s. If the pressure at a certain point is to be reduced to 101 kPa by constricting a section of the pipe, what should the diameter of the constricted section be?

**Picture the Problem** Let $A_1$ represent the cross-sectional area of the 2.00-cm diameter pipe, $A_2$ the cross-sectional area of the constricted pipe, $v_1$ the speed of the water in the 2.00-cm diameter pipe, and $v_2$ the speed of the water in the constricted pipe. We can use the continuity equation to express $d_2$ in terms of $d_1$ and to find $v_1$ and Bernoulli’s equation for constant elevation to find the speed of the water in the constricted pipe.

Using the continuity equation, relate the volume flow rate in the 2.00-cm diameter pipe to the volume flow rate in the constricted pipe:

$$A_1 v_1 = A_2 v_2$$

or

$$\frac{\pi d_1^2}{4} v_1 = \frac{\pi d_2^2}{4} v_2$$

$$d_2 = d_1 \sqrt{\frac{v_1}{v_2}}$$  (1)

Using the continuity equation, relate $v_1$ to the volume flow rate $I_V$:

$$v_1 = \frac{I_V}{A_1} = \frac{I_V}{\frac{1}{4} \pi d_1^2} = \frac{4 I_V}{\pi d_1^2}$$

Using Bernoulli’s equation for constant elevation, relate the pressures in the two segments of the pipe to the velocities of the water in these segments:

$$P_1 + \frac{1}{2} \rho v_1^2 = P_2 + \frac{1}{2} \rho v_2^2$$
Solve for \( v_2 \) to obtain:

\[
v_2 = \sqrt{\frac{2(P_1 - P)}{\rho_w} + v_1^2}
\]

Substituting for \( v_2 \) in equation (1) and simplifying yields:

\[
d_2 = d_1 \sqrt{\frac{v_1}{2(P_1 - P) + v_1^2}} = d_1 \sqrt{\frac{1}{2(P_1 - P) + v_1^2}} = \frac{d_1}{\sqrt{\frac{2(P_1 - P)}{v_1^2 \rho_w} + 1}}
\]

Substitute for \( v_1 \) and simplify to obtain:

\[
d_2 = \sqrt{\frac{2(P_1 - P)}{\frac{4I_v}{\pi d_1^2} + 1}} = \frac{d_1}{\sqrt{\frac{\pi^2 d_1^4 (P_1 - P)}{8I_v \rho_w} + 1}}
\]

Substitute numerical values and evaluate \( d_2 \):

\[
d_2 = \frac{2.00 \text{ cm}}{\sqrt{\frac{\pi^2 \left(0.0200 \text{ m} \right)^4 \left(142 \text{ kPa} - 101.325 \text{ kPa} \right)}{8 \left(2.80 \text{ L/s} \right)^2 \left(1.00 \times 10^3 \text{ kg/m}^3 \right)} + 1}} = 1.68 \text{ cm}
\]

**57 [SSM]** Blood flows at 30 cm/s in an aorta of radius 9.0 mm. (a) Calculate the volume flow rate in liters per minute. (b) Although the cross-sectional area of a capillary is much smaller than that of the aorta, there are many capillaries, so their total cross-sectional area is much larger. If all the blood from the aorta flows into the capillaries and the speed of flow through the capillaries is 1.0 mm/s, calculate the total cross-sectional area of the capillaries. Assume laminar nonviscous, steady-state flow.

**Picture the Problem** We can use the definition of the volume flow rate to find the volume flow rate of blood in an aorta and to find the total cross-sectional area of the capillaries.

(a) Use the definition of the volume flow rate to find the volume flow rate through an aorta:
Substitute numerical values and evaluate \( I_v \):

\[
I_v = \pi \left( 9.0 \times 10^{-3} \text{ m}^3 \right)^2 (0.30 \text{ m/s})
\]

\[
= 7.634 \times 10^{-5} \text{ m}^3 / \text{s} \times \frac{60 \text{ s}}{\text{min}} \times \frac{1 \text{ L}}{10^{-3} \text{ m}^3}
\]

\[
= 4.58 \text{ L/min} = 4.6 \text{ L/min}
\]

\((b)\) Use the definition of the volume flow rate to express the volume flow rate through the capillaries:

\[ I_v = A \, v \Rightarrow A = \frac{I_v}{v} \]

Substitute numerical values and evaluate \( A_{\text{cap}} \):

\[
A_{\text{cap}} = \frac{7.63 \times 10^{-5} \text{ m}^3/\text{s}}{0.0010 \text{ m/s}} = 7.6 \times 10^{-2} \text{ m}^2
\]

58 \hspace{1em} Water flows through a 1.0 m-long conical section of pipe that joins a cylindrical pipe of radius 0.45 m, on the left, to a cylindrical pipe of radius 0.25 m, on the right. If the water is flowing into the 0.45 m pipe with a speed of 1.50 m/s, and if we assume laminar nonviscous steady-state flow, \((a)\) what is the speed of flow in the 0.25 m pipe? \((b)\) What is the speed of flow at a position \(x\) in the conical section, if \(x\) is the distance measured from the left-hand end of the conical section of pipe?

**Picture the Problem** The pictorial representation summarizes what we know about the two pipes and the connecting conical section. We can apply the continuity equation to find \(v_2\). We can also use the continuity equation in \((b)\) provided we first express the cross-sectional area of the transitional conical section as a function of \(x\).

\[
(a)\hspace{1em} \text{Apply the continuity equation to the flow in the two sections of cylindrical pipes:} \hspace{1em} v_1 A_1 = v_2 A_2
\]

Solving for \(v_2\), expressing the cross-sectional areas in terms of their diameters, and simplifying yields:

\[
v_2 = v_1 \frac{A_1}{A_2} = v_1 \frac{\frac{1}{4} \pi d_1^2}{\frac{1}{4} \pi d_2^2} = v_1 \left( \frac{d_1}{d_2} \right)^2
\]

Substitute numerical values and evaluate \(v_2\):

\[
v_2 = (1.50 \text{ m/s}) \left( \frac{0.90\text{ m}}{0.50\text{ m}} \right)^2 = 4.9 \text{ m/s}
\]
Continuity of flow requires that:

\[ v(0)A(0) = v(x)A(x) \]

Solve for \( v(x) \) to obtain:

\[ v(x) = v(0) \frac{A(0)}{A(x)} \] (1)

The diagram to the right is an enlargement of the upper half of the transitional conical section. Use the coordinates shown on the line to establish the following proportion:

\[ \frac{r(x) - 0.45 \text{ m}}{x - 0} = \frac{0.25 \text{ m} - 0.45 \text{ m}}{L - 0} \]

Solving the proportion for \( r(x) \) yields:

\[ r(x) = 0.45 \text{ m} - (0.20 \text{ m}) \frac{x}{L} \]

where \( L \) is the length of the conical section.

The cross-sectional area of the conical section varies with \( x \) according to:

\[ A(x) = \pi (r(x))^2 = \pi \left( 0.45 \text{ m} - (0.20 \text{ m}) \frac{x}{L} \right)^2 \]

Substituting for \( A(x) \) in equation (1) and simplifying yields:

\[ v(x) = v(0) \frac{\frac{1}{4} \pi d_i^2}{\pi \left( 0.45 \text{ m} - (0.20 \text{ m}) \frac{x}{L} \right)^2} = \frac{v(0)d_i^2}{4 \left( 0.45 \text{ m} - (0.20 \text{ m}) \frac{x}{L} \right)^2} \]

Substitute numerical values and simplify further to obtain:

\[ v(x) = \frac{(1.50 \text{ m/s})(0.90 \text{ m})^2}{4 \left( 0.45 \text{ m} - (0.20 \text{ m}) \frac{x}{1.0 \text{ m}} \right)^2} = \frac{0.304 \text{ m}^3/\text{s}}{(0.45 \text{ m} - 0.20x)^2} \]
To confirm the correctness of this equation, evaluate \( v(0) \) and \( v(1.0 \text{ m}) \):

\[
\begin{align*}
  v(0) &= \frac{0.304 \text{ m}^3 / \text{s}}{(0.45 \text{ m})^2} = 1.5 \text{ m/s} \\
  v(1.0 \text{ m}) &= \frac{0.304 \text{ m}^3 / \text{s}}{(0.45 \text{ m} - 0.20 \text{ (1.0 m)})^2} = 4.9 \text{ m/s}
\end{align*}
\]

59  •  The $8-billion, 800-mile long Alaskan Pipeline has a maximum volume flow rate of 240,000 m\(^3\) of oil per day. Along most of the pipeline the radius is 60.0 cm. Find the pressure \( P' \) at a point where the pipe has a 30.0-cm radius. Take the pressure in the 60.0-cm-radius sections to be \( P = 180 \text{ kPa} \) and the density of oil to be 800 kg/m\(^3\). Assume laminar nonviscous steady-state flow.

**Picture the Problem** Let the subscript 60 denote the 60.0-cm-radius pipe and the subscript 30 denote the 30.0-cm-radius pipe. We can use Bernoulli’s equation for constant elevation to express \( P' \) in terms of \( v_{60} \) and \( v_{30} \), the definition of volume flow rate to find \( v_{60} \), and the continuity equation to find \( v_{30} \).

Using Bernoulli’s equation for constant elevation, relate the pressures in the two pipes to the velocities of the oil:

\[
P' = P + \frac{1}{2} \rho \left( \frac{v_{60}^2}{A_{60}} \right) = P + \frac{1}{2} \rho \left( \frac{v_{30}^2}{A_{30}} \right)
\]

Solving for \( P' \) yields:

\[
P' = P + \frac{1}{2} \rho \left( \frac{v_{60}^2}{A_{60}} - \frac{v_{30}^2}{A_{30}} \right)
\]

(1)

Use the definition of volume flow rate to express \( v_{60} \):

\[
v_{60} = \frac{I_v}{A_{60}} = \frac{I_v}{\pi r_{60}^2}
\]

Using the continuity equation, relate the speed of the oil in the half-standard pipe to its speed in the standard pipe:

\[
A_{60} v_{60} = A_{30} v_{30} \Rightarrow v_{30} = \frac{A_{60}}{A_{30}} v_{60}
\]
Substituting for \( v_{60} \) and \( A_{30} \) yields:

\[
v_{30} = \frac{A_{60}}{A_{30}} \frac{I_v}{I_v} = \frac{I_v}{\pi r_{30}^2}
\]

Substitute for \( v_{60} \) and \( v_{30} \) and simplify to obtain:

\[
P' = P + \frac{1}{2} \rho \left( \frac{I_v}{\pi r_{60}^2} \right)^2 - \left( \frac{I_v}{\pi r_{30}^2} \right)^2 = P + \frac{\rho I_v^2}{2\pi^2} \left( \frac{1}{r_{60}^4} - \frac{1}{r_{30}^4} \right)
\]

Substitute numerical values in equation (1) and evaluate \( P' \):

\[
P' = 180 \text{ kPa} + \frac{2\pi^2}{(800 \text{ kg/m}^3) \left( 2.40 \times 10^5 \text{ m}^3 \text{ d} \times \frac{1 \text{ d}}{24 \text{ h}} \times \frac{1 \text{ h}}{3600 \text{ s}} \right)^2}
\]

\[
\times \left( \frac{1}{(0.600 \text{ m})^4} - \frac{1}{(0.300 \text{ m})^4} \right)
\]

\[
= 144 \text{ kPa}
\]

Water flows through a Venturi meter like that in Example 13-11 with a pipe diameter of 9.50 cm and a constriction diameter of 5.60 cm. The U-tube manometer is partially filled with mercury. Find the volume flow rate of the water if the difference in the mercury level in the U-tube is 2.40 cm.

**Picture the Problem** We’ll use its definition to relate the volume flow rate in the pipe to the speed of the water and the result of Example 13-11 to find the speed of the water.

Using its definition, express the volume flow rate:

\[
I_v = A_i v_i = \pi r^2 v_i
\]

Using the result of Example 13-11, find the speed of the water upstream from the Venturi meter:

\[
v_i = \sqrt{\frac{2 \rho_{\text{Hg}} gh}{\rho_u \left( \frac{R_2^2}{R_1^2} - 1 \right)}}
\]

Substituting for \( v_i \) yields:

\[
I_v = \pi r^2 \sqrt{\frac{2 \rho_{\text{Hg}} gh}{\rho_u \left( \frac{R_2^2}{R_1^2} - 1 \right)}}
\]
Substitute numerical values and evaluate \( I_V \):

\[
I_V = \frac{\pi}{4} (0.0950 \text{ m})^2 \sqrt{\frac{2(13.6 \times 10^3 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(0.0240 \text{ m})}{(1.00 \times 10^2 \text{ kg/m}^3) \left( \frac{(0.0950 \text{ m})^2}{0.0560 \text{ m}} - 1 \right)}} = 13.1 \text{ L/s}
\]

61 [SSM] Horizontal flexible tubing for carrying cooling water flows through a large electromagnet used in your physics experiment at Fermi National Accelerator Laboratory. A minimum volume flow rate of 0.050 L/s through the tubing is necessary in order to keep your magnet cool. Within the magnet volume, the tubing has a circular cross section of radius 0.500 cm. In regions outside the magnet, the tubing widens to a radius of 1.25 cm. You have attached pressure sensors to measure differences in pressure between the 0.500 cm and 1.25 cm sections. The lab technicians tell you that if the flow rate in the system drops below 0.050 L/s, the magnet is in danger of overheating and that you should install an alarm to sound a warning when the flow rate drops below that level. What is the critical pressure difference at which you should program the sensors to send the alarm signal (and is this a minimum, or maximum, pressure difference)? Assume laminar nonviscous steady-state flow.

**Picture the Problem** The pictorial representation shows the narrowing of the cold-water supply tubes as they enter the magnet. We can apply Bernoulli’s equation and the continuity equation to derive an expression for the pressure difference \( P_1 - P_2 \).

Apply Bernoulli’s equation to the two sections of tubing to obtain:

\[
P_1 + \frac{1}{2} \rho v_1^2 = P_2 + \frac{1}{2} \rho v_2^2
\]

Solving for the pressure difference \( P_1 - P_2 \) yields:

\[
\Delta P = P_1 - P_2 = \frac{1}{2} \rho v_2^2 - \frac{1}{2} \rho v_1^2 = \frac{1}{2} \rho (v_2^2 - v_1^2)
\]

Factor \( v_1^2 \) from the parentheses to obtain:

\[
\Delta P = \frac{1}{2} \rho v_1^2 \left( \frac{v_2^2}{v_1^2} - 1 \right) = \frac{1}{2} \rho v_1^2 \left( \left( \frac{v_2}{v_1} \right)^2 - 1 \right)
\]

From the continuity equation we have:

\[
A_1 v_1 = A_2 v_2
\]
Solving for the ratio of $v_2$ to $v_1$, expressing the areas in terms of the diameters, and simplifying yields:

$$\frac{v_2}{v_1} = \frac{A_1}{A_2} = \frac{\frac{1}{4} \pi d_1^2}{\frac{1}{4} \pi d_2^2} = \left(\frac{d_1}{d_2}\right)^2$$

Use the expression for the volume flow rate to express $v_1$:

$$v_1 = \frac{I_v}{A_1} = \frac{I_v}{\frac{1}{4} \pi d_1^2} = \frac{4I_v}{\pi d_1^2}$$

Substituting for $v_1$ and $v_2/v_1$ in the expression for $\Delta P$ yields:

$$\Delta P = \frac{1}{2} \rho \left(\frac{4I_v}{\pi d_1^2}\right)^2 \left(\frac{d_1}{d_2}\right)^4 - 1$$

Substitute numerical values and evaluate $\Delta P$:

$$\Delta P = \frac{1}{2} \left(1.00 \times 10^3 \text{ kg/m}^3\right) \left(\frac{4 \left(0.050 \frac{\text{L}}{\text{s}} \times 10^{-3} \text{ m}^3\right)}{\pi (0.0250 \text{ m})^2}\right)^2 \left(\frac{2.50 \text{ cm}}{1.00 \text{ cm}}\right)^4 - 1 = 0.20 \text{ kPa}$$

Because $\Delta P \propto I_v^2$ ($\Delta P$ as a function of $I_v$ is a parabola that opens upward), this pressure difference is the minimum pressure difference.

62 Figure 13-37 shows a Pitot-static tube, a device used for measuring the speed of a gas. The inner pipe faces the incoming fluid, while the ring of holes in the outer tube is parallel to the gas flow. Show that the speed of the gas is given by

$$v = \sqrt{\frac{2gh(\rho_L - \rho_g)}{\rho_g}}$$

by $v = \sqrt{\frac{2gh(\rho_L - \rho_g)}{\rho_g}}$, where $\rho_L$ is the density of the liquid used in the manometer and $\rho_g$ is the density of the gas.

**Picture the Problem** Let the numeral 1 denote the opening in the end of the inner pipe and the numeral 2 to one of the holes in the outer tube. We can apply Bernoulli’s principle at these locations and solve for the pressure difference between them. By equating this pressure difference to the pressure difference due to the height $h$ of the liquid column we can express the speed of the gas as a function of $\rho_L$, $\rho_g$, $g$, and $h$.

Apply Bernoulli’s principle at locations 1 and 2 to obtain:

$$P_1 + \frac{1}{2} \rho_g v_1^2 = P_2 + \frac{1}{2} \rho_g v_2^2$$

where we’ve ignored the difference in elevation between the two openings.

Solve for the pressure difference $\Delta P = P_1 - P_2$:

$$\Delta P = P_1 - P_2 = \frac{1}{2} \rho_g v_2^2 - \frac{1}{2} \rho_g v_1^2$$
Express the speed of the gas at 1: Because the gas is brought to a halt (that is, is stagnant) at the opening to the inner pipe, \( v_1 = 0 \).

Express the speed of the gas at 2: Because the gas flows freely past the holes in the outer ring, \( v_2 = v \).

Substitute to obtain: \( \Delta P = \frac{1}{2} \rho g v^2 \)

Letting \( A \) be the cross-sectional area of the tube, express the pressure at a depth \( h \) in the column of liquid whose density is \( \rho_l \):

\[
P_1 = P_2 + \frac{\text{weight of liquid}}{A} - \frac{B}{A}
\]

where \( B = \rho g Ah \) is the buoyant force acting on the column of liquid of height \( h \).

Substitute to obtain:

\[
P_1 = P_2 + \frac{\rho_l gh A}{A} - \frac{\rho g gh A}{A}
= P_2 + (\rho_l - \rho g)gh
\]

or \( \Delta P = P_1 - P_2 = (\rho_l - \rho g)gh \)

Equate these two expressions for \( \Delta P \):

\[
\frac{1}{2} \rho g v^2 = (\rho_l - \rho g)gh
\]

Solving for \( v \) yields:

\[
v = \sqrt{\frac{2gh(\rho_l - \rho g)}{\rho g}}
\]

Note that the correction for buoyant force due to the displaced gas is very small and that, to a good approximation, \( v = \sqrt{\frac{2gh \rho_l}{\rho g}} \).

Remarks: Pitot tubes are used to measure the airspeed of airplanes.

63 [SSM] Derive the Bernoulli Equation in more generality than done in the text, that is, allow for the fluid to change elevation during its movement. Using the work-energy theorem, show that when changes in elevation are allowed, Equation 13-16 becomes

\[
P_1 + \rho gh_1 + \frac{1}{2} \rho v_1^2 = P_2 + \rho gh_2 + \frac{1}{2} \rho v_2^2
\]

(Equation 13-17).

Picture the Problem Consider a fluid flowing in a tube that varies in elevation as well as in cross-sectional area, as shown in the pictorial representation below. We can apply the work-energy theorem to a parcel of fluid that initially is contained...
between points 1 and 2. During time $\Delta t$ this parcel moves along the tube to the region between point 1’ and 2’. Let $\Delta V$ be the volume of fluid passing point 1’ during time $\Delta t$. The same volume passes point 2 during the same time. Also, let $\Delta m = \rho \Delta V$ be the mass of the fluid with volume $\Delta V$. The net effect on the parcel during time $\Delta t$ is that mass $\Delta m$ initially at height $h_1$ moving with speed $v_1$ is “transferred” to height $h_2$ with speed $v_2$.

Express the work-energy theorem:

$$W_{\text{total}} = \Delta U + \Delta K$$  \hspace{1cm} (1)

The change in the potential energy of the parcel is given by:

$$\Delta U = (\Delta m) g h_2 - (\Delta m) g h_1$$

$$= (\Delta m) g (h_2 - h_1)$$

$$= \rho \Delta V g (h_2 - h_1)$$

The change in the kinetic energy of the parcel is given by:

$$\Delta K = \frac{1}{2} (\Delta m) v_2^2 - \frac{1}{2} (\Delta m) v_1^2$$

$$= \frac{1}{2} (\Delta m) (v_2^2 - v_1^2)$$

$$= \frac{1}{2} \rho \Delta V (v_2^2 - v_1^2)$$

Express the work done by the fluid behind the parcel (to the parcel’s left in the diagram) as it pushes on the parcel with a force of magnitude $F_1 = P_1 A_1$, where $P_1$ is the pressure at point 1:

$$W_1 = F_1 \Delta x_1 = P_1 A_1 \Delta x_1 = P_1 \Delta V$$

Express the work done by the fluid in front of the parcel (to the parcel’s right in the diagram) as it pushes on the parcel with a force of magnitude $F_2 = P_2 A_2$, where $P_2$ is the pressure at point 2:

$$W_2 = -F_2 \Delta x_2 = -P_2 A_2 \Delta x_2 = -P_2 \Delta V$$

where the work is negative because the applied force and the displacement are in opposite directions.

The total work done on the parcel is:

$$W_{\text{total}} = P_1 \Delta V - P_2 \Delta V = (P_1 - P_2) \Delta V$$
Substitute for $\Delta U$, $\Delta K$, and $W_{\text{total}}$ in equation (1) to obtain:

$$ (P_1 - P_2)\Delta V = \rho \Delta V g (h_2 - h_1) + \frac{1}{2} \rho \Delta V (v_2^2 - v_1^2) $$

Simplifying this expression by dividing out $\Delta V$ yields:

$$ P_1 - P_2 = \rho g (h_2 - h_1) + \frac{1}{2} \rho (v_2^2 - v_1^2) $$

Collect all the quantities having a subscript 1 on one side and those having a subscript 2 on the other to obtain:

$$ P_1 + \rho g h_1 + \frac{1}{2} \rho v_1^2 = P_2 + \rho g h_2 + \frac{1}{2} \rho v_2^2 $$

Remarks: This equation is known as Bernoulli’s equation for the steady, nonviscous flow of an incompressible fluid.

64  •••  A large root beer keg of height $H$ and cross-sectional area $A_1$ is filled with root beer. The top is open to the atmosphere. At the bottom is a spigot opening of area $A_2$, which is much smaller than $A_1$. (a) Show that when the height of the root beer is $h$, the speed of the root beer leaving the spigot is approximately $\sqrt{2gh}$. (b) Show that if $A_2 << A_1$, the rate of change of the height $h$ of the root beer is given by $\frac{dh}{dt} = -(A_2/A_1)(2gh)^{1/2}$. (c) Find $h$ as a function of time if $h = H$ at $t = 0$. (d) Find the total time needed to drain the keg if $H = 2.00$ m, $A_1 = 0.800$ m$^2$, and $A_2 = 1.00 \times 10^{-4} A_1$. Assume laminar nonviscous flow.

Picture the Problem  We can apply Bernoulli’s equation to the top of the keg and to the spigot opening to determine the rate at which the root beer exits the tank. Because the area of the spigot is much smaller than that of the keg, we can neglect the speed of the root beer at the top of the keg. We’ll use the continuity equation to obtain an expression for the rate of change of the height of the root beer in the keg as a function of the height and integrate this function to find $h$ as a function of time.

(a) Apply Bernoulli’s equation to the beer at the top of the keg and at the spigot:

$$ P_1 + \rho_{\text{beer}} gh_1 + \frac{1}{2} \rho_{\text{beer}} v_1^2 = P_2 + \rho_{\text{beer}} gh_2 + \frac{1}{2} \rho_{\text{beer}} v_2^2 $$

or, because $v_1 \approx 0$, $h_2 = 0$, $P_1 = P_2 = P_{\text{at}}$, and $h_1 = h$,

$$ gh = \frac{1}{2} v_2^2 \Rightarrow v_2 = \sqrt{2gh} $$

(b) Use the continuity equation to relate $v_1$ and $v_2$:

$$ A_1 v_1 = A_2 v_2 $$
Substitute \(-\frac{dh}{dt}\) for \(v_1\) and \(\sqrt{2gh}\) for \(v_2\) to obtain:

\[-A_1\frac{dh}{dt} = A_2\sqrt{2gh}\]

Solving for \(\frac{dh}{dt}\) yields:

\[
\frac{dh}{dt} = \frac{A_2}{A_1}\sqrt{2gh}
\]

(c) Separate the variables in the differential equation to obtain:

\[-\frac{A_1}{A_2}\sqrt{2g} \frac{dh}{\sqrt{h}} = dt\]

Express the integral from \(H\) to \(h\) and 0 to \(t\):

\[-\frac{A_1}{A_2}\sqrt{2g} \int_{H}^{h} \frac{dh}{\sqrt{h}} = \int_{0}^{t} dt\]

Evaluate the integral to obtain:

\[-\frac{2A_1}{A_2}\sqrt{2g} \left(\sqrt{H} - \sqrt{h}\right) = t\]

Solving for \(h\) yields:

\[h = \left(\sqrt{H} - \frac{A_2}{2A_1}\sqrt{2g} t\right)^2\]

(d) Solve \(h(t)\) for the time-to-drain \(t'\):

\[t' = \frac{A_1}{A_2} \sqrt{\frac{2H}{g}}\]

Substitute numerical values and evaluate \(t'\)

\[t' = \frac{A_1}{1.00 \times 10^{-4} A_1} \sqrt{\frac{2(2.00\text{ m})}{9.81\text{ m/s}^2}}\]

\[= 6.39 \times 10^3 \text{ s} = 1\text{ h 46 min}\]

A siphon is a device for transferring a liquid from container to container. The tube shown in Figure 13-38 must be filled to start the siphon, but once this has been done, fluid will flow through the tube until the liquid surfaces in the containers are at the same level. (a) Using Bernoulli’s equation, show that the speed of water in the tube is \(v = \sqrt{2gd}\). (b) What is the pressure at the highest part of the tube?

**Picture the Problem** Let the letter "a" denote the entrance to the siphon tube and the letter "b" denote its exit. Assuming streamline flow between these points, we can apply Bernoulli’s equation to relate the entrance and exit speeds of the water flowing in the siphon to the pressures at either end, the density of the water, and the difference in elevation between the entrance and exit points. We’ll also use the equation of continuity to argue that, provided the surface area of the beaker is large compared to the area of the opening of the tube, the entrance speed of the water is approximately zero.
(a) Apply Bernoulli’s equation at the entrance to the siphon tube (point a) and at its exit (point b):

\[ P_{at} + \frac{1}{2} \rho v_{a}^2 + \rho g (H - h) = P_{at} + \frac{1}{2} \rho v_{b}^2 + \rho g (H - h - d) \]  

(1)

where \( H \) is the height of the containers.

Apply the continuity equation to a point at the surface of the liquid in the container to the left and to point a:

\[ v_{a} A_{a} = v_{\text{surface}} A_{\text{surface}} \]

or, because \( A_{a} \ll A_{\text{surface}} \),

\[ v_{a} = v_{\text{surface}} = 0 \]

With \( v_{a} = 0 \) and \( v_{b} = v \), equation (1) becomes:

\[ P_{at} + \rho g (H - h) = P_{at} + \frac{1}{2} \rho v^2 + \rho g (H - h - d) \Rightarrow v = \sqrt{2gd} \]

(b) Relate the pressure at the highest part of the tube \( P_{\text{top}} \) to the pressure at point b:

\[ P_{\text{top}} + \rho g (H - \hat{h}) + \frac{1}{2} \rho v_{h}^2 = P_{at} + \rho g (H - h - d) + \frac{1}{2} \rho v_{b}^2 \]

or, because \( v_{h} = v_{b} \),

\[ P_{\text{top}} = P_{at} - \rho gd \]

Remarks: If we let \( P_{\text{top}} = 0 \), we can use this result to find the maximum theoretical height a siphon can lift water.

66  

A fountain designed to spray a column of water 12 m into the air has a 1.0-cm-diameter nozzle at ground level. The water pump is 3.0 m below the ground. The pipe to the nozzle has a diameter of 2.0 cm. Find pump pressure necessary if the fountain is to operate as designed. (Assume laminar nonviscous steady-state flow.)

**Picture the Problem** Let the letter P denote the pump and the 2.0-cm diameter pipe and the letter N the 1-cm diameter nozzle. We’ll use Bernoulli’s equation to express the necessary pump pressure, the continuity equation to relate the speed of the water coming out of the pump to its speed at the nozzle, and a constant-acceleration equation to relate its speed at the nozzle to the height to which the water rises.

Using Bernoulli’s equation, relate the pressures, areas, and velocities in the pipe and nozzle:

\[ P_{p} + \rho_{w} gh_{p} + \frac{1}{2} \rho_{w} v_{p}^2 = P_{N} + \rho_{w} gh_{N} + \frac{1}{2} \rho_{w} v_{N}^2 \]

or, because \( P_{N} = P_{at} \) and \( h_{p} = 0 \),

\[ P_{p} + \frac{1}{2} \rho_{w} v_{p}^2 = P_{N} + \rho_{w} gh_{N} + \frac{1}{2} \rho_{w} v_{N}^2 \]
Solve for the pump pressure:

\[ P_p = P_at + \rho_w g h_N + \frac{1}{2} \rho_w \left( v_N^2 - v_p^2 \right) \quad (1) \]

Use the continuity equation to relate \( v_p \) and \( v_N \) to the cross-sectional areas of the pipe from the pump and the nozzle:

\[ A_p v_p = A_N v_N \]

and

\[ v_p = \frac{A_N}{A_p} v_N = \frac{\frac{1}{4} \pi d_p^2}{\frac{1}{4} \pi d_N^2} v_N = \left( \frac{1.0 \text{ cm}}{2.0 \text{ cm}} \right)^2 v_N = \frac{1}{4} v_N \]

Use a constant-acceleration equation to express the speed of the water at the nozzle in terms of the desired height \( \Delta h \):

\[ v^2 = v_N^2 - 2 g \Delta h \]

or, because \( v = 0 \),

\[ v_N^2 = 2 g \Delta h \]

Substitute for \( v_N \) and \( v_p \) in equation (1) and simplify to obtain:

\[ P_p = P_at + \rho_w g h_N + \frac{1}{2} \rho_w \left[ 2 g \Delta h - \frac{1}{16} \left( 2 g \Delta h \right) \right] = P_at + \rho_w g h_N + \frac{1}{2} \rho_w \left( \frac{15}{16} g \Delta h \right) \]

\[ = P_at + \rho_w g \left( h_N + \frac{15}{16} \Delta h \right) \]

Substitute numerical values and evaluate \( P_p \):

\[ P_p = 101.325 \text{ kPa} + \left( 1.00 \times 10^3 \text{ kg/m}^3 \right) \left( 9.81 \text{ m/s}^2 \right) \left[ 3.0 \text{ m} + \frac{15}{16} \left( 12 \text{ m} \right) \right] = 2.4 \times 10^5 \text{ Pa} \]

67 Water at 20°C exits a circular tap moving straight down with a flow rate of 10.5 cm\(^3\)/s. (a) If the diameter of the tap is 1.20 cm, what is the speed of the water? (b) As the fluid falls from the tap, the stream of water narrows. Find the new diameter of the stream at a point 7.50 cm below the tap. Assume that the stream still has a circular cross section and neglect any effects of drag forces acting on the water. (c) If turbulent flows are characterized by Reynolds numbers above 2300 or so, how far does the water have to fall before it becomes turbulent? Does this match your everyday observations?

**Picture the Problem** Let \( I \) represent the flow rate of the water. Then we can use \( I = Av \) to relate the flow rate to the cross-sectional area of the circular tap and the speed of the water. In (b) we can use the equation of continuity to express the diameter of the stream 7.50 cm below the tap and a constant-acceleration equation to find the speed of the water at this distance. In (c) we can use a constant-acceleration equation to express the distance-to-turbulence in terms of the speed of the water at turbulence \( v_t \) and the definition of Reynolds number \( N_R \) to relate \( v_t \) to \( N_R \).
(a) Express the flow rate of the water in terms of the cross-sectional area $A$ of the circular tap and the speed $v_i$ of the water:

$$I = Av_i = \pi r^2 v_i = \frac{1}{4} \pi d^2 v_i \quad (1)$$

Solving for $v_i$ yields:

$$v_i = \frac{I}{\frac{1}{4} \pi d^2}$$

Substitute numerical values and evaluate $v_i$:

$$v_i = \frac{10.5 \text{ cm}^3/\text{s}}{\frac{1}{4} \pi (1.20 \text{ cm})^2} = 9.28 \text{ cm/s}$$

(b) Apply the equation of continuity to the stream of water:

$$v_f A_f = v_i A_i$$

or

$$v_f \frac{\pi}{4} d_f^2 = v_i \frac{\pi}{4} d_i^2 \Rightarrow d_f = \sqrt{\frac{v_i}{v_f}} d_i \quad (2)$$

Use a constant-acceleration equation to relate $v_f$ and $v_i$ to the distance $\Delta h$ fallen by the water:

$$v_f^2 = v_i^2 + 2g\Delta h \Rightarrow v_f = \sqrt{v_i^2 + 2g\Delta h}$$

Substitute numerical values and evaluate $v_f$:

$$v_f = \sqrt{(9.28 \text{ cm/s})^2 + 2(981 \text{ cm/s}^2)(7.50 \text{ cm})} = 122 \text{ cm/s}$$

Substitute numerical values in equation (2) and evaluate $d_f$:

$$d_f = (1.20 \text{ cm}) \sqrt{\frac{9.28 \text{ cm/s}}{122 \text{ cm/s}}} = 0.331 \text{ cm}$$

(c) Use a constant-acceleration equation to relate the fall-distance-to-turbulence $\Delta d$ to its initial speed $v_i$ and its speed $v_t$ when its flow becomes turbulent:

$$v_t^2 = v_i^2 + 2g\Delta d \Rightarrow \Delta d = \frac{v_t^2 - v_i^2}{2g} \quad (3)$$

Express Reynolds number $N_R$ for turbulent flow:

$$N_R = \frac{2r \rho v_i}{\eta} \quad (4)$$

The volume flow rate equals $Av_i$. Express the volume flow rate at the speed $v_i$:

$$I = \pi r^2 v_i \quad (5)$$

Eliminate $r$ from equations (4) and (5) and solve for $v_i$ (see Table 13-1 for the coefficient of viscosity for water):
Chapter 13

1310  Chapter 13

Substitute numerical values (see Figure 13-1 for the density of water and Table 13-3 for the coefficient of viscosity for water) and evaluate \( v_1 \):

\[
v_1 = \frac{\pi (2300)^2 (1.00 \times 10^{-3} \text{ Pa} \cdot \text{s})^2}{4 (1.00 \times 10^3 \text{ kg/m}^3) (10.5 \text{ cm}^2/\text{s})} = 0.396 \text{ m/s}
\]

Substitute numerical values in equation (3) and evaluate the fall-distance-to turbulence:

\[
\Delta d = \frac{(39.6 \text{ cm/s})^2 - (9.28 \text{ cm/s})^2}{2 (981 \text{ cm/s}^2)} \approx 76 \text{ cm}
\]

in reasonable agreement with everyday experience.

68  To better fight fires in your seaside community, the local fire brigade has asked you to set up a pump system to draw seawater from the ocean to the top of a steep cliff adjacent to the water where most of the homes are. If the cliff is 12.0 m high, and the pump is capable of producing a gauge pressure of 150 kPa, how much water (in L/s) can be pumped using a hose with a radius of 4.00 cm?

Picture the Problem The water being drawn from the ocean is at rest initially and is pumped upward to a height of 12.0 m. Let the variables subscripted 1 correspond to the intake and those subscripted 2 to the outflow and use the volume flow rate equation and Bernoulli’s equation.

Express the volume flow rate in the hose:

\[
I_v = Av = \pi r^2 v_2 \tag{1}
\]

Apply Bernoulli’s equation to the intake and outflow to obtain:

\[
P_1 + \rho g h_1 + \frac{1}{2} \rho v_1^2 = P_2 + \rho g h_2 + \frac{1}{2} \rho v_2^2
\]

Rewriting this equation to isolate the speed terms yields:

\[
\frac{1}{2} \rho (v_2^2 - v_1^2) = P_1 - P_2 + \rho g h_1 - \rho g h_2
\]

Because \( v_1 = 0 \) and \( h_1 = 0 \) and \( P_1 - P_2 = \Delta P \):

\[
\frac{1}{2} \rho v_2^2 = \Delta P - \rho g h_2
\]

Solving for \( v_2 \) yields:

\[
v_2 = \sqrt{\frac{2(\Delta P - \rho g h_2)}{\rho}}
\]

Substitute for \( v_2 \) in equation (1) to obtain:

\[
I_v = \pi r^2 \sqrt{\frac{2(\Delta P - \rho g h_2)}{\rho}}
\]
Substitute numerical values and evaluate $I_V$:

$$I_V = \pi (0.0400 \text{ m})^2 \sqrt{\frac{2(150 \text{ kPa} - (1.00 \times 10^3 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(12.0 \text{ m})}{1.00 \times 10^3 \text{ kg/m}^3}}$$

$$= 4.039 \times 10^{-2} \text{ m}^3/\text{s} \times \frac{1 \text{ L}}{10^{-3} \text{ m}^3} = 40.4 \text{ L/s}$$

69  ...  In Figure 13-39, $H$ is the depth of the liquid and $h$ is the distance from the surface of the liquid to the pipe inserted in the tank’s side. (a) Find the distance $x$ at which the water strikes the ground as a function of $h$ and $H$. (b) Show that, for a given value of $H$, there are two values of $h$ (whose average value is $\frac{1}{2}H$) that give the same distance $x$. (c) Show that, for a given value of $H$, $x$ is a maximum when $h = \frac{1}{2}H$. Find the maximum value for $x$ as a function of $H$.

Picture the Problem  We can apply Bernoulli’s equation to points $a$ and $b$ to determine the rate at which the water exits the tank. Because the diameter of the small pipe is much smaller than the diameter of the tank, we can neglect the speed of the water at the point $a$. The distance the water travels once it exits the pipe is the product of its speed and the time required to fall the distance $H - h$. That there are two values of $h$ that are equidistant from the point $h = \frac{1}{2}H$ can be shown by solving the quadratic equation that relates $x$ to $h$ and $H$. That $x$ is a maximum for this value of $h$ can be established by treating $x = f(h)$ as an extreme-value problem.

(a) Express the distance $x$ as a function of the exit speed of the water and the time to fall the distance $H - h$:

$$x = v_b \Delta t \quad (1)$$

Apply Bernoulli’s equation to the water at points $a$ and $b$:

$$P_a + \rho_w gH + \frac{1}{2} \rho_w v_a^2 = P_b + \rho_w g(H - h) + \frac{1}{2} \rho_w v_b^2$$

or, because $v_a \approx 0$ and $P_a = P_b = P_at$,

$$gH = g(H - h) + \frac{1}{2} v_b^2 \Rightarrow v_b = \sqrt{2gh}$$

Using a constant-acceleration equation, relate the time of fall to the distance of fall:

$$\Delta y = v_{0y} \Delta t + \frac{1}{2} a(\Delta t)^2$$

or, because $v_{0y} = 0$,

$$H - h = \frac{1}{2} g(\Delta t)^2 \Rightarrow \Delta t = \sqrt{\frac{2(H - h)}{g}}$$
Substitute for $\Delta t$ in equation (1) to obtain:

$$x = \sqrt{2gh} \sqrt{\frac{2(H-h)}{g}} = 2\sqrt{h(H-h)}$$

(b) Square both sides of this equation and simplify to obtain:

$$x^2 = 4hH - 4h^2 \text{ or } 4h^2 - 4Hh + x^2 = 0$$

Solving this quadratic equation yields:

$$h = \frac{1}{2} H \pm \frac{1}{2} \sqrt{H^2 - x^2}$$

Find the average of these two values for $h$:

$$h_{av} = \frac{\frac{1}{2} H + \sqrt{H^2 - x^2} + \frac{1}{2} H - \sqrt{H^2 - x^2}}{2} = \frac{1}{2} H$$

(c) Differentiate $x = 2\sqrt{h(H-h)}$ with respect to $h$:

$$\frac{dx}{dh} = 2\left(\frac{1}{2}\right)\left[h(h-h)\right]^{-\frac{1}{2}}(H-2h)$$

$$= \frac{H-2h}{\sqrt{h(H-h)}}$$

Set $dx/dh$ equal to zero for extrema:

$$\frac{H-2h}{\sqrt{h(H-h)}} = 0$$

Solve for $h$ to obtain:

$$h = \frac{1}{2} H$$

Evaluate $x = 2\sqrt{h(H-h)}$ with $h = \frac{1}{2} H$:

$$x_{max} = 2\sqrt{\frac{1}{2} H(H - \frac{1}{2} H)} = H$$

Remarks: To show that this value for $h$ corresponds to a maximum, one can either show that $\frac{d^2x}{dh^2} < 0$ at $h = \frac{1}{2} H$ or confirm that the graph of $f(h)$ at $h = \frac{1}{2} H$ is concave downward.

*Viscous Flow*

70 · Water flows through a horizontal 25.0-cm-long tube with an inside diameter of 1.20 mm at 0.300 mL/s. Find the pressure difference required to drive this flow if the viscosity of water is 1.00 mPa s. Assume laminar flow.

**Picture the Problem** The required pressure difference can be found by applying Poiseuille’s law to the viscous flow of water through the horizontal tube.
Using Poiseuille’s law, relate the pressure difference between the two ends of the tube to its length, radius, and the volume flow rate of the water:

\[ \Delta P = \frac{8\eta L}{\pi r^4} I_v \]

Substitute numerical values and evaluate \( \Delta P \):

\[ \Delta P = \frac{8 \times 1.00 \text{ mPa} \cdot \text{s}}{\pi} \left( \frac{0.250 \text{ m}}{2} \right)^4 \left( 0.300 \text{ mL/s} \right) = 1.47 \text{ kPa} \]

**Picture the Problem** Because the pressure difference is unchanged, we can equate the expressions of Poiseuille’s law for the two tubes and solve for the diameter of the tube that would double the flow rate.

Using Poiseuille’s law, express the pressure difference required for the radius and volume flow rate of Problem 70:

\[ \Delta P = \frac{8\eta L}{\pi r'^4} \]

Express the pressure difference required for the radius \( r' \) that would double the volume flow rate of Problem 70:

\[ \Delta P = \frac{8\eta L}{\pi r''^4} \left( 2I_v \right) \]

Equate these equations and simplify to obtain:

\[ \frac{8\eta L}{\pi r'^4} \left( 2I_v \right) = \frac{8\eta L}{\pi r''^4} I_v \]

or

\[ \frac{2}{r'^4} = \frac{1}{r''^4} \Rightarrow r'^4 = 4r''^4 \]

Because \( d' \) is two times \( r' \):

\[ d' = 2r' = 2\sqrt[4]{2}r = \sqrt[4]{2}d \]

Substitute numerical values and evaluate \( d' \):

\[ d' = \sqrt[4]{2} \times (1.20 \text{ mm}) = 1.43 \text{ mm} \]
Blood takes about 1.00 s to pass through a 1.00-mm-long capillary in the human circulatory system. If the diameter of the capillary is 7.00 \( \mu \text{m} \) and the pressure drop is 2.60 kPa, find the viscosity of blood. Assume laminar flow.

**Picture the Problem** We can apply Poiseuille’s law to relate the pressure drop across the capillary tube to the radius and length of the tube, the rate at which blood is flowing through it, and the viscosity of blood.

Using Poiseuille’s law, relate the pressure drop to the length and diameter of the capillary tube, the volume flow rate of the blood, and the viscosity of the blood:

\[ \Delta P = \frac{8\eta L}{\pi r^4} I_v \Rightarrow \eta = \frac{\pi r^4 \Delta P}{8LI_v} \]

Using its definition, express the volume flow rate of the blood:

\[ I_v = A_{\text{cap}} v = \pi r^2 v \]

Substitute for \( I_v \) and simplify to obtain:

\[ \eta = \frac{r^2 \Delta P}{8L v} \]

Substitute numerical values to obtain:

\[ \eta = \frac{(3.50 \times 10^{-6} \text{ m})^2(2.60 \text{ kPa})}{8(1.00 \times 10^{-3} \text{ m})\left(\frac{1.00 \times 10^{-3} \text{ m}}{1.00 \text{ s}}\right)} = \frac{3.98 \text{ mPa s}}{} \]

An abrupt transition occurs at Reynolds numbers of about \( 3 \times 10^5 \), where the drag on a sphere moving through a fluid abruptly decreases. Estimate the speed at which this transition occurs for a baseball, and comment on whether it should play a role in the physics of the game.

**Picture the Problem** We can use the definition of Reynolds number to find the speed of a baseball at which the drag crisis occurs.

Using its definition, relate Reynolds number to the speed \( v \) of the baseball:

\[ N_R = \frac{2\rho v}{\eta} \Rightarrow v = \frac{\eta N_R}{2\rho} \]

Substitute numerical values (see Figure 13-1 for the density of air and Table 13-3 for the coefficient of viscosity for air) and evaluate \( v \):

\[ v = \frac{(0.018 \text{ mPa s})(3 \times 10^5)}{2(0.05 \text{ m})(1.293 \text{ kg/m}^3)} = 41.8 \text{ m/s} \times \frac{1 \text{ mi/h}}{0.447 \text{ m/s}} \approx 90 \text{ mi/h} \]
Because most major league pitchers can throw a fastball in the low-to-mid-90s, this abrupt decrease in drag may very well play a role in the game.

Remarks: This is a topic that has been fiercely debated by people who study the physics of baseball.

74  ••  A horizontal pipe of radius 1.5 cm and length 25 m is connected to the output that can sustain an output gauge pressure of 19 kPa. What is the speed of 20°C water flowing through the pipe? If the temperature of the water is 60°C, what is the speed of the water in the pipe?

Picture the Problem Assuming laminar flow of the water, we can apply Poiseuille’s law to find the speed of the water coming out the outflow end of the pipe.

Use Poiseuille’s law to relate the pressure difference to the volume flow rate in the pipe: \[ \Delta P = \frac{8\eta L}{\pi r^4} I_v \]

Substituting for \( I_v \) yields: \[ \Delta P = \frac{8\eta L}{\pi r^4} I_v = \frac{8\eta L}{\pi r^4} A v \]

Substitute for the cross-sectional area \( A \) of the pipe and simplify to obtain: \[ \Delta P = \frac{8\eta L}{\pi r^4} \pi r^2 v = \frac{8\eta L}{r^2} v \]

Solving for \( v \) yields: \[ v = \frac{r^2 \Delta P}{8\eta L} \]

Substitute numerical values and evaluate \( v(20°C) \): \[ v(20°C) = \frac{(0.015 \text{ m})^2(10 \text{ kPa})}{8(1.00 \text{ mPa} \cdot \text{s})(25 \text{ m})} = 11 \text{ m/s} \]

Substitute numerical values and evaluate \( v(60°C) \): \[ v(60°C) = \frac{(0.015 \text{ m})^2(10 \text{ kPa})}{8(0.65 \text{ mPa} \cdot \text{s})(25 \text{ m})} = 17 \text{ m/s} \]

75  ••  A very large tank is filled to a depth of 250 cm with oil that has a density of 860 kg/m³ and a viscosity of 180 mPa-s. If the container walls are 5.00 cm thick, and a cylindrical hole of radius 0.750 cm has been bored through the base of the container, what is the initial volume flow rate (in L/s) of the oil through the hole?
**Picture the Problem** Assuming laminar flow, we can apply Poiseuille’s law to relate the pressure difference between the inside and the outside of the container at its base to the volume flow rate of oil out of the hole. We can find the pressure difference from the expression for the pressure as a function of depth in a fluid.

Use Poiseuille’s law to relate the pressure difference to the volume flow rate of oil out of the hole:

\[
\Delta P = \frac{8\eta L}{\pi r^4} I_v \Rightarrow I_v = \frac{\pi r^4 \Delta P}{8\eta L} \quad (1)
\]

The pressure at the bottom of the container of oil is given by:

\[ P = P_0 + \rho_{oil} gh \]

The pressure difference between the inside and outside of the container is:

\[ \Delta P = P - P_0 = \rho_{oil} gh \]

Substituting for \( \Delta P \) in equation (1) yields:

\[ I_v = \frac{\pi r^4 \rho_{oil} gh}{8\eta L} \]

Substitute numerical values and evaluate \( I_v \):

\[
I_v = \frac{\pi (0.0075 \text{ m})^4 (860 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(2.50 \text{ m})}{8(180 \text{ mPa} \cdot \text{s})(0.0500 \text{ m})} = 2.91 \times 10^{-3} \text{ m}^3/\text{s} = 2.91 \text{ L/s}
\]

The drag force on a moving sphere at very low Reynolds number is given by \( F_D = 6\pi \eta a v \), where \( \eta \) is the viscosity of the surrounding fluid and \( a \) is the radius of the sphere. (This relation is called Stokes’ law.) Using this information, find the terminal speed of ascent for a spherical 1.0-mm diameter carbon dioxide bubble of 1.0-mm diameter rising in a carbonated beverage (\( \rho = 1.1 \text{ kg/L} \) and \( \eta = 1.8 \text{ mPa} \cdot \text{s} \)). How long should it take for this bubble to rise 20 cm (the height of a drinking glass)? Is this length of time consistent with your observations?

**Picture the Problem** Let the subscripts “f” refer to “displaced fluid”, “s” to “soda”, and “g” to the “gas” in the bubble. The free-body diagram shows the forces acting on the bubble prior to reaching its terminal speed. We can apply Newton’s 2nd law, Stokes’ law, and Archimedes principle to express the terminal speed of the bubble in terms of its radius, and the viscosity and density of water.
Apply $\sum F_y = m a_y$ to the bubble to obtain:

$$B - m_g g - F_D = ma_y$$

Under terminal speed conditions:

$$B - m_g g - F_D = 0$$

Using Archimedes principle, express the buoyant force $B$ acting on the bubble:

$$B = w_i = m_i g = \rho_i V_g g = \rho_i V_{\text{bubble}} g$$

Express the mass of the gas bubble:

$$m_g = \rho_g V_g = \rho_g V_{\text{bubble}}$$

Substitute to obtain:

$$\rho_i V_{\text{bubble}} g - \rho_g V_{\text{bubble}} g - 6\pi \eta a v_i = 0$$

Solve for $v_i$:

$$v_i = \frac{V_{\text{bubble}} g (\rho_s - \rho_i)}{6\pi \eta a}$$

Substitute for $V_{\text{bubble}}$ and simplify:

$$v_i = \frac{4 \pi a^3 g (\rho_s - \rho_g)}{6\pi \eta a} = \frac{2 a^2 g (\rho_s - \rho_g)}{9 \eta}$$

$$\approx \frac{2 a^2 g \rho_s}{9 \eta}, \text{ since } \rho_s \gg \rho_g.$$ 

Substitute numerical values and evaluate $v_i$:

$$v_i = \frac{2(0.50 \times 10^{-3} \text{ m})^3 (9.81 \text{ m/s}^2)(1.1 \times 10^3 \text{ kg/m}^3)}{9(1.8 \times 10^3 \text{ Pa} \cdot \text{s})} = 0.333 \text{ m/s} = \boxed{0.33 \text{ m/s}}$$

Express the rise time $\Delta t$ in terms of the height of the soda glass $h$ and the terminal speed of the bubble:

$$\Delta t = \frac{h}{v_i}$$

Assuming that a "typical" soda glass has a height of about 15 cm, evaluate $\Delta t$:

$$\Delta t = \frac{0.15 \text{ m}}{0.333 \text{ m/s}} = \boxed{0.45 \text{ s}}$$

Remarks: About half a second seems reasonable for the rise time of the bubble.

General Problems

77 • [SSM] Several teenagers are swimming toward a rectangular, wooden raft that is 3.00 m wide and 2.00 m long. If the raft is 9.00 cm thick, how many 75.0 kg teenage boys can stand on top of the raft without the raft becoming submerged? Assume the wood density is 650 kg/m$^3$. 
**Picture the Problem** If the raft is to be just barely submerged, then the buoyant force on it will be equal in magnitude to the weight of the raft plus the weight of the boys. We can apply Archimedes’ principle to find the buoyant force on the raft.

The buoyant force acting on the raft is the sum of the weights of the raft and the boys:

\[ B = w_{\text{raft}} + w_{\text{boys}} \quad (1) \]

Express the buoyant force acting on the raft:

\[ B = \rho_{\text{water}} V_{\text{displaced water}} g \]
\[ = \rho_{\text{water}} V_{\text{raft}} g \]

Express the weight of the raft:

\[ w_{\text{raft}} = m_{\text{raft}} g = \rho_{\text{raft}} V_{\text{raft}} g \]

Express the weight of the boys:

\[ w_{\text{boys}} = m_{\text{boys}} g = N m_{1\text{boy}} g \]

Substituting for \( B \), \( w_{\text{raft}} \), and \( w_{\text{boys}} \) in equation (1) yields:

\[ \rho_{\text{water}} V_{\text{raft}} g = \rho_{\text{raft}} V_{\text{raft}} g + N m_{1\text{boy}} g \]

Solve for \( N \) and simplify to obtain:

\[ N = \frac{\rho_{\text{water}} V_{\text{raft}} g - \rho_{\text{raft}} V_{\text{raft}} g}{m_{1\text{boy}} g} \]
\[ = \frac{(\rho_{\text{water}} - \rho_{\text{raft}}) V_{\text{raft}}}{m_{1\text{boy}}} \]

Substitute numerical values and evaluate \( N \):

\[ N = \frac{(1.00 \times 10^3 \text{ kg/m}^3 - 650 \text{ kg/m}^3)(3.00 \text{ m})(2.00 \text{ m})(0.0900 \text{ m})}{75.0 \text{ kg}} = 2.52 \]

Hence, a maximum of \( 2 \) boys can be on the raft under these circumstances.

---

78 A thread attaches a 2.7-g Ping-Pong ball to the bottom of a beaker. When the beaker is filled with water so that the ball is totally submerged, the tension in the thread is 7.0 mN. Determine the diameter of the ball.
**Picture the Problem** The forces acting on the ball, shown in the free-body diagram, are the buoyant force, the weight of the ball, and the tension in the string. Because the ball is in equilibrium under the influence of these forces, we can apply the condition for translational equilibrium to establish the relationship between them. We can also apply Archimedes’ principle to relate the buoyant force on the ball to its diameter.

Apply \( \sum F_y = 0 \) to the ball:

\[ B - mg - T = 0 \]

Using Archimedes’ principle, relate the buoyant force on the ball to its diameter:

\[ B = \rho_{\text{water}} V_{\text{ball}} g = \frac{4}{3} \pi \rho_{\text{water}} g r^3 \]

Substituting for \( B \) yields:

\[ \frac{4}{3} \pi \rho_{\text{water}} g r^3 - mg - T = 0 \]

Solve for \( r \):

\[ r = \left( \frac{3(T + mg)}{4\pi \rho_{\text{water}} g} \right)^{\frac{1}{3}} \Rightarrow d = 2 \left( \frac{3(T + mg)}{4\pi \rho_{\text{water}} g} \right)^{\frac{1}{3}} \]

Substitute numerical values and evaluate \( d \):

\[ d = 2 \left( \frac{3 \left[ 7.0 \text{ mN} + (0.0027 \text{ kg})(9.81 \text{ m/s}^2) \right]}{4\pi \left( 1.00 \times 10^3 \text{ kg/m}^3 \right)(9.81 \text{ m/s}^2)} \right)^{\frac{1}{3}} = 1.9 \text{ cm} \]

**Seawater** has a bulk modulus of \( 2.30 \times 10^9 \text{ N/m}^2 \). Find the difference in density of seawater at a depth where the pressure is 800 atm as compared to the density at the surface which is 1025 kg/m\(^3\). Neglect any effects due to either temperature or salinity.

**Picture the Problem** Let \( \rho_0 \) represent the density of seawater at the surface. We can use the definition of density and the fact that mass is constant to relate the fractional change in the density of water to its fractional change in volume. We can also use the definition of bulk modulus to relate the fractional change in density to the increase in pressure with depth and solve the resulting equation for the change in density at the depth at which the pressure is 800 atm.
Using the definition of density, relate the mass of a given volume of seawater to its volume:

\[ m = \rho V \]

Noting that the mass does not vary with depth, evaluate its differential:

\[ \rho dV + V d\rho = 0 \]

Solve for \( d\rho/\rho \):

\[ \frac{d\rho}{\rho} = -\frac{dV}{V} \quad \text{or} \quad \frac{\Delta \rho}{\rho} \approx -\frac{\Delta V}{V} \]

Using the definition of the bulk modulus, relate \( \Delta P \) to \( \Delta \rho/\rho_0 \):

\[ B = -\frac{\Delta P}{\Delta V/V} = \frac{\Delta P}{\Delta \rho/\rho_0} \]

Solving for \( \Delta \rho \) yields:

\[ \Delta \rho = \rho - \rho_0 = \frac{\rho_0 \Delta P}{B} \]

Substitute numerical values and evaluate \( \Delta \rho \):

\[ \Delta \rho = \left( 1025 \text{ kg/m}^3 \right) \left( \frac{799 \text{ atm} \times 101.325 \text{ kPa}}{2.30 \times 10^9 \text{ N/m}^2} \right) = 36.1 \text{ kg/m}^3 \]

A solid cube with 0.60-m edge length is suspended from a spring balance. When the cube is submerged in water, the spring balance reads 80 percent of the reading when the cube is in air. Determine the density of the cube.

**Picture the Problem** When it is submerged, the block is in equilibrium under the influence of the buoyant force due to the water, the force exerted by the spring balance, and its weight. We can use the condition for translational equilibrium to relate the buoyant force to the weight of the block and the definition of density to express the weight of the block in terms of its density.

Apply \( \sum F_y = 0 \) to the block:

\[ B + 0.80mg - mg = 0 \Rightarrow B = 0.20mg \]

Substitute for \( B \) and \( m \) to obtain:

\[ \rho_w V_{\text{block}} g = 0.20 \rho_{\text{block}} V_{\text{block}} g \]
Solve for and evaluate $\rho_{\text{block}}$:

$$\rho_{\text{block}} = \frac{\rho_{\text{water}}}{0.20} = 5.0 \left(1.00 \times 10^3 \text{ kg/m}^3 \right)$$

$$= 5.0 \times 10^3 \text{ kg/m}^3$$

81 ** [SSM] ** A 1.5-kg block of wood floats on water with 68 percent of its volume submerged. A lead block is placed on the wood, fully submerging the wood to a depth where the lead remains entirely out of the water. Find the mass of the lead block.

**Picture the Problem** Let $m$ and $V$ represent the mass and volume of the block of wood. Because the block is in equilibrium when it is floating, we can apply the condition for translational equilibrium and Archimedes’ principle to express the dependence of the volume of water it displaces when it is fully submerged on its weight. We’ll repeat this process for the situation in which the lead block is resting on the wood block with the latter fully submerged. Let the upward direction be the positive $y$ direction.

Apply $\sum F_y = 0$ to floating block:

$$B - mg = 0 \quad (1)$$

Use Archimedes’ principle to relate the density of water to the volume of the block of wood:

$$B = w_{\text{displaced}} = m_{\text{displaced}} \rho_{\text{water}}$$

$$= \rho_{\text{water}} V_{\text{displaced}} g = \rho_{\text{water}} (0.68V) g$$

Using the definition of density, express the weight of the block in terms of its density:

$$mg = \rho_{\text{wood}} Vg$$

Substitute for $B$ and $mg$ in equation (1) to obtain:

$$\rho_{\text{water}} (0.68V) g - \rho_{\text{wood}} Vg = 0$$

Solving for $\rho_{\text{wood}}$ yields:

$$\rho_{\text{wood}} = 0.68 \rho_{\text{water}}$$

Use the definition of density to express the volume of the wood:

$$V = \frac{m}{\rho_{\text{wood}}}$$

Apply $\sum F_y = 0$ to the floating block when the lead block is placed on it:

$$B' - m'g = 0$$, where $B'$ is the new buoyant force on the block and $m'$ is the combined mass of the wood block and the lead block.

Use Archimedes’ principle and the definition of density to obtain:

$$\rho_{\text{water}} Vg -(m_{\text{pb}} + m)g = 0$$
Solve for the mass of the lead block to obtain:

\[ m_{\text{Pb}} = \rho_{\text{water}} V - m \]

Substituting for \( V \) and \( \rho_{\text{water}} \) yields:

\[ m_{\text{Pb}} = \frac{\rho_{\text{wood}} m}{0.68 \rho_{\text{wood}}} - m \]

\[ = \left( \frac{1}{0.68} - 1 \right) m \]

Substitute numerical values and evaluate \( m_{\text{Pb}} \):

\[ m_{\text{Pb}} = \left( \frac{1}{0.68} - 1 \right) (1.5 \text{ kg}) = 0.71 \text{ kg} \]

82  A Styrofoam cube, 25 cm on an edge, is placed on one pan of a balance. The balance is in equilibrium when a 20-g mass of brass is placed on the other pan. Find the mass of the Styrofoam cube. Neglect the buoyant force of the air on the brass mass, but do not neglect the buoyant force of the air on the Styrofoam cube.

**Picture the Problem** The true mass of the Styrofoam cube is greater than that indicated by the balance due to the buoyant force acting on it. The balance is in rotational equilibrium under the influence of the buoyant and gravitational forces acting on the Styrofoam cube and the brass masses. Let \( m \) and \( V \) represent the mass and volume of the cube and \( L \) the lever arm of the balance.

Apply \( \sum \tau = 0 \) to the balance:

\[ (mg - B)L - m_{\text{brass}}gL = 0 \quad (1) \]

Use Archimedes’ principle to express the buoyant force on the Styrofoam cube as a function of volume and density of the air it displaces:

\[ B = w_{\text{displaced fluid}} = m_{\text{displaced fluid}} g \]
\[ = \rho_{\text{displaced fluid}} V_{\text{displaced fluid}} g = \rho_{\text{air}} V g \]

Substitute for \( B \) in equation (1) and simplify to obtain:

\[ m - \rho_{\text{air}} V - m_{\text{brass}} = 0 \]

Solving for \( m \) yields:

\[ m = \rho_{\text{air}} V + m_{\text{brass}} \]

Substitute numerical values and evaluate \( m \):

\[ m = \left( 1.293 \text{ kg/m}^3 \right) (0.25 \text{ m})^3 + 20 \times 10^{-3} \text{ kg} \]
\[ = 40 \text{ g} \]
A spherical shell of copper with an outer diameter of 12.0 cm floats on water with half its volume above the water’s surface. Determine the inner diameter of the shell. The cavity inside the spherical shell is empty.

**Picture the Problem** Let \( d_{\text{in}} \) and \( d_{\text{out}} \) represent the inner and outer diameters of the copper shell and \( V' \) the volume of the spherical shell that is submerged. Because the spherical shell is floating, it is in equilibrium and we can apply a condition for translational equilibrium to relate the buoyant force \( B \) due to the displaced water and its weight \( w \).

Apply \( \sum F_y = 0 \) to the spherical shell:

\[
B - w = 0
\]

Using Archimedes’ principle and the definition of \( w \), substitute to obtain:

\[
\rho_w V' g - mg = 0 \Rightarrow \rho_w V' - m = 0 \quad (1)
\]

Express \( V' \) as a function \( d_{\text{out}} \):

\[
V' = \frac{1}{6} \pi d_{\text{out}}^3 = \frac{\pi}{12} d_{\text{out}}^3
\]

Express \( m \) in terms of \( d_{\text{in}} \) and \( d_{\text{out}} \):

\[
m = \rho_{\text{Cu}} (V'_{\text{out}} - V'_{\text{in}}) = \rho_{\text{Cu}} \left( \frac{\pi}{6} d_{\text{out}}^3 - \frac{\pi}{6} d_{\text{in}}^3 \right)
\]

Substitute in equation (1) to obtain:

\[
\rho_w \frac{\pi}{12} d_{\text{out}}^3 - \rho_{\text{Cu}} \left( \frac{\pi}{6} d_{\text{out}}^3 - \frac{\pi}{6} d_{\text{in}}^3 \right) = 0
\]

Simplifying yields:

\[
\frac{1}{2} \rho_w d_{\text{out}}^3 - \rho_{\text{Cu}} (d_{\text{out}}^3 - d_{\text{in}}^3) = 0
\]

Solve for \( d_{\text{in}} \) to obtain:

\[
d_{\text{in}} = d_{\text{out}} \sqrt{1 - \frac{\rho_w}{2 \rho_{\text{Cu}}}}
\]

Substitute numerical values and evaluate \( d_{\text{in}} \):

\[
d_{\text{in}} = (12.0 \text{ cm}) \sqrt{1 - \frac{1}{2(8.93)}} = 11.7 \text{ cm}
\]

A 200-mL beaker that is half-filled with water is on the left pan of a balance, and a sufficient amount of sand is placed on the right pan to bring the balance to equilibrium. A cube 4.0 cm on an edge that is attached to a string is then lowered into the water so that it is completely submerged, but not touching the bottom of the beaker. A piece of brass of mass \( m \) is then added to the right pan to restore equilibrium. What is \( m \)?
**Determine the Concept** The additional weight on the right side is needed to balance the buoyant force the water in the beaker exerts on the cube.

The buoyant force exerted on the cube by the water is given by:

\[ B = w_{\text{displaced water}} = m_{\text{water}} g = \rho_{\text{water}} V_{\text{cube}} g \]

The weight of the piece of brass is the product of its mass and the gravitational field:

\[ w = mg \]

Equating these forces yields:

\[ \rho_{\text{water}} V_{\text{cube}} g = mg \Rightarrow m = \rho_{\text{water}} V_{\text{cube}} \]

Substitute numerical values and evaluate \( m \):

\[ m = (1.00 \text{ g/cm}^3)(4 \text{ cm})^3 = 64 \text{ g} \]

85 [SSM] Crude oil has a viscosity of about 0.800 Pa·s at normal temperature. You are the chief design engineer in charge of constructing a 50.0-km horizontal pipeline that connects an oil field to a tanker terminal. The pipeline is to deliver oil at the terminal at a rate of 500 L/s and the flow through the pipeline is to be laminar. Assuming that the density of crude oil is 700 kg/m³, estimate the diameter of the pipeline that should be used.

**Picture the Problem** We can use the definition of Reynolds number and assume a value for \( N_R \) of 1000 (well within the laminar-flow range) to obtain a trial value for the radius of the pipe. We’ll then use Poiseuille’s law to determine the pressure difference between the ends of the pipe that would be required to maintain a volume flow rate of 500 L/s.

Use the definition of Reynolds number to relate \( N_R \) to the radius of the pipe:

\[ N_R = \frac{2r \rho v}{\eta} \]

Use the definition of \( I_v \) to relate the volume flow rate of the pipe to its radius:

\[ I_v = A v = \pi r^2 v \Rightarrow v = \frac{I_v}{\pi r^2} \]

Substitute to obtain:

\[ N_R = \frac{2 \rho I_v}{\eta \pi r} \Rightarrow r = \frac{2 \rho I_v}{\eta \pi N_R} \]

Substitute numerical values and evaluate \( r \):

\[ r = \frac{2(700 \text{ kg/m}^3)(0.500 \text{ m}^3/\text{s})}{\pi(0.800 \text{ Pa} \cdot \text{s})(1000)} = 27.9 \text{ cm} \]
Using Poiseuille’s law, relate the pressure difference between the ends of the pipe to its radius:

\[ \Delta P = \frac{8\eta L}{\pi r^4} I_v \]

Substitute numerical values and evaluate \( \Delta P \):

\[ \Delta P = \frac{8(0.800 \text{ Pa} \cdot \text{s})(50 \text{ km})}{\pi(0.279 \text{ m})^4}(0.500 \text{ m}^3/\text{s}) \]

\[ = 8.41 \times 10^6 \text{ Pa} = 83.0 \text{ atm} \]

This pressure is too large to maintain in the pipe. Evaluate \( \Delta P \) for a pipe of 50 cm radius:

\[ \Delta P = \frac{8(0.800 \text{ Pa} \cdot \text{s})(50 \text{ km})}{\pi(0.50 \text{ m})^4}(0.500 \text{ m}^3/\text{s}) \]

\[ = 8.15 \times 10^6 \text{ Pa} = 8.04 \text{ atm} \]

1 m is a plausible diameter for such a pipe.

86  Water flows through the pipe in Figure 13-40 and exits to the atmosphere at the right end of section C. The diameter of the pipe is 2.00 cm at A, 1.00 cm at B, and 0.800 cm at C. The gauge pressure in the pipe at the center of section A is 1.22 atm and the flow rate is 0.800 L/s. The vertical pipes are open to the air. Find the level (above the flow mid-line as shown) of the liquid–air interfaces in the two vertical pipes. Assume laminar nonviscous flow.

**Picture the Problem** We’ll measure the height of the liquid–air interfaces relative to the centerline of the pipe. We can use the definition of the volume flow rate in a pipe to find the speed of the water at point A and the relationship between the gauge pressures at points A and C to determine the level of the liquid-air interface at A. We can use the continuity equation to express the speed of the water at B in terms of its speed at A and Bernoulli’s equation for constant elevation to find the gauge pressure at B. Finally, we can use the relationship between the gauge pressures at points A and B to find the level of the liquid-air interface at B.

Relate the gauge pressure in the pipe at A to the height of the liquid-air interface at A:

\[ P_{\text{gauge,A}} = \rho g h_A \implies h_A = \frac{P_{\text{gauge,A}}}{\rho g} \]

where \( h_A \) is measured from the center of the pipe.

Substitute numerical values and evaluate \( h_A \):

\[ h_A = \frac{(1.22 \text{ atm})(1.01 \times 10^5 \text{ Pa/atm})}{(1.00 \times 10^3 \text{ kg/m}^3)(9.81 \text{ m/s}^2)} \]

\[ = 12.6 \text{ m} \]
Determine the speed of the water at A:

\[ v_A = \frac{I_v}{A_A} = \frac{0.800 \times 10^{-3} \text{ m}^3/\text{s}}{\pi (0.0200 \text{ m})^2} = 2.55 \text{ m/s} \]

Apply Bernoulli’s equation for constant elevation to relate \( P_B \) and \( P_A \):

\[ P_A + \frac{1}{2} \rho v_A^2 = P_B + \frac{1}{2} \rho v_B^2 \quad (1) \]

Use the continuity equation to relate \( v_B \) and \( v_A \):

\[ A_A v_A = A_B v_B \Rightarrow v_B = \frac{A_A}{A_B} v_A \]

Substitute numerical values and evaluate \( v_B \):

\[ v_B = \left( \frac{2.00 \text{ cm}}{1.00 \text{ cm}} \right)^2 v_A = 4v_A \]

Substitute in equation (1) to obtain:

\[ P_A + \frac{1}{2} \rho v_A^2 = P_B + 8 \rho v_A^2 \]

Solving for \( P_B \) yields:

\[ P_B = P_A - \frac{16}{3} \rho v_A^2 = P_{\text{gauge, A}} + P_{\text{at}} - \frac{16}{3} \rho v_A^2 \]

Substitute numerical values and evaluate \( P_B \):

\[ P_B = (1.22 \text{ atm} + 1.00 \text{ atm})(101.325 \text{ kPa/atm}) - \frac{16}{3} (1.00 \times 10^3 \text{ kg/m}^3)(2.55 \text{ m/s})^2 \]

\[ = 1.762 \times 10^5 \text{ Pa} \times \frac{1 \text{ atm}}{101.325 \text{ kPa}} = 1.739 \text{ atm} \]

Relate the gauge pressure in the pipe at B to the height of the liquid-air interface at B:

\[ P_{\text{gauge, B}} = \rho g h_B \]

Solve for \( h_B \):

\[ h_B = \frac{P_{\text{gauge, B}}}{\rho g} = \frac{P_B - P_{\text{at}}}{\rho g} \]

Substitute numerical values and evaluate \( h_B \):

\[ h_B = \frac{(1.739 \text{ atm} - 1.00 \text{ atm})(101.325 \text{ kPa/atm})}{(1.00 \times 10^3 \text{ kg/m}^3)(9.81 \text{ m/s}^2)} = 7.63 \text{ m} \]

---

**SSM** You are employed as a tanker truck driver for the summer. Heating oil is delivered to customers for winter usage by your large tanker truck. The delivery hose has a radius 1.00 cm. The specific gravity of the oil is 0.875, and its coefficient of viscosity is 200 mPa·s. What is the minimum time it will
Picture the Problem  We can use the volume of the drum and the volume flow rate equation to express the time to fill the customer’s oil drum. The Reynolds number equation relates Reynolds number to the speed with which oil flows through the hose to the volume flow rate. Because the upper limit on the Reynolds number for laminar flow is approximately 2000, we’ll use this value in our calculation of the fill time.

The time \( t_{\text{fill}} \) is related to the volume flow rate in the hose:

\[
\frac{t_{\text{fill}}}{I_v} = \frac{V}{A v} \quad (1)
\]

where \( A \) is the cross-sectional area of the hose and \( V \) is the volume of the oil drum.

Reynolds number is defined by the equation:

\[
N_R = \frac{2r \rho v}{\eta} \Rightarrow v = \frac{\eta N_R}{2r \rho}
\]

Substitute for \( v \) and \( A \) in equation (1) to obtain:

\[
t_{\text{fill}} = \frac{V}{\pi r^2 \left( \frac{\eta N_R}{2r \rho} \right)} = \frac{2 \rho V}{\pi r \eta N_R}
\]

Substitute numerical values and evaluate \( t_{\text{fill}} \):

\[
t_{\text{fill}} = \frac{2 \left(875 \text{ kg/m}^3 \right) \left(55 \text{ gal} \times \frac{3.785 \text{ L}}{\text{gal}}\right)}{\pi (0.010 \text{ m})(200 \text{ mPa} \cdot \text{s})(2000)}
\]

\[
t_{\text{fill}} = [29 \text{ s}]
\]

88  

A U-tube is filled with water until the liquid level is 28 cm above the bottom of the tube (Figure 13-41a). Oil, which has a specific gravity 0.78, is now poured into one arm of the U-tube until the level of the water in the other arm of the tube is 34 cm above the bottom of the tube (Figure 13-41b). Find the levels of the oil–water and oil–air interfaces in the other arm of the tube.

Picture the Problem  We can use the equality of the pressure at the bottom of the U-tube due to the water on one side and that due to the oil and water on the other to relate the various heights. Let \( h \) represent the height of the oil above the water. Then \( h_o = h_1w + h \).
Using the constancy of the amount of water, express the relationship between $h_{1w}$ and $h_{2w}$:

\[ h_{1w} + h_{2w} = 56 \text{ cm} \]

Find the height of the oil-water interface:

\[ h_{tw} = 56 \text{ cm} - 34 \text{ cm} = 22 \text{ cm} \]

Express the equality of the pressure at the bottom of the two arms of the U tube:

\[ \rho_w g (34 \text{ cm}) = \rho_w g (22 \text{ cm}) + 0.78 \rho_w g h_{oil} \]

Solve for and evaluate $h_{oil}$:

\[ h_{oil} = \frac{\rho_w g (34 \text{ cm}) - \rho_w g (22 \text{ cm})}{0.78} \]
\[ = \frac{(34 \text{ cm}) - (22 \text{ cm})}{0.78} = 15.4 \text{ cm} \]

The height of the air-oil interface $h_o$ is:

\[ h_o = 22 \text{ cm} + 15.4 \text{ cm} = 37 \text{ cm} \]

[SSM] A helium balloon can just lift a load that weighs 750 N and has a negligible volume. The skin of the balloon has a mass of 1.5 kg. (a) What is the volume of the balloon? (b) If the volume of the balloon were twice that found in Part (a), what would be the initial acceleration of the balloon when released at sea level carrying a load weighing 900 N?

**Picture the Problem** Because the balloon is in equilibrium under the influence of the buoyant force exerted by the air, the weight of its basket and load $w$, the weight of the skin of the balloon, and the weight of the helium. Choose upward to be the $+y$ direction and apply the condition for translational equilibrium to relate these forces. Archimedes’ principle relates the buoyant force on the balloon to the density of the air it displaces and the volume of the balloon.

(a) Apply $\sum F_y = 0$ to the balloon:

\[ B - m_{\text{skin}} g - m_{\text{He}} g - w = 0 \]

Letting $V$ represent the volume of the balloon, use Archimedes’ principle to express the buoyant force:

\[ \rho_{\text{air}} V g - m_{\text{skin}} g - m_{\text{He}} g - w = 0 \]

Substituting for $m_{\text{He}}$ yields:

\[ \rho_{\text{air}} V g - m_{\text{skin}} g - \rho_{\text{He}} V g - w = 0 \]
Solve for $V$ to obtain:

$$V = \frac{m_{\text{skin}} g + w}{(\rho_{\text{air}} - \rho_{\text{He}})g}$$

Substitute numerical values and evaluate $V$:

$$V = \frac{(1.5 \text{ kg})(9.81 \text{ m/s}^2) + 750 \text{ N}}{(1.293 \text{ kg/m}^3 - 0.1786 \text{ kg/m}^3)(9.81 \text{ m/s}^2)} = 70 \text{ m}^3$$

(b) Apply $\sum F_x = ma$ to the balloon:

$$B - m_{\text{tot}}g = m_{\text{tot}}a \implies a = \frac{B}{m_{\text{tot}}} - g \quad (1)$$

Assuming that the mass of the skin has not changed and letting $V'$ represent the doubled volume of the balloon, express $m_{\text{tot}}$:

$$m_{\text{tot}} = m_{\text{load}} + m_{\text{He}} + m_{\text{skin}}$$

$$= \frac{w_{\text{load}}}{g} + \rho_{\text{He}}V' + m_{\text{skin}}$$

Express the buoyant force acting on the balloon:

$$B = w_{\text{displaced fluid}} = \rho_{\text{air}}V'g$$

Substituting for $m_{\text{tot}}$ and $B$ in equation (1) yields:

$$a = \frac{\rho_{\text{air}}V'g}{\frac{w_{\text{load}}}{g} + \rho_{\text{He}}V' + m_{\text{skin}}} - g$$

Substitute numerical values and evaluate $a$:

$$a = \frac{(1.293 \text{ kg/m}^3)(140 \text{ m}^3)(9.81 \text{ m/s}^2)}{900 \text{ N}} + (0.1786 \text{ kg/m}^3)(140 \text{ m}^3) + 1.5 \text{ kg} - 9.81 \text{ m/s}^2 = 5.2 \text{ m/s}^2$$

90 A hollow sphere with an inner radius $R$ and an outer radius $2R$. It is made of material of density $\rho_0$ and is floating in a liquid of density $2\rho_0$. The interior is now completely filled with material of density $\rho'$ such that that the sphere just floats completely submerged. Find $\rho'$.

**Picture the Problem** When the hollow sphere is completely submerged but floating, it is in translational equilibrium under the influence of a buoyant force and its weight. The buoyant force is given by Archimedes’ principle and the weight of the sphere is the sum of the weights of the hollow sphere and the material filling its center.

Apply $\sum F_y = 0$ to the hollow sphere:

$$B - w = 0 \quad (1)$$
Express the buoyant force acting on the hollow sphere:

\[ B = 2\rho_0 V_{\text{sphere}} g = 2\rho_0 \left[ \frac{4}{3} \pi (2R)^3 \right] g \]
\[ = \frac{64}{3} \rho_0 \pi R^3 g \]

Express the weight of the sphere when the hollow in it is filled with a material of density \( \rho' \):

\[ w = \rho_0 V_{\text{hollow sphere}} g + \rho' V_{\text{hollow}} g \]
\[ = \rho_0 \left[ \frac{4}{3} \pi (2R)^3 - R^3 \right] g + \rho' \left[ \frac{4}{3} \pi R^3 \right] g \]
\[ = \frac{28}{3} \rho_0 \pi R^3 g + \frac{4}{3} \rho' \pi R^3 g \]

Substitute for \( B \) and \( w \) in equation (1) to obtain:

\[ \frac{64}{3} \rho_0 \pi R^3 g - \frac{28}{3} \rho_0 \pi R^3 g - \frac{4}{3} \rho' \pi R^3 g = 0 \]

Solving for \( \rho' \) yields:

\[ \rho' = 9 \rho_0 \]

According to the law of atmospheres, the fractional decrease in atmospheric pressure is proportional to the change in altitude. Expressed as a differential equation we have \( \frac{dP}{P} = -C dh \), where \( C \) is a positive constant.

(a) Show that \( P(h) = P_0 e^{-Ch} \) where \( P_0 \) is the pressure at \( h = 0 \), is a solution of the differential equation. (b) Given that the pressure 5.5 km above sea level is half that at sea level, find the constant \( C \).

**Picture the Problem** We can differentiate the function \( P(h) \) to show that it satisfies the differential equation \( \frac{dP}{P} = -C \, dh \) and in Part (b) we can use the approximation \( e^{-x} \approx 1 - x \) and \( \Delta h << h_0 \) to establish the given result.

(a) Differentiate \( P(h) = P_0 e^{-Ch} \):

\[ \frac{dP}{dh} = -CP_0 e^{-Ch} = -CP \]

Separating variables yields:

\[ \frac{dP}{P} = -Cdh \]

(b) Take the logarithm of both sides of the function \( P(h) \):

\[ \ln P = \ln \left( P_0 e^{-Ch} \right) = \ln P_0 + \ln e^{-Ch} \]
\[ = \ln(P_0) - Ch \]

Solving for \( C \) yields:

\[ C = \frac{1}{h} \ln \left( \frac{P_0}{P} \right) \]

Substitute numerical values and evaluate \( C \):

\[ C = \frac{1}{5.5 \text{ km}} \ln \left( \frac{P_0}{\frac{1}{2} P_0} \right) = \frac{1}{5.5 \text{ km}} \ln 2 \]
\[ = 0.13 \text{ km}^{-1} \]
A submarine has a total mass of $2.40 \times 10^6$ kg, including crew and equipment. The vessel consists of two parts, the pressure hull, which has a volume of $2.00 \times 10^3$ m$^3$, and the ballast tanks, which have a volume of $4.00 \times 10^2$ m$^3$. When the boat cruises on the surface, the ballast tanks are filled with air at atmospheric pressure; to cruise below the surface, seawater must be admitted into the tanks. (a) What fraction of the submarine’s volume is above the water surface when the tanks are filled with air? (b) How much water must be admitted into the tanks to give the submarine neutral buoyancy? Neglect the mass of any air in the tanks and use 1.025 as the specific gravity of seawater.

**Picture the Problem** Let $V$ represent the volume of the submarine and $V'$ the volume of seawater it displaces when it is on the surface. The submarine is in equilibrium in both parts of the problem. Hence we can apply the condition for translational equilibrium (neutral buoyancy) to the submarine to relate its weight to the buoyant force acting on it. We’ll also use Archimedes’ principle to connect the buoyant forces to the volume of seawater the submarine displaces. Let upward be the $+y$ direction.

(a) Express $f$, the fraction of the submarine’s volume above the surface when the tanks are filled with air:

$$f = \frac{V - V'}{V} = 1 - \frac{V'}{V} \quad (1)$$

Apply $\sum F_y = 0$ to the submarine when its tanks are full of air:

$$B - w = 0$$

Use Archimedes’ principle to express the buoyant force on the submarine in terms of the volume of the displaced water:

$$B = \rho_{sw} V' g$$

Substituting for $B$ and $w$ yields:

$$\rho_{sw} V' g - mg = 0 \Rightarrow V' = \frac{m}{\rho_{sw}}$$

Substitute in equation (1) to obtain:

$$f = 1 - \frac{m}{\rho_{sw} V}$$

Substitute numerical values and evaluate $f$:

$$f = 1 - \frac{2.40 \times 10^6 \text{ kg}}{(1.025 \times 10^3 \text{ kg/m}^3)(2.40 \times 10^3 \text{ m}^3)} = 2.44\%$$
(b) Express the volume of seawater in terms of its mass and density:

\[ V_{sw} = \frac{m_{sw}}{\rho_{sw}} \]  

(2)

Apply \( \sum F_y = 0 \), the condition for neutral buoyancy, to the submarine:

\[ B - w_{sub} - w_{sw} = 0 \]

Use Archimedes’ principle to express the buoyant force on the submarine in terms of the volume of the displaced water:

\[ B = \rho_{sw} V g \]

Substituting for \( B \), \( w_{sub} \), and \( w_{sw} \) yields:

\[ \rho_{sw} V g - m_{sub} g - m_{sw} g = 0 \]

Solve for \( m_{sw} \) to obtain:

\[ m_{sw} = \rho_{sw} V - m_{sub} \]

Substituting for \( V_{sw} \) in equation (2) yields:

\[ V_{sw} = \frac{\rho_{sw} V - m_{sub}}{\rho_{sw}} = V - \frac{m_{sub}}{\rho_{sw}} \]

Substitute numerical values and evaluate \( V_{sw} \):

\[ V_{sw} = 2.40 \times 10^3 \text{ m}^3 - \frac{2.40 \times 10^6 \text{ kg}}{1025 \text{ kg/m}^3} \]

\[ = 60 \text{ m}^3 \]

Most species of fish have expandable sacs, commonly known as "swim bladders," that enable fish to rise in the water by filling the bladders with oxygen collected by their gills and to sink by emptying the bladders into the surrounding water. A freshwater fish has an average density equal to 1.05 kg/L when its swim bladder is empty. How large must the volume of oxygen in the fish’s swim bladder be if the fish is to have neutral buoyancy? The fish has a mass of 0.825 kg. Assume the density of oxygen in the bladder is equal to air density at standard temperature and pressure.

**Picture the Problem** For the fish to achieve neutral buoyancy, its overall density must be lowered to 1.00 kg/L. In order to lower its density, the buoyant force acting on the fish needs to be increased by increasing the fish’s volume. It can accomplish this by filling its swim bladder.

Apply the condition for translational equilibrium to the fish:

\[ \sum F_y = B - F_g = 0 \]
Substitute for $B$ and $F_g$ to obtain:

$$\rho_{\text{water}} (V + \delta V)g - mg = 0$$

where $V$ is the fish’s volume and $\delta V$ is the increase in the fish’s volume resulting from the additional oxygen in its swim bladder.

Solving for $\delta V$ yields:

$$\delta V = \frac{m}{\rho_{\text{water}}} - V$$

Express the density $\rho$ of the fish:

$$\rho = \frac{m}{V} \Rightarrow V = \frac{m}{\rho}$$

Substitute for $V$ to obtain:

$$\delta V = \frac{m}{\rho_{\text{water}}} - \frac{m}{\rho} = m \left( \frac{1}{\rho_{\text{water}}} - \frac{1}{\rho} \right)$$

The specific gravity of the fish with its swim bladder full is:

$$\delta V = (0.825 \text{ kg}) \left( \frac{1}{1.00 \text{ kg/L}} - \frac{1}{1.05 \text{ kg/L}} \right)$$

$$= 39 \text{ cm}^3$$

Remarks: In this solution, we’ve neglected the additional mass of oxygen that should be added to the fish’s mass. Consider how small that mass is for a volume of 39 cm$^3$: $m_{\text{oxygen}} = \rho_{\text{air}} \delta V = (1.293 \text{ kg/m}^3) (39 \times 10^{-6} \text{ m}^3) = 50 \mu g$. 