

MATH 136, Review Test 1, Fall 09

$$\begin{aligned}
 1. \int_0^1 \frac{1}{3-2x} dx &= \int_3^1 \frac{-\frac{1}{2} du}{u} \\
 &= -\frac{1}{2} \ln|u| \Big|_3^1 = -\frac{1}{2} (-\ln 3) \\
 &= \frac{1}{2} \ln 3
 \end{aligned}$$

$$\begin{aligned}
 u &= 3-2x \\
 du &= -2 dx \\
 dx &= -\frac{1}{2} du
 \end{aligned}$$

2. $\int \sin^3 x \cos^2 x dx =$

$$\begin{aligned}
 &= \int u^2 du + \int u^4 du = \\
 &= \frac{1}{3} u^3 + \frac{1}{5} u^5 + C \\
 &= \frac{1}{3} \cos^3 x + \frac{1}{5} \cos^5 x + C
 \end{aligned}$$

$$\begin{aligned}
 \sin x (1 - \cos^2 x) \cos^2 x &= \\
 \sin x \cos^2 x - \sin x \cos^4 x & \\
 u = \cos x & \\
 du = -\sin x dx &
 \end{aligned}$$

3. $\int x \cos x dx = x \sin x - \int \sin x dx$
 $= x \sin x + \cos x + C$

(By parts)

4. $\int x \cos(x^2) dx =$

$$\begin{aligned}
 \int \cos u \frac{1}{2} du &= \frac{1}{2} \sin u + C \\
 &= \frac{1}{2} \sin(x^2) + C
 \end{aligned}$$

$$\begin{aligned}
 u &= x^2 \\
 du &= 2x dx \\
 x dx &= \frac{1}{2} du
 \end{aligned}$$

$$\begin{aligned}
 5. \int_0^{\pi/2} \sin^2 t dt &= \int_0^{\pi/2} \frac{1 - \cos 2t}{2} dt \\
 &= \frac{1}{2} t \Big|_0^{\pi/2} - \frac{1}{2} \int_0^{\pi/2} \cos 2t dt \\
 &= \frac{\pi}{4} - \frac{1}{2} \frac{1}{2} \sin 2t \Big|_0^{\pi/2} = \frac{\pi}{4}
 \end{aligned}$$

$$6. \int x \sin(3x) dx = -\frac{1}{3} x \cos 3x + \frac{1}{3} \int \cos 3x dx = -\frac{1}{3} x \cos 3x + \frac{1}{9} \sin 3x + C$$

$$\begin{aligned}
 7. \int \frac{x-1}{x^2+1} dx &= \int \frac{x}{x^2+1} dx - \int \frac{dx}{x^2+1} \\
 &= \frac{1}{2} \int \frac{du}{u} - \text{Arctan } x \\
 &= \frac{1}{2} \ln(x^2+1) - \text{Arctan } x
 \end{aligned}$$

$u = x^2 + 1$
 $du = 2x dx$
 $x dx = \frac{1}{2} du$

$$\begin{aligned}
 8. \int \frac{x^3}{1+x^4} dx &= \\
 u &= 1+x^4 \\
 du &= 4x^3 dx \\
 x^3 dx &= \frac{1}{4} du \\
 \int \frac{\frac{1}{4} du}{u} &= \frac{1}{4} \ln|u| = \frac{1}{4} \ln(1+x^4) + C
 \end{aligned}$$