Solutions to Sample Quiz

FALL 2007

- (a) $A = \begin{pmatrix} 1 & 1 & 0 & 3 \\ -2 & 1 & 1 & -1 \\ 0 & -1 & 1 & -3 \end{pmatrix} \xrightarrow{2R_1+R_2 \to R_2} \begin{pmatrix} 1 & 1 & 0 & 3 \\ 0 & 3 & 1 & 5 \\ 0 & -1 & 1 & -3 \end{pmatrix} \xrightarrow{\frac{1}{3}R_2 \to R_2}$

$$\to \begin{pmatrix} 1 & 1 & 0 & 3 \\ 0 & 1 & 1/3 & 5/3 \\ 0 & -1 & 1 & -3 \end{pmatrix} \xrightarrow{R_2+R_3 \to R_3} \begin{pmatrix} 1 & 1 & 0 & 3 \\ 0 & 1 & 1/3 & 5/3 \\ 0 & 0 & 4/3 & -4/3 \end{pmatrix} \xrightarrow{\frac{3}{4}R_3 \to R_3} \begin{pmatrix} 1 & 1 & 0 & 3 \\ 0 & 1 & 1/3 & 5/3 \\ 0 & 0 & 1 & -1 \end{pmatrix}$$

Row Echelon Form

$$\xrightarrow{-\frac{1}{3}R_3+R_2 \to R_2} \begin{pmatrix} 1 & 1 & 0 & 3 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & -1 \end{pmatrix} \xrightarrow{-R_2+R_1 \to R_1} \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & -1 \end{pmatrix}$$

Reduced Row Echelon Form

(b) $\text{rank}(A) = 3$ (max. number of linearly independent rows)

(c) C_1, C_2, C_3, C_4 = columns of A $\Rightarrow C_1x_1 + C_2x_2 + C_3x_3 = C_4$

$\Rightarrow \boxed{x_1 = 1,\ x_2 = 2,\ x_3 = -1}$

(d) Based on the reduced row echelon form, the max. number of lin. ind. columns equals the # of leading 1's in the matrix, which also equals the # of lin. ind. rows.

- (a) Since the transpose has $n \times m$ dimension (where M has $m \times n$) then $m = n$.

(b) $M = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$ Diagonal entries must be zero (Since they stay unchanged under transposition)

(c) $(B + B^T)^T = B^T + B \Rightarrow B + B^T$ is symmetric.

(d) Similarly $(B - B^T)^T = B^T - B = -(B - B^T) \Rightarrow B - B^T$ is skew-symm.

$B = \frac{1}{2}(B + B^T) + \frac{1}{2}(B - B^T)$.