Solutions to Sample Problems for Exam 2
Math 135 - Spring 2008 - Instructor: Dr. Radu C. Cascaval

1. Find the derivatives of the following functions. Simplify when possible.

   (a) $f(x) = \frac{\sin 3x}{2x}$ (b) $g(x) = (x+1)^2(x^2+2)$ (c) $y = \ln(x-1)$
   (d) $x = \sqrt{t}(2t-3)$ (e) $r = \sqrt{\cos\theta}$ (f) $y = \sin^3(\ln(2x-1))$

   ANS: (a) $f'(x) = \dfrac{3x\cos 3x - \sin 3x}{2x^2}$ (b) $g'(x) = 2(x+1)(2x^2+x+2)$ (c) $y' = \dfrac{1}{x-1}$
   (d) $\dfrac{dx}{dt} = \dfrac{3(2t-1)}{2\sqrt{t}}$ (e) $\dfrac{dr}{d\theta} = -\dfrac{\sin\theta}{2\sqrt{\cos\theta}}$ (f) $\dfrac{dy}{dx} = 3\sin^2(\ln(2x-1))\cos(\ln(2x-1))\dfrac{2}{2x-1}$.

2. Find the equation of the tangent line to the curve at the indicated point $y = xe^{1-2x}$ at $x = \frac{1}{2}$.
   ANS: $y' = (1-2x)e^{1-2x}$. Horizontal tangent at $x = \frac{1}{2}$ since $y'(\frac{1}{2}) = 0$. Equation of tangent line $y = \frac{1}{2}$.

3. (a) Find the derivative of the following function $f(x) = \frac{\sqrt{x-1}}{x^2+1}$.
   (b) Find the points where the graph $y = f(x)$ has horizontal tangents.
   ANS: $f'(x) = \frac{-3x^2+4x+1}{2\sqrt{x-1}(x^2+1)^2}$. Tangent line is horizontal where $f'(x) = 0$, or $-3x^2+4x+1 = 0$. $x_{1,2} = \frac{2\pm\sqrt{7}}{3}$. One value is extraneous (since only $x > 1$ is allowed). So $x = \frac{2+\sqrt{7}}{3}$.

4. Find the second order derivative of each of the folloquing functions
   (a) $f(x) = x^{5/3}$ (b) $g(x) = e^x \sin x$ (c) $h(x) = (\ln x)^2$
   ANS: (a) $f''(x) = 10/9x^{-1/3}$ (b) $g''(x) = 2e^x\cos x$ (c) $h''(x) = 2\frac{1-\ln x}{x^2}$

5. Use implicit differetiation to compute $\frac{dy}{dx}$, where $x^3 - 3xy + y^3 = 1$.

   ANS: $\frac{dy}{dx} = \frac{x^2-y}{x-y^2}$.

6. Coffee is draining from a conical filter into a cylindrical coffee pot at a rate of 10 in^3/min. The relevant dimensions of the cone and cylinder are given in the picture below. (a) How fast is the level in the pot rising? (b) How fast is the level in the cone falling when the cone is 5 in deep? [Hint: The volume of a cone is $V_{cone} = \frac{1}{3}\pi r^2 h$, where r is the radius of the base and h is its height. The volume of a cylinder is $V_{cyl} = \pi r^2 h$, where the variables r, h have same meaning as above.]
   ANS: (a) $\frac{dh_{cyl}}{dt} = \frac{10}{9\pi}\ in/min$, (b) $\frac{dh_{cone}}{dt} = -\frac{8}{5\pi}\ in/min$

7. A light shines from the top of a pole 50 ft high. A ball is dropped from the same height but from a point 30 ft away from the light (see picture). How fast is the shadow of the ball moving along the ground after 1 $sec.$? (Assume the ball falls a distance $s = 16t^2$ in t sec.)

   ANS : $\frac{dx}{dt} = -187.5\, ft/s$

8. Given the function $y = \frac{1}{\sqrt{1-x}}$, find the differential dy when $x = 0$. Evaluate dy for the values $x = 0$ and $dx = 0.01$

   ANS : At $x = 0$, $dy = \frac{1}{2}dx$. For $dx = 0.01$, $dy = 0.005$.

9. Using logarithmic differentiation, compute $\frac{dy}{dx}$, where $y = \frac{x^2-4}{e^{2x}(1-x)^5}$.
   ANS: $\frac{dy}{dx} = \frac{x^2-4}{e^{2x}(1-x)^5}\left(\frac{2x}{x^2-4} - 2 + \frac{5}{1-x}\right)$.