Math 381 Test 1 Professor Carlson

Show all your work

1. Seniors at Albert Einstein High School obtain scores on the Math SAT which are summarized in the following table.

score	200	300	400	500	600	700	800
number of students	10	30	30	40	50	20	20

Compute the mean and median grade on the Math SAT for this group of students. Compare the changes to the mean and median grades if it turns out that there was a reporting error, and 10 of the scores initially reported as 300 were really scores of 200.

The mean μ of the scores is just the sum of the scores for all students divided by the total number of students. That is

$$\mu = \frac{103,000}{200} = 515.$$

The median M is the score below (or above) which half of the students scored. In this case

$$M = 500.$$

If there were a grading error as indicated, the mean would change to

$$\mu_1 = \frac{102,000}{200} = 510,$$

but the median would be unchanged. The median is insensitive to changes in outlying data.

2. *(a) How many distinct permutations can be made from the letters of the word columns ?*

There are 7 letters, and any permuation is allowed, for a total of 7! distinct permutations.

(b) In how many ways can 4 *boys and* 5 *girls sit in a row if the boys and girls must alternate?*

To alternate, the seating pattern must be $G_1B_1G_2B_2G_3B_3G_4B_4G_5$. Any of the 5! permutations of the girls and the 4! permutations of the boys is allowed, giving a total of $5! * 4!$ seating arrangements.

3. *(a) You draw 5 cards from a standard 52 card deck. The order in which cards are drawn does not matter. What is the probability of getting a 5 card hand with 4 kings? There is no need to evaluate expressions of the form $N!$.*

There are 48 hands with 4 kings, since the fifth card can be any card other than a king. There are $\binom{52}{5}$ ways to choose 5 cards from a 52 card deck, so the probability of getting a 5 card hand with 4 kings is

$$48/\binom{52}{5} = \frac{48 * 5! * 47!}{52!} = \frac{120}{52 * 51 * 50 * 49}$$

(b) *You roll 3 dice. What is the probability that at least one of the dice will be a 4?*

Let A be the event that at least one of the 3 dice is a 4. The complementary event A' is the event that none of the 3 dice is a 4. The number of ways to have an outcome in A' is $5 * 5 * 5$, while there are 6^3 total rolls. Thus $P(A') = 5^3/6^3$, and

$$P(A) = 1 - P(A') = 1 - 5^3/6^3.$$

4. *(a) Give the definition of $P(B|A)$, the conditional probability of B given A.*

$$P(B|A) = \frac{P(B \cap A)}{P(A)}.$$

(b) You have two bags. The first bag initially contains 5 pennies and 5 nickels. The second bag initially contains 4 pennies and 6 nickels. A coin is drawn from the first bag and placed unseen into the second bag. What is the probability that a coin later drawn from the second bag is a nickel?

Let P_1 be the event that the first coin drawn is a penny, while N_1 is the event that the first coin drawn is a nickel. Similarly, N_2 will be the event that the second coin drawn is a nickel. Then we have

$$P(N_2) = P(P_1)P(N_2|P_1) + P(N_1)P(N_2|N_1) = \frac{5}{10} * \frac{6}{11} + \frac{5}{10} * \frac{7}{11} = \frac{13}{22}.$$

5. *(a) John interviews for a job with companies A,B,C. After the interviews, he estimates that the probability of getting a job offer from A is* 0.9*, from B is* 0.5*, and from C is* 0.2*. Assuming that the companies make independent hiring decisions, what is the probability that John will get at least one offer?*

Let A be the event that John will get at least one offer, and let A' be the complementary event that John will get no offers. Since the decisions of the companies are independent,

$$P(A') = (1 - 0.9)(1 - 0.5)(1 - 0.2) = .1 * .5 * .8 = 0.04,$$

and

$$P(A) = 1 - P(A') = 0.96.$$

(b) A patient either has the seasonal flu or the swine flu, but not both. The probability of having the swine flu is 0.8*, while the probability of having the seasonal flu is* 0.2*. A new flu test reports seasonal flu for* 90% *of the patients with seasonal flu, and reports seasonal flu for* 20% *of the patients with swine flu. What is the probability that a patient has the swine flu, given a test result that is positive for the seasonal flu?*

Hint: Use Bayes' Rule

$$P(B_r|A) = \frac{P(B_r)P(A|B_r)}{\sum_{i=1}^{k} P(B_i)P(A|B_i)}.$$

Let A be the event that the test is positive for seasonal flu, while B_1 is the event that the patient has seasonal flu, and B_2 is the event that the patient has swine flu. By Bayes rule

$$P(B_2|A) = \frac{P(B_2)P(A|B_2)}{P(B_1)P(A|B_1) + P(B_2)P(A|B_2)} = \frac{.8 * .2}{.2 * .9 + .8 * .2} = \frac{16}{34}$$