1.3 Matrix operations

1.3.2 Solve for a, b, c, d.

$$\begin{pmatrix} a-b & b+c \\ 3d+c & 2a-4d \end{pmatrix} = \begin{pmatrix} 8 & 1 \\ 7 & 6 \end{pmatrix}$$

For the matrices to be equal, the entries must be equal. Thus we have

$$a-b=8, \quad b+c=1,$$

$$3d+c=7, \quad 2a-4d=6.$$

The solution to the system of equations is

$$a=5, \quad b=-3, \quad c=4, \quad d=1.$$

1.3.5 For the matrices

$$A = \begin{pmatrix} 3 & 0 \\ -1 & 2 \\ 1 & 1 \end{pmatrix}, \quad B = \begin{pmatrix} 4 & -1 \\ 0 & 2 \end{pmatrix}, \quad C = \begin{pmatrix} 1 & 4 & 2 \\ 3 & 1 & 5 \end{pmatrix},$$

$$D = \begin{pmatrix} 1 & 5 & 2 \\ -1 & 0 & 1 \\ 3 & 2 & 4 \end{pmatrix}, \quad E = \begin{pmatrix} 6 & 1 & 3 \\ -1 & 1 & 2 \\ 4 & 1 & 3 \end{pmatrix},$$

compute the following:

$$(a) \quad AB = \begin{pmatrix} 12 & -3 \\ -4 & 5 \\ 4 & 1 \end{pmatrix},$$

$$(b) \quad BA \; is \; not \; definded,$$

since the number of columns of B is 2, but the number of rows of A is 3.

$$(c) \quad 3ED = \begin{pmatrix} 42 & 108 & 75 \\ 12 & -3 & 21 \\ 36 & 78 & 63 \end{pmatrix},$$

$$(d) \quad (AB)C = \begin{pmatrix} 3 & 45 & 9 \\ 11 & -11 & 17 \\ 7 & 17 & 13 \end{pmatrix},$$

1.4 Matrix inverses

1.4.8 *Let A be the matrix*

$$\begin{pmatrix} 2 & 0 \\ 4 & 1 \end{pmatrix}.$$

Compute A^3, A^{-3}, *and* $A^2 - 2A + I$.

We have

$$A = \begin{pmatrix} 2 & 0 \\ 4 & 1 \end{pmatrix}, \quad A^2 = \begin{pmatrix} 4 & 0 \\ 12 & 1 \end{pmatrix},$$

$$A^3 = \begin{pmatrix} 8 & 0 \\ 28 & 1 \end{pmatrix}, \quad A^{-3} = \frac{1}{8}\begin{pmatrix} 1 & 0 \\ -28 & 8 \end{pmatrix},$$

$$A^2 - 2A + I = \begin{pmatrix} 1 & 0 \\ 4 & 0 \end{pmatrix}.$$

1.4.11 *Find the inverse of*

$$A = \begin{pmatrix} \cos(\theta) & \sin(\theta) \\ -\sin(\theta) & \cos(\theta) \end{pmatrix}.$$

Using the formula in Theorem 1.4.5 we first find $\det(A) = \cos^2(\theta) + \sin^2(\theta) = 1$ and then

$$A^{-1} = \begin{pmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{pmatrix}.$$

1.4.16 *Is the sum of two invertible matrices necessarily invertible?*

No - look at the example

$$A = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad B = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}.$$

Then $A^{-1} = A$, $B^{-1} = B$, but

$$A + B = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix},$$

which is not invertible.