JOHN F. MAHONEY

What Is the Name of This Game?

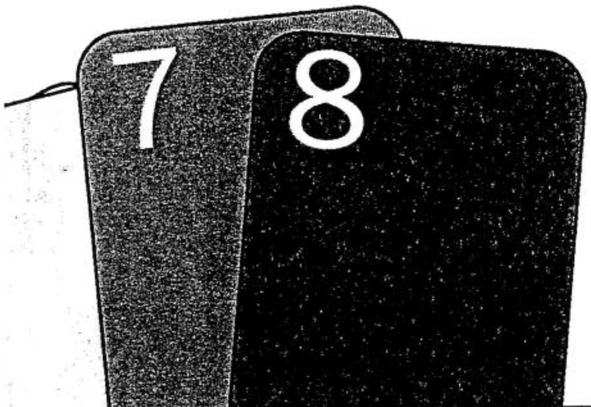
STUDENTS OF ALL AGES ENJOY PLAYING games. Many games illustrate or reinforce mathematical principles. The following game does both.

This two-person game is played with nine cards. Each card has a different number on it, as shown in **figure 1.** Players alternate picking up the cards. The winner is the first who has exactly three cards whose sum is 15. An example of play is shown in **figure 2.**

Although player 1 had four cards at the end of the game, that player won because the sum of a set of three of those cards was exactly 15. Another round of the game is shown in **figure 3.**

When I teach my students how to play the game, I put the numbers on nine index cards, and two students at a time come to the front of the room to play. While they take turns picking cards, I write the numbers of their cards on the board, so that the rest of the class can see which choices are made. I try not to permit the rest of the class to give advice, but often it is hard to hide their reactions. I tell the students that it is the first person who *realizes* that he or she has a sum of 15 in three cards who wins. Often, a student will have four cards like 1, 5, 6, and 9 and not realize that a winning combination of three cards—1, 5, 9—is being held.

JOHN MAHONEY, *mahoneyj@sidwell.edu, teaches mathematics at Benjamin Banneker Academic High School, a public school in Washington, D.C. He has been teaching high school for thirty-four years and loves to learn, read, teach, and write about mathematics.*



1	2	3	4	5	6	7	8	9

Fig. 1 The game cards

PLAYER 1	PLAYER 2	PLAYER 1'S CARDS	PLAYER 2'S CARDS	NOTES
Picks the 8		8		
	Picks the 4	8	4	
Picks the 2		2, 8	4	
	Picks the 5	2, 8	4, 5	If player 2 had not picked the 5, player 1 might have picked it and won with 2, 5, and 8.
Picks the 6		2, 6, 8	4, 5	If player 1 had not picked the 6, player 2 might have picked it and won with 4, 5, and 6.
	Picks the 7	2, 6, 8	4, 5, 7	If player 2 had not picked the 7, player 1 might have picked it and won with 2, 6, and 7.
Picks the 1		1, 2, 6, 8	4, 5, 7	Player 1 wins with the 1, 6, and 8.

Fig. 2 An example of play

PLAYER 1	PLAYER 2	PLAYER 1'S CARDS	PLAYER 2'S CARDS	NOTES
Picks the 4		4		
	Picks the 6	4	6	
Picks the 1		1, 4	6	
	Picks the 8	1, 4	6, 8	
Picks the 3		1, 3, 4	6, 8	
	Picks the 2	1, 3, 4	2, 6, 8	
Picks the 5		1, 3, 4, 5	2, 6, 8	If player 1 had not picked the 5, player 2 might have picked it and won with 2, 5, and 8.
	Picks the 7		2, 6, 7, 8	Player 2 wins with the 2, 6, and 7.

Fig. 3 Another round of play

5	12	19	26	33	40	47	54	61

Fig. 4 The cards used in the extension version of the game

When students play the game, ask them who is more likely to win—the first player or the second. Does someone always win in this game? (No.) Are there ways to make sure that one does not lose? (Maybe, but review the two games discussed on the previous page.) Try to get your students to compare this game with other games they know. Listen to what they say. Tell them that the key to the winning strategy is to find out the name of the game!

Here is an extension of this game for older students. Again, nine cards are used, but this time the cards are numbered as shown in **figure 4.** The winner is the first player who has exactly three cards that sum to 99. An example of play is shown in **figure 5.**

Many games illustrate or reinforce mathematical principles

This version is harder because the numbers are larger. The strategy is exactly the same as the basic version. Ask your students how many winning combinations of three numbers are in this game. (Answer: 8.) Again, the strategy is based on learning the name of the game. This version of the game can be changed to percentages by adding the percent sign to each number. The object then would be to get exactly three cards that add to 99 percent.

Another extension of the same game reviews fractions. **Figure 6** shows the nine cards that are used. This time the winner is the first player who has exactly three cards whose sum is 1. One can win with cards such as $1/6 + 1/3 + 1/2 = 1$, or $5/24 + 7/24 + 1/2 = 1$, or $3/8 + 5/12 + 5/24$, or five other combinations. Again, the strategy is based on the name of the game.

So, what is the name of the game and what is the strategy? Why is this game mathematically interesting? Consider the following chart.

8	1	6
3	5	7
4	9	2

This chart is a magic square. The sum of the three numbers in each row and column is 15. Each of the two diagonals also sums to 15. The game with the cards outlined above is simply ticktacktoe played on a magic square! Consider the first illustration of the game (shown in **fig. 1**), and now consider player 1 being X and player 2 being O. See **figure 7,** which illustrates a round of play.

Ticktacktoe and the nine-card game are isomorphic. An *isomorphism* is a mathematical term that describes two structures that have exactly the same rules and behavior. In this case, the game played with the nine cards is exactly the same as ticktacktoe. The relationship is given by the magic square. The strategy of ticktacktoe can be used by students to win in the game of the nine cards. Therefore, the name of the game of nine cards is ticktacktoe!

PLAYER 1	PLAYER 2	PLAYER 1'S CARDS	PLAYER 2'S CARDS	NOTES
Picks the 54		54		
	Picks the 40	54	40	
Picks the 19		19, 54	40	
	Picks the 26	19, 54	26, 40	If player 2 had not picked the 26, player 1 might have picked it and won with 19, 26, and 54.
Picks the 12		12, 19, 54	26, 40	
	Picks the 33	12, 19, 54	26, 33, 40	Player 2 wins with 26, 33, and 40, which sum to 99.

Fig. 5 A round of play with an extension of the original game

$\frac{1}{6}$	$\frac{5}{24}$	$\frac{1}{4}$
$\frac{7}{24}$	$\frac{1}{3}$	$\frac{3}{8}$
$\frac{5}{12}$	$\frac{11}{12}$	$\frac{1}{2}$

Fig. 6 The fraction game's cards

The magic squares for the two other versions of the game are shown in **figure 8.** In box (a), the sum of each row, column, and diagonal is 99. In box (b), the sum of each row, column, and diagonal is 1. What is the relationship among the three versions of the game illustrated in the article? First, they are all based on the magic square whose sum of each row, column, and diagonal is 15. In the second version, each number (1–9) in the original magic square is operated on by the function $f(x) = 7x - 2$. In the third version, each number (1–9) in the original magic square is operated on by the function $f(x) = 1/24x + 1/8$. Your students can easily devise additional versions of the game by using appropriate transformations.

The first game in this article was originally described by Martin Gardner in an article in *Scientific American* magazine. It has been reprinted in his book *Mathematical Carnival: A New Round-up of Tantalizers and Puzzles from "Scientific American"* (1975). One chapter, "Jam, Hot, and Other Games," describes the game "hot," which was devised by the mathematician Leo Moser (1961). This game is played with nine cards; sound familiar? Each card contains a single word. The words are *hot, hear, tied, horn, wasp, brim, tank, ship,* and *woes.* "The nine cards are placed faceup on the table. Players take turns removing a card. The first to hold three cards that bear the same letter is the winner." Notice that exactly three of the words contain an "h"; three have a "t"; three have an "a"; and so on. Have your students determine whether this is the same game as the others in this article. The hint is shown in **figure 9,** where the same letter is found in each row, column, and diagonal.

Figure 10 is a version of the game for algebra students. Label the nine cards $x, x^2, x^3, \ldots, x^9$. The object in this version is to find a product equaling x^{15} with exactly three factors. For example, $x^8 \times x \times x^6 = x^{15}$. This version is based on the additive property of exponents that is used when calculating

PLAYER 1: X	PLAYER 2: O		
Picks the 8		X 1 6 3 5 7 4 9 2	
	Picks the 4		X 1 6 3 5 7 O 9 2
Picks the 2		X 1 6 3 5 7 O 9 X	
	Picks the 5		X 1 6 3 O 7 O 9 X
Picks the 6		X 1 X 3 O 7 O 9 X	
	Picks the 7		X 1 X 3 O O O 9 X
Picks the 1		~~X X X~~ 3 O O O 9 X	**Player 1 wins!**

Fig. 7 A round of play with X's and O's

54	5	40
19	33	47
26	61	12

(a)

$\frac{11}{24}$	$\frac{1}{6}$	$\frac{3}{8}$
$\frac{1}{4}$	$\frac{1}{3}$	$\frac{5}{12}$
$\frac{7}{24}$	$\frac{1}{2}$	$\frac{5}{24}$

(b)

Fig. 8 Summing to 99 (a), and summing to 1 (b)

TIED	TANK	HOT
BRIM	HEAR	HORN
SHIP	WASP	WOES

Fig. 9 A version of the game using words

x^8	x	x^6
x^3	x^5	x^7
x^4	x^9	x^2

Fig. 10 An algebraic version of the game

9	6	3	16
4	15	10	5
14	1	8	11
7	12	13	2

Fig. 11 A version using 16 numbers

the powers of expressions with the same base. **Figure 10** shows the relationship between these factors and a standard ticktacktoe layout.

Finally, one is not limited to nine cards. With sixteen cards, for example, numbered 1–16, the object of the game would be to find a sum of 34 with exactly four cards. This game is equivalent to playing ticktacktoe on a 4 × 4 magic square. See **figure 11.**

A simple ticktacktoe "board" is all that is needed to practice some mathematics skills.

Bibliography

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1	2	3
4	5	6
7	8	9