Theory of microwave propagation in dielectric/magnetic film multilayer structures

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We explore the theory of microwave propagation in dielectric films, on which thin metallic ferromagnetic films have been deposited. Our aim is to study coupling between the microwave electromagnetic fields, and spin excitations in the ferromagnetic films. We present quantitative studies of attenuation provided by coupling to spin excitations, for various model structures including superlattices. We find strong attenuation of the microwaves, for frequencies near the ferromagnetic resonance frequency of Fe. Modest magnetic fields place this resonance above 20 GHz, and allow its frequency to be tuned. We note a transmission minimum occurs near the frequency $\gamma(H_0 + 4\pi M_s)$, which is in the 70 GHz range for external magnetic fields H_0 of a few kilograms. We explore the dependence of these phenomena on film thicknesses, and argue that such structures will move suitably for high frequency microwave devices. [S0021-8979(97)08717-3]

I. INTRODUCTION

During the past decade, there has been impressive progress in the growth of very high quality thin metallic films, and multilayer structures such as superlattices formed from such films. Multilayers can be synthesized from diverse constituents, and growth by either sputtering techniques or molecular beam epitaxy (MBE) provide samples with interfaces of very high quality.[1]

There has been particularly strong interest in structures which contain films of metallic ferromagnets such as Fe, Co, or Ni and their alloys. Phenomena such as giant magnetoresistance (GMR)[2] and spin dependent tunneling make such structures suitable for various applications, such as magnetic sensors, or elements in high density memory devices. For this reason, there has been a very high level of activity in recent years, devoted to the synthesis and characterization of new multilayer structures.

While the remarks above have in mind metallic films, and multilayers formed from them, it is the case that very high quality metallic films may be grown on semiconductors as well, by methods such as MBE. There is a good lattice match between Fe, and the (100) surface of GaAs. Also, high quality Fe films may be grown on ZnSe. Progress in this area has been summarized in a review article by Prinz.[4]

Such semiconductor/ferromagnetic film combinations offer new device possibilities. The semiconductor, viewed here as simply a dielectric film, may support the propagation of a microwave signal, or perhaps also an optical beam. In addition, the magnetic film possesses collective excitations referred to as spin waves. These are magnetic analogs of the sound waves in elastic media. Optical beams or microwaves may couple to the spin waves in the magnetic film, since their electric and magnetic fields penetrate the metal film by virtue of its finite skin depth. One may envision possible device applications, made possible through use of the spin waves as a means of modifying the propagation characteristics of the electromagnetic wave supported by the dielectric film.

For many years, garnet films have been used as the basis for microwave and integrated optics devices. A recent example is the development of the magneto-optic Bragg cell.[5] In the garnet films, the maximum spin wave frequencies which may be realized for device applications are in the range of 10 GHz, or slightly above. Large external magnetic fields must be applied to exceed this frequency range, and these are difficult to realize in device geometries.

The use of films such as Fe offer the possibility, at least in principle, of operating at much higher frequencies. The reason is as follows. When spin motions are excited in a ferromagnet, the spin precession frequency, and hence that of the spin wave or collective excitation, is the Larmor frequency of the spin in the externally applied magnetic field H_0, supplemented by an internal field generated from the ferromagnetically aligned spin array. A measure of the strength of this internal field is $4\pi M_s$, with M_s the saturation magnetization of the ferromagnet. In the garnets, $4\pi M_s$ is roughly 2 kG, whereas in ferromagnetic Fe, this internal field is 21 kG at room temperature. In the absence of anisotropy, the ferromagnetic resonance frequency Ω_{FM} of thin ferromagnetic films is given by $\gamma[H_0(H_0+4\pi M_s)]^{1/2}$, where γ is the gyromagnetic ratio. Application of a 2 kG field to a Fe film provides a resonance frequency a bit above 20 GHz, while the same field applied to a garnet film gives a resonance frequency of roughly 8 GHz.

The obvious disadvantage of utilizing Fe and other metallic ferromagnets in devices is the Ohmic dissipation necessarily introduced into the structure. For this reason, dielectric/ferromagnetic multilayers are most attractive, since the electromagnetic energy is stored mainly in the dielectric components, where the electrical conductivity is extremely

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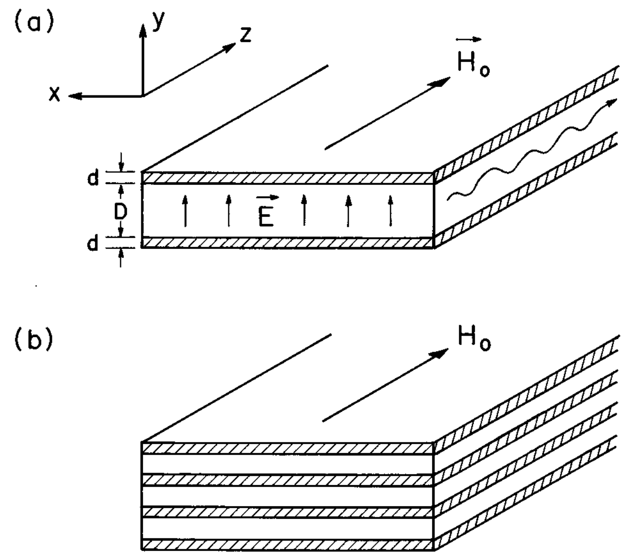

FIG. 1. Two examples of the structures explored in the present paper. In (a), we have a dielectric film of thickness D, with a ferromagnetic film of thickness d deposited on both the top and the bottom. We consider microwaves launched down the structure, which propagate in the z direction. In (b) we have a superlattice formed from ferromagnetic films (shaded), and dielectric films. Again the microwaves propagate in the z direction.

low. There is then the question of achieving strong coupling to the spin waves. This paper is devoted to a theoretical study of this question, for several model structures, and for microwave propagation in the 20 GHz frequency range.

The possibility of utilizing Fe films deposited on GaAs (100) as the basis for a notch filter was considered some years ago by Schlömann *et al.*[7] The GaAs film serves as a dielectric waveguide, and as mentioned above coupling to spins in the Fe film is achieved through the skin effect. Microwaves are absorbed as they propagate down the structure, in a frequency band centered around Ω_{FM}, with width controlled by the ferromagnetic resonance linewidth. Schlömann and his co-workers presented both theoretical studies of this structure, and data in the 10 GHz frequency range on samples. The calculations presented here are in very good accord with his, when we examine the structures explored earlier.

This paper is organized as follows. In Sec. II, we summarize our theoretical approach, with emphasis on physical considerations that enter importantly. In Sec. III, we present results of our studies of microwave attenuation in various model structures, and in Sec. IV we summarize our principal conclusions.

II. ANALYSIS

Two examples of the model structures explored here are illustrated in Fig. 1. In Fig. 1(a), we have a system which is patterned after that used previously by Schlömann and his colleagues. We have a dielectric film of thickness D, with metallic ferromagnetic films of thickness d deposited on both the top and bottom surface. The only difference between this configuration, and that explored in Ref. 7 is that these authors had only one magnetic film, and not two. We shall also explore other forms of multilayer structure, such as the superlattice illustrated in Fig. 1(b).

We suppose a magnetic field is applied in the plane of the magnetic film, as illustrated in Fig. 1. The microwaves propagate parallel to the z direction, along which the magnetic field is directed. All quantities thus exhibit the spatial variation $\exp(ikz)$. If Ω is the frequency of the disturbance, both the real and imaginary part of the wave vector k are determined from an implicit dispersion relation described below for the various structures of interest.

If we consider a single isolated ferromagnetic film, and examine the spin waves which propagate in this geometry, the configuration is such that one realizes entities referred to in the literature as ''backward volume waves.'' These have standing wave character in the direction perpendicular to the film surfaces, and they propagate down the film, with group velocity that is antiparallel to the phase velocity. It will be apparent that our interest will center on very thin ferromagnetic films, for which $kd \ll 1$. In this limit, the backward volume waves have frequency $\Omega_{\mathrm{BVW}}(k)$ described by a simple dispersion relation. If Ω_{FM} is the ferromagnetic resonance frequency of the thin film discussed in Sec. I, we have

$$\Omega_{\mathrm{BVW}}(k)^2 = \Omega_{\mathrm{FM}}^2 - 2\pi\gamma^2 H_0 M_s kd, \tag{1}$$

where M_s is the saturation magnetization and γ the gyromagnetic ratio. Recall that

$$\Omega_{\mathrm{FM}} = \gamma[H_0(H_0 + 4\pi M_s)]^{1/2}. \tag{2}$$

When $kd \ll 1$, the modes of interest to which the microwaves couple all lie very close in frequency to Ω_{FM}.

We will also be interested in structures formed from thin dielectric films, so we have $kD \ll 1$ as well. The microwave mode of interest is then the lowest frequency TM mode of the structure. For this mode the magnetic field **H** is parallel to the x direction. When $kD \ll 1$, this **H** field is almost spatially uniform throughout the dielectric film. Since tangential components of **H** are conserved across the dielectric/metal interface, the full **H** field penetrates into the metal films, to excite the spins. For the TM mode, the dominant component of electric field is parallel to the y direction, as illustrated in Fig. 1(a). There is a small longitudinal (z) component of electric field as well. The structure illustrated will support the TM mode just discussed, for all frequencies down to zero frequency.

There are higher order TM modes, which only exist for frequencies above a cutoff frequency the order of $(c/\sqrt{\epsilon}) \times (n\pi/D)$, with ϵ the dielectric constant of the dielectric film. For a GaAs film with thickness in the 300 μm range ($\epsilon \cong 12$), the cutoff frequency of the $n=1$ mode is roughly 150 GHz, well above the frequency range of interest here. The structure also supports TE modes, in which the electric field is parallel to the x direction. The TE modes all have a cutoff frequency in the range just estimated, and thus will not propagate.

We now turn to our analysis. First we explore the very simple case where the metal films are not ferromagnetic. This will allow us to assess the influence of Ohmic dissipation on microwave propagation down the structure. This is a concern for any device which incorporates metallic overlayers. This discussion is straightforward, and will enable us to establish the notation and approach. We remark that in this section, we just obtain the implicit dispersion relations for our various models. We present results based on their solution in Sec. III.

A. Microwave attenuation in a dielectric waveguide cladded with metal films

1. The structure depicted in Fig. 1(a)

Here we explore the particular structure illustrated in Fig. 1(a), wherein a dielectric waveguide has deposited on both its top and bottom surface a thin film of conducting material, with air outside each metal film. If these films are made from a material (such as Fe) which may oxidize, quite commonly one adds an overlayer of a noble metal such as Ag or Au, whose role is to suppress oxidation. We shall consider the influence of such overlayers in the next subsection. We shall see, when the results in Sec. III are presented, that the presence of such overlayers strongly influences the nature of the Ohmic damping present in such systems.

The dielectric waveguide occupies the regime $0<y<D$, while the lower metal film lies in the region $-d<y<0$, and the upper metal film $D<y<D+d$.

It is straightforward to synthesize fields within the dielectric waveguide from Maxwell's equations applied to the TM mode of interest. We write the electric field $\mathbf{E}$ and magnetic field $\mathbf{H}$ in the form

$$\mathbf{E}=E_{\perp}\left\{\hat{y}\cos\left[Q\left(y-\frac{1}{2}D\right)\right]-i\frac{Q}{k}\hat{z}\sin\left[Q\left(y-\frac{1}{2}D\right)\right]\right\}e^{ikz}e^{-i\Omega t} \tag{3a}$$

and

$$\mathbf{H}=-\frac{\epsilon\Omega}{ck}\hat{x}E_{\perp}\cos\left[Q\left(y-\frac{1}{2}D\right)\right]e^{ikz}e^{-i\Omega t}. \tag{3b}$$

We note that, in accord with our earlier discussion, H is an even function about the midplane of the structure. Here, ϵ is the dielectric constant of the waveguide, assumed real for the numerical calculations reported below. Then c is the velocity of light, and Q along with the propagation constant k are related through requiring each Cartesian component of the fields to satisfy the wave equation. One has

$$Q^2=\epsilon\frac{\Omega^2}{c^2}-k^2. \tag{4}$$

Given the frequency Ω, our aim is to solve for the propagation constant k. We shall do this through an implicit dispersion relation derived below.

We must find the electromagnetic fields within the metal films, and then match them to the fields in Eqs. (3) through the appropriate boundary conditions. With microwave frequencies in mind, we neglect the displacement current term in Maxwell's equation, since its influence in metals is quite negligible at such frequencies. The structure in Fig. 1(a) has reflection symmetry through the plane $y=D/2$. Thus, we may confine our attention only to one of the two metal films, which we take to be that between $y=-d$ and $y=0$. The most general forms for the electric and magnetic fields in the metal is then

$$\mathbf{E}=\left[E_{\perp}^{(+)}\left(\hat{y}-\frac{\kappa}{k}\hat{z}\right)e^{i\kappa y}+E_{\perp}^{(-)}\left(\hat{y}+\frac{\kappa}{k}\hat{z}\right)e^{-i\kappa y}\right]e^{ikz-i\Omega t} \tag{5a}$$

and

$$\mathbf{H}=-\hat{x}\frac{c\kappa^2}{\Omega k}\left(E_{\perp}^{(+)}e^{i\kappa y}+E_{\perp}^{(-)}e^{-i\kappa y}\right)e^{ikz}e^{-i\Omega t}. \tag{5b}$$

In these expressions:

$$\kappa=\frac{1}{\delta_0}(1+i). \tag{6}$$

Here δ_0 is the classical skin depth of the metal:

$$\delta_0=\frac{c}{(2\pi\sigma_0\Omega)^{1/2}} \tag{7}$$

with σ_0 its conductivity.

Relations between the three amplitudes $E_{\perp}$, $E_{\perp}^{(+)}$, and $E_{\perp}^{(-)}$ follow upon applying the electromagnetic boundary conditions at $y=0$. Conservation of tangential components of $\mathbf{E}$ provide us with

$$\sin\left(\frac{1}{2}QD\right)E_{\perp}=i\frac{\kappa}{Q}(E_{\perp}^{(+)}-E_{\perp}^{(-)}), \tag{8a}$$

while conservation of tangential $\mathbf{H}$ yields

$$\cos\left(\frac{1}{2}QD\right)E_{\perp}=\frac{c^2\kappa^2}{\epsilon\Omega^2}(E_{\perp}^{(+)}+E_{\perp}^{(-)}). \tag{8b}$$

No new information follows from the requirement that normal components of $\mathbf{D}$ be conserved.

We must now consider the region below $y=-d$, which we assume is occupied by air with dielectric constant unity. The fields in this region will be evanescent in character, for waves confined to and guided by the structure. For $y<-d$, we have fields which we write as

$$\mathbf{E}=E_{\perp}^{(<)}\left(\hat{y}+i\frac{\alpha_0}{k}\hat{z}\right)e^{\alpha_0(y+d)}e^{ikz}e^{-i\Omega t} \tag{9a}$$

and

$$\mathbf{H}=-\frac{\Omega}{ck}E_{\perp}^{(<)}\hat{x}e^{\alpha_0(y+d)}e^{ikz}e^{-i\Omega t} \tag{9b}$$

where the wave equation in vacuum gives

$$\alpha_0=\left(k^2-\frac{\Omega^2}{c^2}\right)^{1/2}. \tag{10}$$

For all the modes we consider, the imaginary part of k, namely k_2, will be very small compared to its real part. Waves are bound or guided by the structure when k_1, the real part of k, is larger than Ω/c. We always choose the square root in Eq. (10) so that

$$\mathrm{Re}(\alpha_0) > 0. \tag{11}$$

Once again we require tangential **E** and **H** be conserved, but now at the interface $y = -d$. Conservation of tangential **E** gives

$$E_\perp^{(+)} e^{-i\kappa d} - E_\perp^{(-)} e^{i\kappa d} = -i \frac{\alpha_0}{k} E_\perp^{(<)} \tag{12a}$$

while conservation of tangential **H** requires

$$E_\perp^{(+)} e^{i\kappa d} + E_\perp^{(-)} e^{i\kappa d} = \frac{\Omega^2}{c^2 \kappa^2} E_\perp^{(<)}. \tag{12b}$$

In Eqs. (8) and (12), we have four homogeneous equations in the four field amplitudes $E_\perp$, $E_\perp^{(+)}$, $E_\perp^{(-)}$, and $E_\perp^{(<)}$. Once the frequency Ω is chosen, these equations admit nonzero solutions only for one, or perhaps a discrete set of propagation constants k. It is a straightforward matter to derive an implicit equation from which the allowed propagation constants may be obtained. This has the form

$$\cot\left(\frac{1}{2} QD\right) = i \frac{\kappa Q}{\epsilon k_0^2} \frac{(1 - z e^{i2\kappa D})}{(1 + z e^{i2\kappa d})}, \tag{13}$$

where

$$z = \frac{\alpha_0 \kappa + i k_0^2}{\alpha_0 \kappa - i k_0^2} \tag{14a}$$

and we have defined

$$k_0 = \frac{\Omega}{c}. \tag{14b}$$

In Sec. III, we shall discuss numerical solutions of Eq. (13) for structures of interest, and we shall also obtain approximate analytic solutions applicable to particular regimes.

2. The influence of metallic caps on the structure in Fig. 1(a)

As noted above, the expression in Eq. (13) provides us with the implicit dispersion relation of microwaves which propagate down the structure illustrated in Fig. 1(a), where it is assumed that we have air outside the two metallic films. In practice, most particularly if the thin films are a metal such as Fe, a noble metal overlayer will be deposited over the Fe films to prevent oxidation.

In what follows, we assume such overlayers are present, and furthermore that they are sufficiently thick that they may each be supposed to be of infinite thickness. Let the conductivity of the overlayer material be σ_1, and its skin depth be δ_1. It is straightforward to modify the discussion given in the previous subsection to describe this case. When this is done, the implicit dispersion relation has precisely the form given in Eq. (13), except the quantity z is replaced by z_1, where

$$z_1 = \left(\frac{\kappa - \kappa_1}{\kappa + \kappa_1}\right) \tag{15}$$

and

$$\kappa_1 = \frac{1}{\delta_1}(1+i). \tag{16}$$

Again we present calculations which address this structure in Sec. III.

B. Microwave attenuation in a dielectric waveguide cladded with ferromagnetic metal films

1. The structure depicted in Fig. 1(a)

We now turn to the case where the metal films in Fig. 1(a) are not only metallic, as in the discussion above, but ferromagnetic as well. Here we discuss the situation illustrated in Fig. 1(a) where the ferromagnetic films are uncapped, with air above. In the next subsection, we discuss the extension to the case where each ferromagnetic film is covered by a thick metallic film.

The electromagnetic fields within the dielectric waveguide, and those in the air outside the ferromagnetic film are described as in Sec. (II A). Thus, we do not display their form here, but we will use the expressions given in the previous section. We do discuss the influence of the ferromagnetism on the fields within the two ferromagnetic films. As we proceed, we will invoke an approximation described below which we believe to be quite accurate, for the systems studied here.

The two principal Maxwell equations we explore are

$$\nabla \times \mathbf{E} = i k_0 \mathbf{B} \tag{17a}$$

and

$$\nabla \times \mathbf{H} = \frac{4\pi\sigma_0}{c} \mathbf{E} \tag{17b}$$

where once again we ignore the displacement current contribution to Eq. (17b), since in the frequency regime of interest, its influence is quite negligible. Recall that $k_0 = \Omega/c$, from Eq. (14b). The conditions $\nabla \cdot \mathbf{E} = \nabla \cdot \mathbf{B} = 0$ are appended to Eqs. (17).

To proceed, we require a constitutive relation between **B** and **H**. This takes the form, for a ferromagnet oriented such as that in Fig. 1:

$$B_x = \mu_1 H_x + i \mu_2 H_y, \tag{18a}$$

$$B_y = -i \mu_2 H_x + \mu_1 H_y, \tag{18b}$$

and

$$B_z = H_z. \tag{18c}$$

Expressions for the frequency dependent magnetic response functions are derived standardly from the Landau–Lifshitz equations.[8] Let γ be the gyromagnetic ratio, and define $\Omega_M = \gamma M_S$ and $\Omega_H = \gamma H_0$. One then finds[8]

$$\mu_1 = 1 + \frac{4\pi\Omega_M(\Omega_H - i\Gamma\Omega)}{(\Omega_H - i\Gamma\Omega)^2 - \Omega^2} \tag{19a}$$

and

$$\mu_2 = \frac{4\pi\Omega_M\Omega}{(\Omega_H - i\Gamma\Omega)^2 - \Omega^2}. \tag{19b}$$

The dimensionless parameter Γ in Eqs. (19) has its origin in dissipation in the spin system.[9] Its value controls the ferromagnetic resonance linewidth ΔH, defined by Heinrich and Cochran as[8]

$$\Delta H = 1.16\left(\frac{\Omega}{\gamma}\right)\Gamma. \tag{20}$$

We proceed by combining Eqs. (18) with (17), and seeking eigensolutions with the form $\exp(\pm i\kappa y)\exp(ikz)\exp(-i\Omega t)$. There are two eigensolutions, neither of which has the character of a pure TM mode, or a pure TE mode. The gyrotropic character of the response of the ferromagnet, with origin in μ_2, produces normal modes in which all the Cartesian components of the field are nonzero. It follows from this that the electromagnetic fields within the dielectric waveguide are also no longer of pure TE or TM character, but are mixtures of the TE and TM mode.

If, in the metal film, we take the mathematical (but unphysical) limit $\mu_2 \to 0$, one of the two modes reduces to a TM mode, and one reduces to a TE mode. For ease of discussion, when $\mu_2 \neq 0$, we refer to one of the exact modes as the TM mode, and the second the TE mode, labeling each by their behavior in the limit $\mu_2 \to 0$. This nomenclature is appropriate, for reasons we shall appreciate below.

Consider the electric and magnetic fields associated with the TM mode of the ferromagnetic film. These have the form

$$\mathbf{E}^{(\pm)} = E_\perp^{(\pm)}\left(\frac{i\beta}{(\mu_v - 1)}\,\hat{x} + \hat{y} \mp \frac{\tilde{\kappa}}{k}\,\hat{z}\right) e^{\pm i\tilde{\kappa}y} e^{ikz} e^{-\Omega t} \tag{21a}$$

$$\mathbf{B}^{(\pm)} = E_\perp^{(\pm)}\left[-\left(\frac{\tilde{\kappa}^2}{kk_0}\right)\hat{x} - i \times \frac{k}{k_0}\left(\frac{\beta}{\mu_v - 1}\right)\hat{y} \pm i\,\frac{\tilde{\kappa}}{k_0}\left(\frac{\beta}{\mu_v - 1}\right)\hat{z}\right] e^{\pm i\tilde{\kappa}y} e^{ikz} e^{-i\Omega t} \tag{21b}$$

and

$$\mathbf{H}^{(\pm)} = E_\perp^{(\pm)}\left[-\left(\frac{\tilde{\kappa}^2}{kk_0\mu_v}\right)(\hat{x} + i\beta\hat{y}) + \frac{k}{k_0}\,\frac{1}{\mu_v}\left(\frac{i\beta}{\mu_v - 1}\right) \times \left(-i\beta\hat{x} + \hat{y} \mp \frac{\tilde{\kappa}}{k}\,\mu_v\hat{z}\right)\right] e^{\pm i\tilde{\kappa}y} e^{ikz} e^{i\Omega t}. \tag{21c}$$

In these expressions:

$$\mu_v = \frac{\mu_1^2 - \mu_2^2}{\mu_1} \tag{22a}$$

is often referred to as the Voigt permeability:

$$\beta = \frac{\mu_2}{\mu_1} \tag{22b}$$

and

$$\tilde{\kappa} = \frac{(\mu_v)^{1/2}}{\delta_0}\,(1 + i). \tag{22c}$$

Some general comments on the structure of these expressions are in order. First note, as remarked above, that when μ_2 and consequently β are nonzero, as mentioned earlier, the fields do not have pure TM character. As a consequence, the mode as a whole is no longer a TM mode, by virtue of the gyrotropic response of the ferromagnet. We argue below, however, that under circumstances of interest to us, deviations from pure TM character are very small.

The microwave skin depth is affected strongly by the magnetic response of the film, as one sees from Eq. (22c). The effective skin depth is

$$\delta_{\text{eff}} = \frac{\delta_0}{(\mu_v)^{1/2}}. \tag{23}$$

If, in the interest of simplicity, we set the damping constant Γ to zero, then

$$\mu_v = \frac{\Omega_B^2 - \Omega^2}{\Omega_{\text{FM}}^2 - \Omega^2} \tag{24}$$

where Ω_{FM} is the ferromagnetic resonance frequency of the film discussed in Sec. I $[\Omega_{\text{FM}}^2 = \Omega_H(\Omega_H + 4\pi\Omega_M)]$, and $\Omega_B = \Omega_H + 4\pi\Omega_M$.

As Ω approaches the ferromagnetic resonance frequency, μ_v increases dramatically, and the skin depth decreases by a large amount. This will have important consequences for the calculations presented in Sec. III. This is an unfortunate situation, because the reduced skin depth ''cuts off'' coupling between the microwave field and the spins, precisely when it is most desired, on resonance. Notice that μ_v has a zero, and thus the metallic films ''open up'' near the frequency Ω_B, which for Fe is in the 70 GHz range, when a 2 kg field is present. We shall explore consequences of this as well.

We next consider the order of magnitude of the various parameters that enter Eqs. (21), with the 20 GHz frequency range in mind. We have $k_0 = \Omega/c \cong 4\ \text{cm}^{-1}$. One expects $k \cong k_0\sqrt{\epsilon} \sim 8-10\ \text{cm}^{-1}$, for a typical semiconducting waveguide. The parameter Q will be in the same range, since $Q^2 + k^2 = k_0^2\epsilon$ for these propagating modes.

However, κ and $\tilde{\kappa}$ are very much larger indeed than the three parameters just described. For Fe at 20 GHz, the skin depth δ_0 off resonance is very close to 10^{-4} cm, or 1 μm. Hence, $\kappa \approx 10^4\ \text{cm}^{-1}$, $\gg k_0$, Q, or k. The argument given above suggests that near ferromagnetic resonance, $\tilde{\kappa}$ is in fact much larger than κ.

If one examines Eq. (21a) with the above numerical estimates in mind, one sees that the $\hat{z}$ component of the electric field (the component parallel to the biasing field $\mathbf{H}_0$) is larger than the $\hat{x}$ and $\hat{y}$ components by roughly three or four orders of magnitude. For the magnetic field $\mathbf{H}$, whose tangential components are conserved across the interface, the $\hat{x}$ component (parallel to the interface) and the $\hat{y}$ component (normal to the interface) are larger than the $\hat{z}$ component by three or four orders of magnitude.

It is the presence of the $\hat{x}$ component of $\mathbf{E}$, and the $\hat{z}$ component of $\mathbf{H}$ which are responsible for ''mixing in'' fields of the TE character in the dielectric waveguide. We have just seen that these two components are three to four orders of magnitude smaller than the dominant components of $\mathbf{E}$ and $\mathbf{H}$, which can be matched appropriately to fields of TM character in the dielectric.

With these remarks in mind, we shall proceed by approximating the fields in the dielectric film by fields of pure TM character, as given in Eq. (3). We match them to fields in the metal film, which are linear combinations of the fields $\mathbf{E}^{(\pm)}$, and $\mathbf{H}^{(\pm)}$ given in Eqs. (21a) and (21c). When we match the fields, we require only that tangential components of $\mathbf{E}$ and $\mathbf{H}$ be conserved across the boundary, and ignore the very small quantitative errors introduced by requiring continuity of the other small components.

Once this approximation is accepted, the implicit dispersion relation may be derived by a discussion that follows that given in the previous section. We thus simply quote the result:

$$\cot\left(\frac{1}{2}\,QD\right)=i\,\frac{\tilde{\kappa}Q}{\epsilon k_0^2\mu_v}\left(\frac{1-\tilde{z}\,e^{i2\tilde{\kappa}d}}{1+\tilde{z}\,e^{i2\tilde{\kappa}d}}\right) \tag{25}$$

where

$$\tilde{z}=\frac{\alpha_0\tilde{\kappa}+ik_0^2\mu_v}{\alpha_0\tilde{\kappa}-ik_0^2\mu_v}. \tag{26}$$

2. The influence of metallic caps on the structure in Fig. 1(a)

We handle this with the approximation described in the previous subsection, in regard to the fields within the ferromagnetic films, presently capped by a thick conducting film. The thick conducting films are treated as in Sec. II A 1. The derivation is straightforward, and the effective dispersion relation has form identical to Eq. (25), with the factor $\tilde{z}$ replaced by

$$\tilde{z}=\left(\frac{\tilde{\kappa}-\mu_v\kappa_1}{\tilde{\kappa}+\mu_v\kappa_1}\right), \tag{27}$$

with κ_1 defined by Eq. (16).

C. Microwave propagation in the superlattice structure depicted in Fig. 1(b)

As one sees from the figure, one has a superlattice structure whose basic unit cell consists of a dielectric film of thickness D, and a ferromagnetic film of thickness d. The unit cells are stacked together as indicated in the figure. We shall assume here that we have an infinite number of unit cells, so the structure fills the entire space from $y=-\infty$ to $y=+\infty$. It should be remarked that our interest will be in the case where the dielectric film thickness D is rather small. Thus, a practical sample will consist of many unit cells.

We shall treat the fields within the scheme described in Sec. II B, where we regard the mode as very well approximated by one TM character. We match tangential components of $\mathbf{E}$ and $\mathbf{H}$ in the dielectric film to the very large tangential components of $\mathbf{E}$ and $\mathbf{H}$ in the ferromagnetic film.

For the superlattice which consists of the infinite stack of unit cells, from the perspective of any individual dielectric film, the structure has reflection symmetry through the midplane of the film. Thus, within each dielectric film, the electromagnetic field may be taken to have a form identical to that described by Eqs. (3). These apply to the particular film located between $y=0$ and $y=D$, and through the appropriate translation describe the remaining dielectric films. If we sit in one of the ferromagnetic films, the structure also has reflection symmetry through the midpoint of the ferromagnetic film. A consequence is that the Cartesian components of the electric and magnetic field have well-defined parity in these films. Within the ferromagnetic film centered between $y=0$ and $y=-d$, for the tangential components of $\mathbf{E}$ and $\mathbf{H}$ we have

$$\mathbf{E}_t=-i\,\frac{\tilde{\kappa}}{k}\,E^{(M)}\hat{z}\,\sin\left[\tilde{\kappa}\left(y+\frac{1}{2}\,d\right)\right] \tag{28a}$$

and

$$\mathbf{H}_t=-\frac{\tilde{\kappa}^2}{kk_0\mu_v}\,E^{(M)}\hat{x}\,\cos\left[\tilde{\kappa}\left(y+\frac{1}{2}\,d\right)\right]. \tag{28b}$$

Since these field forms are repeated throughout the structure unchanged in shape, we obtain the implicit dispersion relation by tangential components of $\mathbf{E}$ and $\mathbf{H}$ to be conserved across the interface at $y=0$. A short calculation gives us

$$\cot\left(\frac{1}{2}\,QD\right)=-\frac{\tilde{\kappa}Q}{\epsilon k_0^2\mu_v}\cot\left(\frac{1}{2}\,\tilde{\kappa}d\right). \tag{29}$$

In the next section, we present both analytical results in special limits, and numerical results for the various structures considered in this section.

III. RESULTS AND DISCUSSION

Next we turn to a discussion of the results of a series of calculations based on the implicit dispersion relations obtained in Sec. II. It should be noted that all calculations are performed for a frequency of 20 GHz. We have in mind dielectric waveguide thicknesses in the range of a few tens to a few hundred microns. The skin depth of Fe is very close to one micron at 20 GHz, so Fe film thicknesses will be at most a few microns. As we shall see, in fact, a few hundred angstroms of Fe will allow one to achieve optimum coupling.

The first question we address is the influence of the metal films on the microwave propagation length, in the absence of magnetism. That is, we inquire the extent to which the presence of the metal films leads to attenuation over and above that provided by losses in the dielectric film itself.

If the two metal films are very thick compared to the skin depth, then one may derive a very simple analytic formula for the attenuation length. One uses Eq. (13), and takes the limit $d\rightarrow\infty$, where $\exp(i2\kappa d)\rightarrow 0$. We recall from Sec. II that under the conditions of interest $(\kappa/k_0)\sim 10^4$. The right-hand side of Eq. (13) is then very large compared to unity, and we are in the limit $QD\ll 1$. Thus, $\cot(1/2QD)$ is well approximated by $2/QD$, and thus $Q^2\cong-(2i\epsilon k_0^2/\kappa D)$, then recall that

$$k=(\epsilon k_0^2-Q^2)^{1/2}\cong\epsilon^{1/2}k_0\left(1+\frac{i}{\kappa D}\right). \tag{30}$$

We write $k=k_1+ik_2$, and note that $k_0=2\pi/\lambda_0$, where λ_0 is the free space wavelength of the radiation field, at the wavelength of interest. We then have a very simple result:

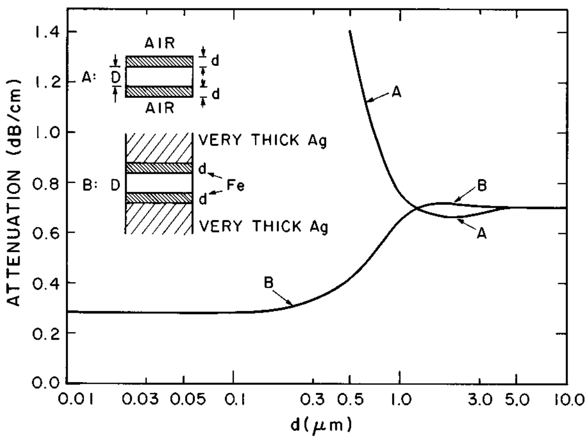

FIG. 2. The microwave attenuation at 20 GHz, as a function of Fe film thickness, for two cases. Case (A) is a lossless dielectric film ($\epsilon=12$) with a thickness of 100 μm, and air outside the Fe films on the top and bottom of the dielectric. Case (B) is the same dielectric film, but now deposited on each Fe film are overlayers of Ag, whose thickness is large compared to the skin depth in Ag.

$$k_2=\frac{\pi\epsilon^{1/2}}{\lambda_0}\frac{\delta_0}{D}, \quad d\gg\delta_0. \tag{31}$$

From Eq. (13), we may calculate k_2, for the case where the metal films have finite thickness. We have done this numerically. For the structure illustrated in Fig. 1(a), where we have a dielectric waveguide with a metal film deposited on top and on bottom, the results are surprising. As the thickness of the metal films is decreased, the Ohmic dissipation increases rather than decreases, as one might expect intuitively. Of course, as $d\rightarrow 0$, as one sees from Eq. (13), Q and consequently k are purely real, as they must be for the lossless dielectric assumed here. But, as just remarked, initially the attenuation rises, as d decreases.

We illustrate this in the curve labeled A (see Fig. 3). The calculations assume the dielectric waveguide has a thickness of 100 μm, with a (real) dielectric constant of 12, a value typical for the common semiconductors. The metal films have the conductivity of Fe at room temperature.

From curve A in Fig. 2, we see the conductivity damping depends weakly on film thickness when $d>2$ μm, as expected from the value of the skin depth mentioned above. As the film thicknesses drop below 1 μm or so, quite surprisingly once again, we see a dramatic increase. If one examines the electric field within the metal film, the dominant component of the electric field is the longitudinal component (z component). The combined effect of the boundary conditions at the air/metal interface, and metal/dielectric interfaces is to cause the strength of the z component of electric field to increase as $1/d$ when $\kappa d\ll 1$. In the end, k_2 increases as $1/d$ as well, as a consequence of this field enhancement.

One may extract the behavior just described from Eq. (13), and derive as well an estimate of the film thickness below which k_2 will eventually fall to zero as $d\rightarrow 0$. When $\kappa d\ll 1$, we make the replacement $\exp(2i\kappa d)\cong 1+2i\kappa d$ to find

$$\cot\left(\frac{1}{2}QD\right)=\frac{\kappa Q}{\epsilon k_0^2}\frac{[k_0^2+(\alpha_0\kappa+ik_0^2)\kappa d]}{[\alpha_0\kappa+i(\alpha_0\kappa+ik_0^2)\kappa d]}. \tag{32}$$

In both the numerator and the denominator of this expression, $(\alpha_0\kappa+ik_0^2)$ may be replaced by $\alpha_0\kappa$, and in the denominator, the term in κd may be dropped altogether. When this is done, Eq. (32) may be written

$$\cot\left(\frac{1}{2}QD\right)=\frac{Q}{\epsilon\alpha_0}\left(1+\frac{2i\alpha_0 d}{k_0^2\delta_0^2}\right). \tag{33}$$

The metal overlayers continue to assert their presence so long as $2\alpha_0 d/k_0^2\delta_0^2$ is large compared to unity. Recall that $k_0=2\pi/\lambda_0$, with λ_0 the free space wavelength of the radiation. The wave vector k is always close in value to $\epsilon^{1/2}k_0$, as one sees from the example in Eq. (30). Hence the metal films control the behavior of the structure so long as

$$d>\frac{\pi}{(\epsilon-1)^{1/2}}\frac{\delta_0^2}{\lambda_0}\equiv d_c. \tag{34}$$

For Fe, as noted, $\delta_0\cong 10^{-4}$ cm, and at 20 GHz, $\delta_0/\lambda_0\sim 10^{-4}$. Hence, the conducting overlayer has a very strong influence on the propagation characteristics until the coverage is down to the atomic monolayer level! We are reminded of a study of ultrathin Ag films on GaAs some years ago, which demonstrated that monolayer quantities of Ag completely screened electric fields generated by atomic motions in the GaAs from the outside world.[10]

In the regime $d_c\ll d\ll\delta_0$, we may ignore the factor of unity on the right-hand side of Eq. (33), and we still have $QD\ll 1$. Upon proceeding as in the derivation of Eq. (31), we have

$$k_2\cong\frac{\pi\epsilon^{1/2}}{\lambda_0}\frac{\delta_0^2}{Dd}\cdot(d_c\ll d\ll\delta_0). \tag{35}$$

As discussed above, the attenuation rate increases inversely with the metal film thickness. This expression provides a good account of curve A in Fig. 2, when $d\ll\delta_0$.

If, as is commonly done to prevent oxidation, the Fe films are covered with a noble metal film, the behavior of k_2 differs qualitatively from the case just described. Curve B in Fig. 2 are calculations for a dielectric film 100 μm in thickness, where now the Fe films of thickness d are covered with very thick Ag films. We now see that as the Fe films are made progressively thinner, the attenuation decreases substantially.

These calculations show that capping the Fe films with thick metallic overlayers plays a most important role in limiting the conductivity damping, as the Fe films are made thinner. For reasons discussed below, Fe films in the 300–500 Å range will be proven to be of primary interest. In the absence of capping, the conductivity damping would be severe, to the point where the metal coated dielectric waveguide would be of limited usefulness.

We now turn our attention to the coupling between the microwave fields, and spin motions in ferromagnetic films located on the top and bottom of the dielectric waveguide. We shall confine our attention to the case where thin ferro-

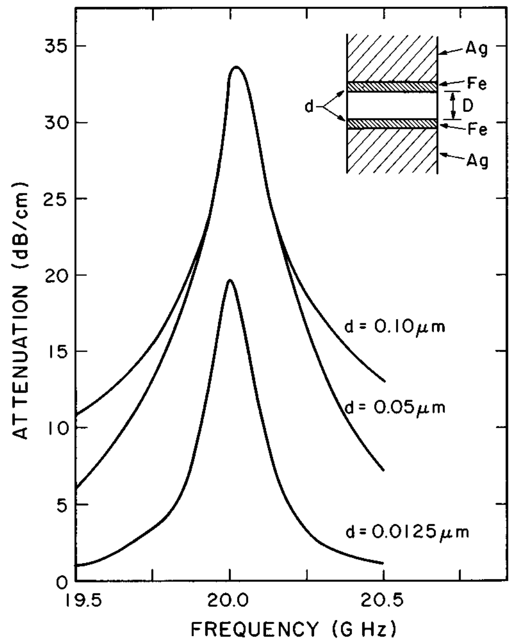

FIG. 3. The frequency dependent attenuation, for a dielectric waveguide ($\epsilon=12$) of thickness $D=100$ μm, upon which two Fe films of thickness d have been deposited. Each Fe film is assumed capped by a thick layer of Ag, as illustrated in the inset. We assume an external field $H_0=1.85$ kG is present, which gives a ferromagnetic resonance frequency very near 20 GHz in the Fe films.

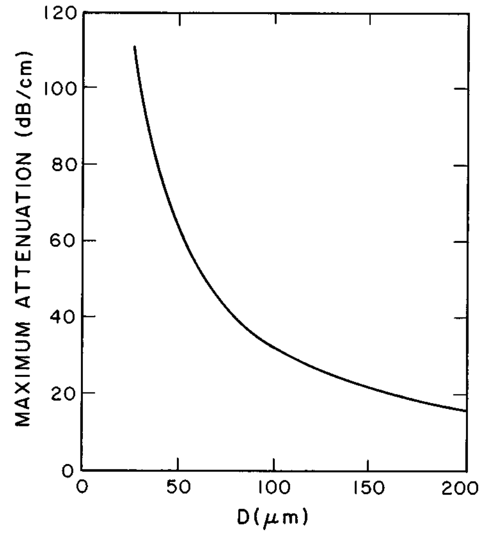

FIG. 4. For the structure studied in Fig. 3, we plot the peak attenuation as a function of the thickness D of the dielectric waveguide. We have taken the thickness of the Fe film to be 500 Å.

magnetic films of thickness d are placed on the waveguide, and these are each capped by very thick metallic overlayers which we take to be silver.

In Fig. 3, we show the frequency dependence of the attenuation introduced by coupling between the microwaves, and the ferromagnetic films. These calculations explore various Fe film thicknesses, for a dielectric waveguide whose thickness is 100 μm. An external field of 1.85 kG renders the ferromagnetic resonance frequency to be 20 GHz.

One might believe that to achieve maximal coupling, the Fe films must have a thickness of a few microns, since the skin depth δ_0 is the order of 1 μm. We have seen, however, that near resonance, the effective skin depth is very much smaller. Right on resonance, for the parameters we have used in these calculations,[9] $\sqrt{\mu_v}\cong 35$, so in fact the skin depth is reduced to only about 300 Å. Thus, maximal coupling is achieved even with very thin Fe films.

We illustrate this in Fig. 2, where we display the attenuation introduced by coupling to the ferromagnetic resonance response of the spin system. We see almost no difference between the peak attenuation produced by a film 0.1 μm (1000 Å) in thickness, and that produced by a film 0.05 μm (500 Å) in thickness. It is not until the Fe film thickness drops well below 300 Å that one begins to see a falloff in the peak attenuation. This is illustrated by the curve labeled 0.0125 μm (125 Å) in Fig. 3. We remark that conclusions very similar to these are evident in the calculations displayed in the paper by Schlömann and co-workers.[7] Indeed, our calculations are in very good accord with theirs in all regards, if one realizes we have two ferromagnetic films deposited on the dielectric waveguide, while they have only a single film. Our calculated peak attenuation rates are thus quite close, as expected, to twice theirs.

One virtue of the strong dependence of the skin depth on frequency is that it reduces the sensitivity of the peak attenuation to the linewidth of the ferromagnetic resonance line of the ferromagnetic film. In our treatment, this is controlled by the parameter Γ which enters Eqs. (19). In general, on resonance, the absorption rate is inversely proportional to Γ. Here, the peak scales as $\Gamma^{-1/2}$, so the peak absorption is somewhat less sensitive to linewidth than one might expect. If one increases the linewidth of the ferromagnet, the amplitude of the spin response on resonance is of course, reduced. However, the skin depth is larger at resonance; this allows the microwave field to sample more spins than before to partially compensate for the loss in amplitude of the spin response.

The peak attenuation realized in the geometry employed in Fig. 3 is affected sensitively by the thickness of the dielectric waveguide. As D, the thickness of the waveguide decreases, the peak attenuation increases dramatically as illustrated in Fig. 4. Note, that if the dielectric waveguide thickness is decreased from 100 to 50 μm, then the calculated peak attenuation rate increases to a value in excess of 75 dB/cm. If strong coupling between the microwave fields and spins in the ferromagnetic films is highly desirable, quite clearly one should fabricate samples from the thinnest possible dielectric waveguide.

The results presented suggest that to obtain strong coupling, one wishes to make the dielectric waveguide very thin, as just discussed. However, in practice, of course, there is a lower limit to the thickness that may be utilized. In the particular example of a GaAs based structure explored here, it is our understanding that a structure based on the use of a 50 μm thick GaAs film would be quite fragile.

These remarks suggest one should utilize a superlattice structure such as that illustrated in Fig. 1(b). The notion is

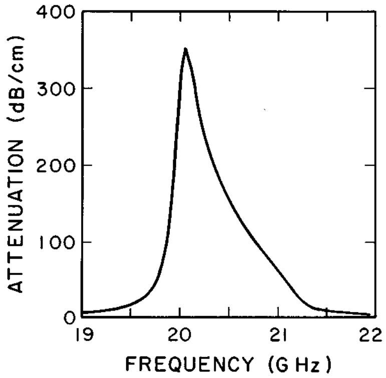

FIG. 5. The attenuation as a function of frequency near the ferromagnetic resonance frequency, for a superlattice structure such as that depicted in Fig. 1(b). The thickness of the dielectric film ($\epsilon=12$) is 1 μm, and that of the Fe film is 100 Å.

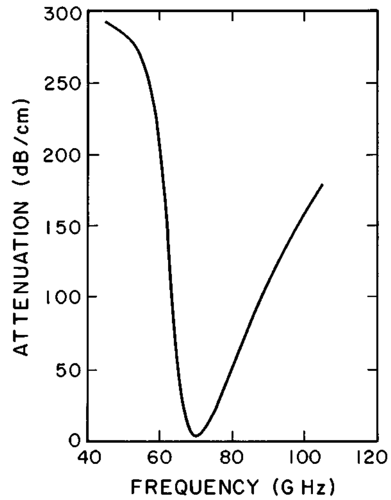

FIG. 6. The frequency variation of the propagation length near the Fe film antiresonance, at the frequency $\Omega_H+4\pi\Omega_M$. The dielectric films ($\epsilon=12$) have a thickness of 1 μm, and the Fe films a thickness of 3 μm.

that one can create a macroscopic sample by stacking together many unit cells, as depicted in Fig. 1(b).

In Fig. 5, we show calculations for such a superlattice, in which the dielectric films have a thickness of 1 μm, and the intervening Fe films have a thickness of only 100 Å. The attenuation at the peak is now quite enormous, in excess of 300 dB/cm. Thus, by fabricating such a structure, one can realize very strong coupling between microwaves and spin excitations. Off resonance, in the presence of the metal films, the attenuation remains modest. In the 15 GHz range, for instance, one realizes 0.3 dB/cm, with a similar value at 25 GHz.

In Sec. II, it was noted that the Voigt susceptibility μ_v has a zero in the near vicinity of the frequency $\Omega_B=\Omega_H+4\pi\Omega_M$, which is near 70 GHz, for Fe exposed to an external magnetic field in the 2 kG range. In this frequency regime, the skin depth in the ferromagnetic film opens up, and becomes very large, as one sees from Eq. (23). In ferromagnetic resonance studies of thin films, there is a transition resonance, discovered some years ago by Heinrich and Mescharyakov.[11] This feature is referred to as an antiresonance of the film.

If one prepares a superlattice such as that displayed in Fig. 1(b) that is metal rich, then at frequencies removed from the antiresonance, the conductivity damping is very strong. However, near Ω_B, in the antiresonance region, the structure opens up and transmits. We illustrate this in Fig. 6, where we show the frequency dependence of the transmissivity of a structure fabricated from dielectric films 1 μm thick, with Fe films 3 μm thick interspersed between them. We see the dramatic attenuation minimum near 70 GHz. At the minimum, the attenuation falls to 4 dB/cm; the depth of the minimum is controlled by the damping parameter Γ. This structure may be appropriate for use as a tunable band pass filter. Since $\Omega_B=\Omega_H+4\pi\Omega_M$ the frequency of the dip may be tuned by varying the externally applied magnetic field.

In the limit that both constituents in the superlattice are very thin, a simple analytic expression for the propagation constant k follows from Eq. (29). If $QD\ll1$, and also $\tilde{\kappa}d\ll1$, each cotangent may be replaced by its small argument limit. This yields

$$Q^2=-\frac{d}{D}\,\epsilon k_0^2\mu_v\,, \tag{36}$$

from which one finds the simple result

$$k^2=\epsilon k_0\left(1+\frac{d}{D}\,\mu_v\right). \tag{37}$$

An expression equivalent to this emerges from the effective medium theory of magnetic superlattices.[12]

IV. SUMMARY AND CONCLUSIONS

The calculations presented in Sec. III have elucidated a number of features of the microwave propagation characteristics of structures such as those depicted in Fig. 1. Our principal conclusions may be summarized as follows:

(a) Capping of the Fe films by a nonmagnetic conductor does more than simply prevent oxidation of the Fe film. It controls the dependence of the off-resonance conductivity damping on Fe film thickness, when the Fe films become considerably thinner than the off-resonant skin depth. Without the capping, as the Fe films become very thin, one realizes a very strong conductivity damping off resonance, as illustrated by curve A in Fig. 2. If the films are capped by a thick conducting layer, then the off-resonance conductivity damping remains quite small, for all Fe film thicknesses considered, for the structures explored here.

(b) One would think that to achieve maximal coupling of microwaves to spins, one needs Fe films a few microns thick. That is, they should be thicker than the nominal skin depth, so the microwave field comes into contact with as many spins as possible. This is not the case. The fact that the apparent skin depth decreases dramatically on resonance allows one to achieve maximal coupling with rather thin (500 Å) Fe films, as illustrated in Fig. 3.

(c) One can increase absorption on resonance by using the thinnest possible dielectric film. The range of 50 μm seems interesting, if such a thin film structure can be fabricated.

(d) The use of multilayer or superlattice structures seems of great interest, if one seeks strong coupling between microwaves and the spins in the structure, for the following reasons:

(i) One can achieve very large attenuation on resonance, as illustrated in Fig. 5, by making such a structure with rather thin (order of 1 μm) dielectric films.

(ii) There is a dramatic attenuation dip at high frequencies, due to the ''opening up'' of the skin depth at antiresonance. This is illustrated in Fig. 6. The development of metal-rich structures will present very large attenuation away from the antiresonance region, with attenuation dips here as illustrated.

(iii) Both effects (i) and (ii) just described can be achieved in samples made with rather low quality Fe films. For (i), the attenuation maxima are very high, so relatively low quality films with only modest linewidths can provide strong coupling to spins, if such films are incorporated into a superlattice. For (ii), the overall shape of the attenuation dip is controlled by the real part of the Voigt susceptibility, which is not so sensitive to linewidth, save quite near the zero in the real part which drives the phenomenon. These considerations suggest sputtered samples should prove quite adequate, for the superlattice structures.

It is our hope that the calculations presented here provide an orientation of the influence of sample geometry and microstructure, for combinations of semiconductor and ferromagnetic metal films which may prove useful for high frequency microwave devices.

ACKNOWLEDGMENTS

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[1] For a discussion of the growth of high quality ultrathin films and the means of characterizing them see: *Ultrathin Magnetic Structures*, edited by J. A. C. Bland and B. Heinrich (Springer, Heidelberg, 1994), Vol. 1, Chap. 5, p. 177.

[2] M. N. Babich, J. M. Broto, A. Fert, F. Nguyen Van Dau, F. Petroff, P. Eitenne, G. Creuzet, A. Frederick, and J. Chazelas, Phys. Rev. Lett. **61**, 2472 (1988).

[3] For a discussion of the application of spin dependent tunneling to magnetic memory devices see Z. Wang and Y. Nakamura, J. Magn. Magn. Mater. **159**, 233 (1996).

[4] G. A. Prinz, *Ultrathin Magnetic Structures*, edited by J. A. C. Bland and B. Heinrich (Springer, Heidelberg, 1994), Vol. II, p. 1.

[5] C. S. Tsai, IEEE Trans. Magn. **32**, 4118 (1996).

[6] C. Kittel, Phys. Rev. **71**, 270 (1947); *ibid.* **73**, 155 (1948).

[7] E. Schlömann, R. Tutison, J. Weissman, H. J. Van Hook, and T. Vatimos, J. Appl. Phys. **63**, 3140 (1988).

[8] B. Heinrich and J. F. Cochran, Adv. Phys. **42**, 523 (1993).

[9] In the conventional form of the Landau–Lifshitz equations, one encounters the Gilbert damping constant G (Ref. 8). We have $\Gamma = G/\gamma M_s \equiv G/\Omega_M$. For Fe at room temperature, $G = 0.8\times 10^8\ s^{-1}$.

[10] L. H. Dubois, G. P. Swartz, R. E. Camley, and D. L. Mills, Phys. Rev. B **29**, 3208 (1984).

[11] B. Heinrich and V. F. Mescharyakov, Sov. Phys. JETP **32**, 232 (1971).

[12] N. S. Almeida and D. L. Mills, Phys. Rev. B **38**, 6698 (1988); *ibid.* **39**, 12 339 (1989).