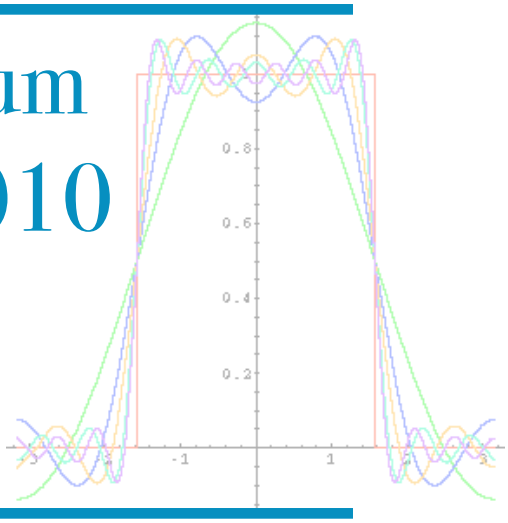
UNIVERSITY OF COLORADO AT COLORADO SPRINGS

UCCS Math Colloquium
Thursday, Sept 23, 2010
12:30-1:30pm
UC 307 (refreshments at 12:15)



A PROOF OF CARLESON'S THEOREM BASED ON A NEW CHARACTERIZATION OF THE LORENTZ SPACES $L(p,1)$ FOR $1 < p < \infty$ AND OTHER APPLICATIONS

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ABSTRACT. In his 1950 Annals of Mathematics paper entitled "Some New Functional Spaces", G. G. Lorentz introduced the function spaces denoted by $\Lambda(\alpha), 0 < \alpha < 1$, defined as the set of real measurable functions for $0 < x < 1$ for which

$$\|f\|_{\Lambda(\alpha)} = \alpha \int_0^1 x^{\alpha-1} f^\star(x) dx < \infty \,,$$

where $f^\star$ is the decreasing rearrangement of f. In this paper we give two simple characterizations for $\Lambda(1/p)$ for $1 < p < \infty$ based on a generalization of the special atom space introduced by G. De Souza in earlier works De Souza (1980), Bloom and De Souza (1994), Bloom and De Souza(1989), and De Souza (1989). The space $\Lambda(1/p)$ is nowadays denoted by $L(p,1)$. As an application, we give a proof of Carleson's Theorem on the convergence of Fourier series on $L(p,1)$ and, more generally, on $L(p,r)$ for $p, r > 1$. Also we have a simple proof of a theorem of Stein and Weiss on operators in $L(p,1)$.

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