UCCS Mathematics Colloquium

Thursday March 10th, 2011

UC 307

12:30 pm – 1:30 pm

(Refreshments at 12:15 pm)

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A Numerical Methodology for the Painlevé equations

Abstract: The six Painlevé transcendents P_I to P_VI have both applications and analytic properties that make them stand out from most other classes of special functions. Although they have been the subject of extensive theoretical investigations for about a century, they still have a reputation of being numerically challenging. In particular, their extensive pole fields in the complex plane have often been perceived as 'numerical mine fields'. We note in this present work that, on the contrary, the Painlevé property in fact provides excellent opportunities for very fast and accurate numerical solutions across the full complex plane. The numerical method will be illustrated for the P_I equation.

The present work was carried out in collaboration with Prof. André Weideman (University of Stellenbosch). Jonah Reeger (CU Boulder) has assisted vith creating movie clips.

$$\frac{d^2y}{dt^2} = 6y^2 + t \qquad \frac{d^2y}{dt^2} = 2y^3 + ty + \alpha$$

$$ty\frac{d^2y}{dt^2} = t\left(\frac{dy}{dt}\right)^2 - y\frac{dy}{dt} + \delta t + \beta y + \alpha y^3 + \gamma t y^4$$

$$y\frac{d^2y}{dt^2} = \tfrac{1}{2}\left(\frac{dy}{dt}\right)^2 + \beta + 2(t^2 - \alpha)y^2 + 4ty^3 + \tfrac{3}{2}y^4$$

$$\frac{d^2y}{dt^2} = \left(\frac{1}{2y} + \frac{1}{y-1}\right)\left(\frac{dy}{dt}\right)^2 - \frac{1}{t}\frac{dy}{dt} + \frac{(y-1)^2}{t}\left(\alpha y + \frac{\beta}{y}\right) + \gamma\frac{y}{t} + \delta\frac{y(y+1)}{y-1}$$

$$\frac{d^2y}{dt^2} = \tfrac{1}{2}\left(\frac{1}{y} + \frac{1}{y-1} + \frac{1}{y-t}\right)\left(\frac{dy}{dt}\right)^2 - \left(\frac{1}{t} + \frac{1}{t-1} + \frac{1}{y-t}\right)\frac{dy}{dt} + \frac{y(y-1)(y-t)}{t^2(t-1)^2}\left(\alpha + \beta\frac{t}{y^2} + \gamma\frac{t-1}{(y-1)^2} + \delta\frac{t(t-1)}{(y-t)^2}\right)$$