

UCCS Mathematics

Colloquium

Thursday, November 3rd, 2011

UC 307

12:30 pm – 1:30 pm

(Refreshments at 12:15 pm)

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Solitons, boundary value problems and a nonlinear method of images

Abstract: It is well known that, for integrable nonlinear evolution equations (NLEEs), each soliton is associated to a discrete eigenvalue of the scattering problem via the inverse scattering transform. Boundary value problems (BVPs) present additional complications compared to initial value problems (IVPs), however. In this talk I will characterize BVPs for both discrete and continuous nonlinear Schroedinger (NLS) systems on semi-infinite domains with linearizable boundary conditions (BCs), with either zero or non-zero BCs at infinity. In all of these situations, the relation between solitons and discrete eigenvalues existing in the IVP is preserved in the BVP. Soliton solutions, however, generically do not satisfy the linearizable BCs. Moreover, numerical solutions of the BVP clearly show that solitons are reflected at the boundary -- even though the soliton velocity is determined by the discrete eigenvalue, which is constant in time.

I will show that the resolution of these apparent paradoxes is that discrete eigenvalues in the BVP appear in symmetric pairs/quartets/octets, and corresponding relations exist for the associated norming constants. Thus, for each soliton in the physical domain a symmetric counterpart exists, with equal amplitude and opposite velocity, whose presence ensures that the whole solution satisfies the BCs. The ostensible reflection of the soliton at the boundary of the physical domain then corresponds simply to the interchanging of roles between the "physical" and "mirror" solitons. These results represents a nonlinear analogue of the method of images in electrostatics. Here, however, the soliton reflection comes accompanied by a corresponding position shift, which is a reminder of the nonlinear nature of the problem. I will show how these phenomena arise for the focusing and defocusing NLS and for the the Ablowitz-Ladik lattice.

