SIAM

Society for Industrial and Applied Mathematics

Front Range Applied Mathematics Student Conference

University of Colorado at Denver

PROGRAM AND ABSTRACTS

Saturday
March 5th, 2005

Sponsors:
THE SIAM STUDENT CHAPTERS AT
University of Colorado, Boulder
University of Colorado, Colorado Springs
University of Colorado, Denver

Program for the Front Range Applied Mathematics Student Conference

All sessions are being held in the CU-Building, on the corner of 14th St and Lawrence St in Denver.

8:30 – 9:30am: Breakfast and Registration, Room 470

Morning Session – Room 470
9:30 – 11:00am

9:30 – 9:55	Shane Kirkbride *University of Colorado, Colorado Springs*	Riemann's Hypothesis, Quantum Chaos, and the Universe in General
10:00 – 10:25	Dan Cooley *University of Colorado, Boulder*	Precipitation Return Levels for Colorado's Front Range
10:30 – 10:42	Zhongben Wang *Colorado School of Mines*	Optimal Superconvergent Alternating Direction Implicit Nodal Cubic Spline Collocation Methods for Helmholtz Problems
10:45 – 10:57	Travis King *Colorado State University*	Detecting and Countering Instabilities Arising from Operator Splitting in Reaction-Diffusion Equations

MCM/ICM Session – Room 480
9:30 – 10:45am

9:30 – 9:55	Matthew J. Kaspari, Jeremy J. Noe, and Barry J. O'Reilly *University of Colorado, Denver*	Relieving Toll Booth Congestion
10:00 – 10:25	Kurt Cordle and Jon Stranske *University of Colorado, Denver*	Modeling Toll Plazas: Traffic Flow and Optimum Configurations
10:30 - 10:42	Sanghui Lee *University of Colorado, Colorado Springs*	Oil, Black Gold, Texas Tea What Do You Mean We're Out?

Break: 11:00 – 11:15

Plenary Address, Stan Osher: 11:15 – 12:15, Room 470

Mathematics in the Real World and the Fake World

Lunch and Poster Session: 12:15 – 1:15, Room 480

	Alberto Villarreal *Colorado School of Mines*	Accurate Computation of Optical Flow

Afternoon Session I – Room 470
1:15 – 3:30pm

1:15 – 1:40	Sean Eastman *Colorado State University*	Linearization Error for Computational Error Estimates, and the Perturbed Power Method
1:45 – 1:57	Josh Nolting *University of Colorado, Boulder*	Parallel FOSPACK
2:00 – 2:12	Christopher Harder *University of Colorado, Denver*	Iterative Methods for Solving Non-Symmetric Systems
2:15 – 2:40	Eunjung Lee *University of Colorado, Boulder*	$FOSLL^*$ Method for Eddy Current Problem with 3D Edge Singularities
2:45 – 2:57	Brad Klingenberg *University of Colorado, Boulder*	A Common Framework and Initialization Strategy for Non-negative Matrix Factorization
3:00 – 3:25	Que Nguyen *Colorado School of Mines*	Matrix Decomposition Algorithms for Modified Quadratic Spline Collocation for Helmholtz Problems

Afternoon Session II – Room 480
1:15 – 3:30pm

1:15 – 1:40	Xilin Shen *University of Colorado, Boulder*	Analysis of Event-Related fMRI Data using Diffusion Maps
1:45 – 1:57	Jutta Bikowski *Colorado State University*	Electrical Impedance Tomography
2:00 – 2:12	Adam Ringler *Colorado School of Mines*	The Total Homotopy Operator on the Jet Space: A Theoretical Approach Made Concrete
2:15 – 2:40	Steve Ogden *University of Colorado, Denver*	Automating Escher Tiling and Coloring
2:45 – 2:57	Tessa Weinstein *University of Colorado, Denver*	Upscaling Via the Hybrid Mixture Theoretic Approach: Governing Equations for a Swelling Porous Medium
3:00 – 3:12	Pascal Getreuer *University of Colorado, Boulder*	Nonlinear Interpolation
3:15 – 3:27	Sada Narayanappa *Denver University*	An Improved Approximation Factor for Disk Covering Problem

Plenary Speaker (11:15-12:15pm)

MATHEMATICS IN THE REAL WORLD AND THE FAKE WORLD

Dr. Stanley Osher (sjo@math.ucla.edu)
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UCLA

The level set method for capturing moving fronts was introduced in 1987 by Osher and Sethian. It has proven to be phenomenally successful as a numerical device. For example, typing in ”Level Set Methods” on Google’s search engine gives roughly 25,000 responses. Applications range from capturing multiphase fluid dynamical flows, to special effects in Hollywood to visualization, image processing, control, epitaxial growth, computer vision and many more. In this talk we shall give a quick overview of the numerical technology, its relation with the field of PDE based imaging science and some applications, including visibility (joint work with Tsai, Cheng, Burchard and Sapiro), image decomposition joint work with Vese and Sole) and inverse scale space (with Burger, Goldfarb, Xu and Yin. The real world is exemplified by image analysis and the fake world is exemplified by computer graphics.

MORNING SESSION I

RIEMANN'S HYPOTHESIS, QUANTUM CHAOS, AND THE UNIVERSE IN GENERAL

Shane Kirkbride (skirkbri@uccs.edu)
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Recent advances in complex number theory and quantum mechanics have revealed a profound relationship between the nature of the quantum atomic structure and the Riemann Zeta function, specifically the non-trivial zeros of this function. This presentation will inform the audience about this relationship. First a general account of what Riemann's Hypothesis is given, why it is the most relevant unproven hypothesis of our time and what is being done to change the unproven part of this statement. Second, Chaos theory and its relevance to Riemann's Hypothesis will be explored. Third, a brief outline of quantum theory as it relates to thee afore mentioned topics will be presented. Lastly we will present a brief outline of the correlation between Riemann's Hypothesis, Chaos Theory and Quantum mechanics and why this correlation is both sublime and relevant.

PRECIPITATION RETURN LEVELS FOR COLORADO'S FRONT RANGE

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To aid with the planning for flooding along Colorado's Front Range, we are developing a map of precipitation return levels for the region. To develop the map, we are relying on the theory of extreme values. Specifically, we use the generalized pareto distribution (GPD) to model precipitation above a threshold at 56 weather stations throughout the region. By constructing a Bayesian hierarchical model which relates each station's GPD parameters to a spatial model we are able to pool the data from all the stations and obtain parameter and return-level estimates which have more spatial consistency. The parameter estimates also take into account the available covariates we have for the model. In this talk, I will give a brief background in extreme value theory, and discuss the methods we are using to develop the map.

OPTIMAL SUPERCONVERGENT ALTERNATING DIRECTION IMPLICIT NODAL CUBIC SPLINE COLLOCATION METHODS FOR HELMHOLTZ PROBLEMS

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Colorado School of Mines
Advisors: Dr. Graeme Fairweather and Dr. Bernard Bialecki

In the classical nodal spline collocation method, one seeks a function which satisfies i) the differential equation at the partition nodes, and ii) the boundary conditions. It is well known that this method is suboptimal and provides approximations which are no better than second order globally. Rice et al., developed two modified nodal cubic spline methods, a one step method (OSM) involving a perturbation of the Laplacian, and a two step method (TSM) based on a deferred correction approach. While OSM is of optimal global accuracy for the Dirichlet problem, it is no better than third order accurate for the Neumann, mixed and periodic problems. The TSM is optimal accurate and superconvergent but computationally expensive.

For the solution of the collocation equations, iterative methods have been proposed by several authors. In particular, Tsompanopoulou and Vavalis generalized the OSM to the Dirichlet problem for the Helmholtz equation in several space variables and formulated and analyzed an alternating direction implicit (ADI) method for the solution of the collocation equations. However the results of the numerical experiments presented in the paper exhibit only third order global accuracy; superconvergence is not mentioned.

Moreover, none of the other boundary conditions are considered.

The purpose of our research is to develop modified OSM nodal cubic spline collocation methods for all four boundary value problems for the Helmholtz equation which are of optimal global accuracy and exhibit expected superconvergence phenomena at the partition nodes. These methods involve perturbations of both the differential operator and the right hand side. In each case, the collocation method is designed so that the resulting collocation equations can be solved iteratively using an ADI method. A computational study confirms the theoretical analysis and the results of numerical experiments show expected global optimal orders of convergence as well as desired superconvergence.

DETECTING AND COUNTERING INSTABILITIES ARISING FROM OPERATOR SPLITTING IN REACTION-DIFFUSION EQUATIONS

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Colorado State University
Donald Estep (advisor), Colorado State University
John Shadid, Sandia National Laboratories
David Ropp, Sandia National Laboratories

Several operator splitting integrators for reaction-diffusion equations exhibit a dangerous form of instability when the reaction is destabilizing. This instability, characterized by nonphysical chaotic behavior and even blowup, is impossible to detect using standard error estimators. We describe a technique for detecting this kind of instability using *a posteriori* error estimates based on generalized Green's functions and new step selection mechanisms for reducing this instability that are based on stability, rather than accuracy, considerations. Our goal is to develop a method that can analyze the solution at periodic intervals to test the current solution for stability and change the time step as necessary.

We will present results of our current tests for this instability on a series of test problems. In these trials, we test the solution after every time step for stability. There are two main problems we will be presenting results for. These are the chemical Brusselator problem, and a PDE version of the Lorenz problem. With the parameters we use, both the Brusselator and the Lorenz problem should exhibit a stable, semi-periodic solution that remains smooth through time. However, if the time step is too large, the solution starts to destabilize, causing high frequency, high wave number oscillations to develop at the boundaries that eventually pollute the entire solution.

MORNING SESSION II
MCM/ICM TEAMS

RELIEVING TOLL BOOTH CONGESTION

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Barry O'Reilly (gundaetiapo@yahoo.com)
University of Colorado at Denver

When traveling on a toll road, a traveler is interrupted at certain intervals by toll plazas. In the case of a barrier-toll road, every vehicle must stop to pay the toll, and as a result traffic congestion occurs. Our conclusion in regards to this is that congestion is unavoidable. Since congestion is inevitable, we want to confine or limit the congestion in some way. There are three possible scenarios to this point of view:

- allow congestion upon entry to the toll plaza, and ensure that little congestion occurs when exiting the toll plaza,
- attempt to ensure that there is very little or no congestion upon entry to the toll plaza and allow congestion in the exit, or
- allow congestion at both the entry and exit of the toll plaza.

The primary goal is to minimize the annoyance of the motorists by minimizing traffic congestion. We must determine how many toll booths to deploy in the toll plaza to minimize motorist annoyance by limiting congestion and maximizing traffic flow. As a secondary objective, we deal with the issue of maximizing a schedule of opening toll booths.

To this end, we present here four models in an attempt to answer the question of how many is an optimal number of tollbooths to deploy in the toll plaza. The first model is simply a rough sketch of the problem presented in order to get a framework on which to build slightly more sophisticated models. We call this the Primitive model, so called because of the liberal assumptions made in its formulation. The other three models build upon the Primitive model, in that some of the assumptions made in the formulation of the Primitive model are discarded and more appropriate methods are utilized. The 2-Queue model presents a more realistic approach to the problem by making use of a Poisson distribution to estimate traffic flow, as well as giving consideration to parts of the problem ignored in the Primitive model. With the Toll Booth Utilization model, we strive to present the random nature of traffic flow and take in to account the notion of actual wait times, and thereby determine an optimal number of toll booths. Finally, the Dual-Flux model presents a possible alternative to a barriertoll system by offering motorists a way to avoid the toll plaza altogether.

MODELING TOLL PLAZAS: TRAFFIC FLOW AND OPTIMUM CONFIGURATIONS

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Co-author: Mathew Burman
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For the Mathematical Contest in Modeling we were tasked with exploring how traffic interacts in a large toll collection plaza. Specifically, we assessed the current configuration of the Ft. McHenry Toll Plaza on I-95. Our first goal was to determine the minimum number of tollbooths required by such a plaza. Secondly, we evaluated the worthiness of operating only as many tollbooths as incoming lanes of traffic. Our model only describes manually operated tollbooths; newer electronic collection systems are not considered.

We developed one primary stochastic model, using a series of M/M/1 queues and adjusting for vehicular movement, to estimate the optimal number of tollbooths for Ft. McHenry Plaza and extended the model to consider a worst-case scenario for traffic processing. We incorporated a Poisson distribution to model the possibility that we have both an unexpectedly large number of vehicles arriving and that our servers in the tollbooths happen to be having a particularly slow day. Not surprisingly, our model suggested increasing the number of servers to accommodate the demand. Clearly, this cannot be expanded indefinitely without consideration for the downstream flow of traffic so we incorporated a maximal-flow model to determine a cap on the number of tollbooths that may be deployed.

Assuming an entirely manually operated booth scenario, Ft. McHenry Plaza is under-equipped. Presently the Plaza has a maximum of 12 operational booths per direction of traffic. By extending this to 19, driver times could be significantly reduced. At peak times the total drive time would be only 8 minutes from the time the driver begins to decelerate to the time the driver merges from the tollgates and accelerates to full cruise speed.

OIL, BLACK GOLD, TEXAS TEA WHAT DO YOU MEAN WE'RE OUT?

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Co-authors: Justin Gray & Catherine Mann
Electrical and Computer Engineering, UCCS

In the year 2082, the world will no longer have any fuel for cars, heat during the winter, electricity, plastic, and certain medicines. This is because at the present rate of consumption, the world will not have any more crude oil, or it will be too rare for common use at an affordable price. We devise a model to illustrate this by gathering data of past production and consumption of crude oil. In order to prevent the total depletion of oil before new technologies are developed to break our addiction to this limited commodity, we develop another model to illustrate how availability of crude oil can be prolonged. We do this by decreasing production by a small percentage every year, which will in turn increase the price. We then collect a percentage of the increased profit to fund the development of new technologies.

In order to create these two models, we use a number that is a rough estimate of how much oil the world had to begin with. We obtain this number from research done by the United States Geographical Survey (USGS). We also assume that new oil will not be discovered or created naturally, and that loss of oil through natural seepage is negligible. Another assumption is that supply and demand are linearly related, so that lowering production ultimately raises cost. Although there may be factors that make this untrue, these factors, if significant, are unknown. The most unrealistic but necessary assumption made is that there will be no resistance to the policies set due to ”human” differences (emotional, cultural, religious etc.), and these policies are assumed to be implemented by an international organization consisting of all oil producers worldwide.

A best-fit curve of known data was used to project the future availability of the resource. Deviations from the gathered data from this line are used to calculate error. Because the data did not cover the entire history of oil production, we were forced to generate data based on previous trends. We cannot take into account all political considerations involved. Also, the policies created may be difficult to enforce.

Poster Presentation (12:15-1:15pm)

ACCURATE COMPUTATION OF OPTICAL FLOW

Alberto Villarreal (avillarr@mines.edu)
Colorado School of Mines

Automated quantitative determination of motion in image sequences is a challenging application in image processing. In particular, this applies to human and vehicle surveillance, and MRI or PET/SPECT data of a human beating heart, among other problems.

One way to estimate the motion in a sequence is to compute the optical flow field (or image velocity) that approximates the image motion assuming that changes in the intensities are due only to the motion of objects in the scene.

This problem of recovering the optical flow can be cast as an ill-posed inverse problem (computing the velocity field between two images given the brightness in the images), which can be solved only by introducing additional assumptions through regularization, to chose one solution among all the possible ones.

The forward problem is to compute the brightness in the second image given the velocity field and an initial condition, which is the brightness in the first image.

The inverse problem can be solved using Tikhonov regularization, which involves solving a discrete optimization problem where the necessary conditions for the minimum include a hyperbolic PDE representing the forward problem (the optical flow equation).

However, numerical discretization methods for hyperbolic PDEs with discontinuities (e.g., upwind methods) are not differentiable with respect to the velocity. In the optical flow problem, discontinuities in image brightness (occlusion, for example) can generate a situation like this one.

In that case, standard numerical optimization tools cannot be used to solve the inverse problem. Thus, the goal of the proposed research is to solve the inverse problem of optical flow computation accurately by modifying numerical discretization methods for hyperbolic PDEs (both low order and high order) so as to maintain their overall approximation properties but allow for differentiable objective functions.

AFTERNOON SESSION I

LINEARIZATION ERROR FOR COMPUTATIONAL ERROR ESTIMATES, AND THE PERTURBED POWER METHOD

Sean Eastman (eastman@math.colostate.edu)
Colorado State University

A-posteriori error estimates for nonlinear equations based on residuals and variational analysis are subject to an error of linearization. It is not well understood what effect the linearization has on the estimate, and in this talk I will present some ideas for ways in which to bound the effect of linearization. The primary computational tool is very closely related to the Power Method for finding the dominant eigenvalue and eigenvector of a square matrix.

PARALLEL FOSPACK

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Co-authors: John Ruge, Tom Manteuffel, Hans DeSterk, Jeff Heys

FOSPACK is a general partial differential equation solver. It uses finite element methodology in a FOSLS (first order system least squares) setting. The system of equations is then solved with some algebraic multigrid algorithm. In order to solve very large finite element meshes it is important to use parallel processes. Over the last year I have been working to add certain parallel features to this already written extensive serial program. The goal is to use as much existing code as possible; each processor should feel as though it is solving the serial problem on a smaller grid. The project covers research in partitioning algorithms and parallel AMG algorithms as well as implementation of the parallel code.

ITERATIVE METHODS FOR SOLVING NON-SYMMETRIC SYSTEMS

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University of Colorado at Denver

Recently, the left conjugate direction method (LCD) has been proposed as an iterative method for solving non-symmetric systems of linear equations. A numerical assessment of its performance on systems arising from the stream-line diffusion finite element discretization (SDFEM) of advection diffusion equations will be presented. The results on these systems will be compared to the results obtained from solving those same systems using GMRES, a well-known iterative scheme for solving non-symmetric systems of linear equations.

FOSLL* METHOD FOR EDDY CURRENT PROBLEM WITH 3-D EDGE SINGULATIRIES

Eunjung Lee and Thomas A. Manteuffel
University of Colorado at Boulder

Maxwell's equations are a set of fundamental equations governing all macroscopic electromagnetic phenomena. It is known that the numerical resolution of the full system of Maxwell's equations can be extremely expensive. However it is possible to use a simplified model which approximates Maxwell's equations and explains particular problems encountered in electromagnetism. In many cases, one can use the so-called eddy current model. The eddy current model is obtained by neglecting the displacement current in the Maxwell's equations. Here we consider the following two basic laws of electricity and magnetism which form the eddy current model

$$\text{Faraday's Law} \qquad \frac{\partial \mu \boldsymbol{H}}{\partial t} + \nabla \times \boldsymbol{E} = \boldsymbol{0}$$
$$\text{Ampère's Law} \qquad \nabla \times \boldsymbol{H} - \sigma \boldsymbol{E} = \boldsymbol{0}$$

and two types of boundary conditions

$$\boldsymbol{n} \times \boldsymbol{E},\ \boldsymbol{n} \cdot \boldsymbol{H} \quad \text{or} \quad \boldsymbol{n} \cdot \boldsymbol{E},\ \boldsymbol{n} \times \boldsymbol{H},$$

where $\boldsymbol{E}$ is the electric field intensity, $\boldsymbol{H}$ is the magnetic field intensity, μ is the permeability, σ is the conductivity, and $\boldsymbol{n}$ is the unit external normal vector. The electric and magnetic field intensities, $\boldsymbol{E}$ and $\boldsymbol{H}$, which follow the Faraday's and Ampère's laws with homogeneous boundary conditions satisfy

$$\boldsymbol{E} \in H(\nabla\times) \cap H(\nabla\cdot\sigma) \text{ and } \boldsymbol{H} \in H(\nabla\times) \cap H(\nabla\cdot\mu).$$

In addition, if $i)$ μ and σ are smooth, $ii)$either the domain is a convex polyhedron or the boundary is $\mathcal{C}^{1,1}$, and $iii)$different types of boundary conditions do not meet at the edge with the internal angle $> \pi/2$, then $\boldsymbol{E}$ and $\boldsymbol{H}$ satisfy $\boldsymbol{E} \in (H^1)^3$ and $\boldsymbol{H} \in (H^1)^3$. Standard numerical techniques can be used to approximately solve the equations under the above assumptions. For example, the first-order system least squares($FOSLS$) method with H^1-finite element spaces can solve the equations using multigrid method. The $FOSLS$ method is based on minimization of the residual $||L\boldsymbol{V} - \boldsymbol{F}||_0$ of the system $L\boldsymbol{U} = \boldsymbol{F}$, where L represents a system of linear first order equations, $\boldsymbol{U}$ a vector of unknowns, and $\boldsymbol{F}$ a vector of known functions. The standard least squares method approximates the unknown $\boldsymbol{U}$ in the given H^1-finite element space when the bilinear form of $||L\boldsymbol{V} - \boldsymbol{F}||_0$ is equivalent to the product H^1 norm. But this H^1-equivalence is provided only under sufficient smoothness assumptions on the original problem like the domain, coefficients, and data. In the presence of discontinuous coefficients, non-convex domain, or certain irregular boundary conditions, the solution may not be in H^1. This precludes the use of standard H^1-conforming finite element spaces in a $FOSLS$ formulation.

The first-order system LL^*($FOSLL^*$) method was introduced to overcome the difficulty coming from the discontinuous coefficients. The $FOSLL^*$ method solves the dual problem $L^*\boldsymbol{W} = \boldsymbol{U}$ with the dual variable $\boldsymbol{W}$ and the L^2-adjoint operator L^* of L in the rewritten system $LL^*\boldsymbol{W} = \boldsymbol{F}$. Now the original problem is recast as the minimization of the functional $||L^*\boldsymbol{W} - \boldsymbol{U}||_0$ which has the same minimizer of the functional $||L^*\boldsymbol{W}||_0^2 - 2\langle \boldsymbol{W}, \boldsymbol{F}\rangle$. Minimizing $||L^*\boldsymbol{W} - \boldsymbol{U}||_0$ over $\boldsymbol{W}$ in the domain of L^* is accomplished by solving the weak problem of finding $\boldsymbol{W}$ such that

$$\begin{aligned} \langle\, L^*\boldsymbol{W}, L^*V \,\rangle \;&=\; \langle\, \boldsymbol{U}, L^*V \,\rangle \;=\; \langle\, L\boldsymbol{U}, V \,\rangle \\ &=\; \langle\, \boldsymbol{F}, V \,\rangle\,, \end{aligned}$$

for every V in the domain of L^*. Then the solution we seek is $\boldsymbol{U} = L^*\boldsymbol{W}$. A modified $FOSLL^*$ method was developed that allows an accurate approximation using H^1-conforming finite elements for the equations having singular boundary points in two dimension.

It is well-known that the finite element scheme works poorly in the circumstances that the problem does not obey one or more than one of the assumptions i), ii), and iii). Many efforts have been made to overcome this situation, for instance, people modify Maxwell's equations into second order elliptic equations and use mesh refinements, weighted regularization, or edge elements.

Here, we focus on the problem in the three dimensional domain that has discontinuous coefficients and singular boundaries, such as reentrant edges. We use the standard $FOSLL^*$ method to abate the difficulty from the discontinuous coefficients and a modification will be made to deal with the reentrant edges. Here we do not consider the case that different types of boundary conditions meet at the edge with the internal angle $> \pi/2$. However, we believe that our modification can be applied for the above case either.

A COMMON FRAMEWORK AND INITIALIZATION STRATEGY FOR NON-NEGATIVE MATRIX FACTORIZATION

Brad Klingenberg
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University of Colorado at Boulder

With numerous applications to feature extraction, component analysis, and numerical linear algebra, non-negative matrix factorization (NMF) can provide insight into a variety of problems. Unfortunately, while several iterative algorithms for performing the factorization exist, convergence is slow. The investigation provides a common framework for understanding the underlying dynamics of the problem, and the considerations which affect the rate of convergence. Specifically, the constrained orthogonality of basis vectors is explored, leading to an initialization strategy. An effort to cast the algorithms as constrained least squares problems is also undertaken.

MATRIX DECOMPOSITION ALGORITHMS FOR MODIFIED QUADRATIC SPLINE COLLOCATION FOR HELMHOLTZ PROBLEMS

Que Nguyen (qnguyen@mines.edu)
Colorado School of Mines
Advisor: Dr. Graeme Fairweather

A new one-step method for modified quadratic spline collocation is developed in order to numerically solve Helmholtz equations satisfying Dirichlet, Neumann, mixed (Neumann-Dirichlet and Dirichlet-Neumann), and periodic boundary conditions so that (a) optimal order accuracy and superconvergence are obtained and (b) the quadratic spline collocation equations can be solved by matrix decomposition algorithms. Appropriate quadratic spline collocation methods for second-order two-point boundary value problems are first formulated and then extended to the two-dimensional problem.

For Dirichlet boundary conditions, a new one-step modified quadratic spline collocation method is formulated and implemented for solving a second-order two-point boundary value problem. We demonstrate numerically that its accuracy is optimal and comparable to existing optimal modified methods such as the methods of Archer and Houstis et al. The collocation points are taken to be the midpoints of the subintervals of the partition. Optimal accuracy is shown to be of order $3-k$ globally for the k^{th} derivative, $k=0,1,2$, and superconvergence of order $4-k$, $k=0,1,2$, at the nodal points, the collocation points, and the Gauss points. This method is then extended to a Helmholtz-Dirichlet problem and it is shown how the collocation equations are solved using matrix decomposition algorithms.

Using a similar approach, a new one-step modified quadratic spline collocation method has also been developed for Neumann boundary conditions. Preliminary numerical results show optimal accuracy and superconvergence as expected. Future research will involve the treatment of mixed and periodic boundary conditions. In each case, extensive numerical experimentation will be conducted to demonstrate the optimality and superconvergence of the quadratic spline collocation methods and the efficiency of the matrix decomposition algorithms.

AFTERNOON SESSION II

ANALYSIS OF EVENT-RELATED fMRI DATA USING DIFFUSION MAPS

Xilin Shen and François G. Meyer
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University of Colorado at Boulder

The goal of functional neuroimaging is to map the activity of the brain in space and time. Event-related fMRI makes it possible to study the transient changes triggered by cognitive and sensory stimulation. Unlike block paradigm, event-related fMRI allows mixing of different task conditions on a trial-by-trial basis and provides a means of examining the dynamics and time-course of neural activity under various conditions.

Here, we regard the fMRI data as a very large set of time series $x_i(t)$ in $\mathbb{R}^T$, indexed by their position i. After removing the low frequency components from the time series in the preprocessing, we assume any significant changes in the fMRI data is related to the experimental paradigm. Although the experiment could recruit several cortical regions with different temporal responses, we restrict our attention to scenarios with no more than one type of temporal response. We now consider the set of all activated time series taken from the same cortical region. We assume that this set constitutes a manifold in $\mathbb{R}^T$. Clearly, if we were to use a parametric model for the haemodynamic response, the set of all haemodynamic responses generated from all the possible values of the parameters would form a manifold. The fact that the activated time series belong to a manifold in $\mathbb{R}^T$ has two implications. First, the activated time series reside only in a very small part of $\mathbb{R}^T$. Second, the activated time series are similar to each other, and one can go smoothly from one to the other one. In practice, the activated time series are corrupted by noise, and the associated manifold may exhibit some roughness.

A meaningful geometric description of the data in $\mathbb{R}^T$ would exhibit the presence of the activated manifold, and thus would have the power to discriminate between the activated and the non-activated time series. The diffusion maps method is known to be capable of generating efficient representations of complex geometric structures. In particular, it can be applied to describe the geometry of a low dimensional manifold in high dimensions. In this work, we apply this technique to fMRI data. The diffusion maps provide us with the embedding of the dataset. We then use a graph partitioning technique called the normalized cut to separate the activated time series from the background time series. Because the normalized cut is closely related to the diffusion maps, it provides a natural method to perform the clustering of the time series.

ELECTRICAL IMPEDANCE TOMOGRAPHY

Jutta Bikowski (bikowski@math.colostate.edu)
Colorado State University
Advisor: Dr. Jennifer Mueller

Electrical Impedance Tomography (EIT) is a fairly new method to construct an image of the internal structure of an object by determining the spatial impedance distribution of the object. Of special interest is the medical application since EIT is a non-invasive method and has no known side effects. In order to determine the conductivity distribution inside the region of interest, currents are applied on the surface and the corresponding voltages are measured.

The problem can be modeled by the generalized Laplace equation where the conductivity is a coefficient and the current density on the boundary leads to a Neumann condition. Knowing the conductivity distribution and the current density at the boundary, it is possible to find the voltage on the boundary. This is the forward problem. In EIT we have the voltage on the boundary and we want to know the conductivity inside. This is called an inverse problem and is nonlinear and ill posed.

There are different types of reconstruction methods: linearization, iterative, stochastic and direct methods. The most understood methods are linearization and iterative methods. Stochastic methods are relatively new and direct methods are barely known. A. Nachman [Annals of Mathematics, 128 (1988), 531-576] published a direct method for R^n, $n \geq 3$, based on D-bar techniques.

In this talk I will give a short overview about EIT in general, a linearization method for 3D geometry (application: breast cancer detection) as well as some comments about Nachmans D-bar method.

THE TOTAL HOMOTOPY OPERATOR ON THE JET SPACE: A THEORETICAL APPROACH MADE CONCRETE

Adam T. Ringler (aringler@mines.edu)
Colorado School of Mines

In this talk we give a quick overview of the general framework of the total homotopy operator and the generalized Euler operators. We then present a method for testing for the existence of a flux given a density for a conservation law for a system of nonlinear partial differential equations. We then extend this existence test to a method for recovering the flux using the homotopy operator. In solving for the associated flux we see how the total homotopy operator yields an algorithmic method for integration by parts. We also show how it gives a more general result than that of integration by parts by inverting the divergence of a conservation law of the form $D_t \rho + \bigtriangledown \mathbf{J} = 0$, where ρ is the density and $\mathbf{J}$ is the associated flux.

This work has been part of an ongoing project including other undergraduates, under the guidance of Professor Hereman and Professor Colagrosso at the Colorado School of Mines.

AUTOMATING ESHER TILING AND COLORING

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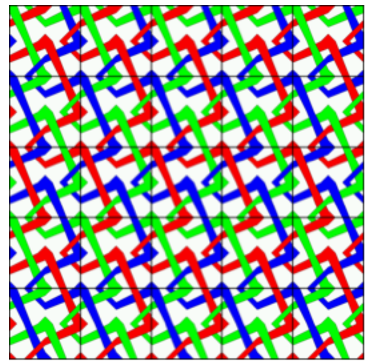
Advisors: Professor Ellen Gethner, Professor Min Choi

An Escher tile is a creation most notably attributed to the famous artist M. C. Escher. The tile is a square that contains a unique design. It is used to cover a surface much as tiles cover a wall, for instance. The design in the tile may seem to be unremarkable because it appears to contain only a group of random polygons. But, when the tiles are placed adjacent to one another to cover a surface, patterns appear on the surface. These patterns are repeated in cycles, or periods, to create wallpaper patterns on the surface. In addition, the patterns may or may not be overlapping. The patterns may not be apparent just by looking at the designs, or motifs, in the square tile.

The patterns are interesting but even more interesting when colored. A question that begs an answer is: ”How can the patterns be colored so as to receive unique colors?” Unique coloring, in this case, means that each wallpaper pattern receives a color that is different from patterns it may overlap. There must also be a discrete number of colors used on the wallpaper. This allows the color patterns are repeated as well. Fortunately, Ellen Gethner, has discovered a method for the solution to this problem.



Another question is: ”How can this method be automated on a computer?” That is the question that this thesis seeks to answer. Each step in the process of coloring Escher tiles in a wallpaper is mathematically sound and serves a purpose in the solution. However, for an implementation, there are issues of data structures, algorithms, and procedures that are necessary for engineering a solution. There is also the problem of simply defining the problem in concrete terms so that a solution may be implemented. The theoretical solution many times does not account for the practical solution. There must be some translation from the realm of theory to the realm of practice. These are the issues that are summarized in the presentation.

UPSCALING VIA THE HYBRID MIXTURE THEORETIC APPROACH: GOVERNING EQUATIONS FOR A SWELLING POROUS MEDIUM

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At the microscale the field equations: conservation of mass, linear momentum, angular momentum, and energy are known to hold. Additionally, the second law of thermodynamics, which states that entropy is always increasing, can also be formulated as a conservation equation. Experiments, however, take place on a much larger scale, herein called the macroscale, which is several magnitudes larger than the microscale. Thus, it becomes necessary to upscale to obtain equations that hold at the macroscale. Hybrid mixture theory (HMT) is one of many averaging techniques used to accomplish this. We will discuss the averaging procedure used in HMT and the underlying assumptions, such as the existence of a representative elementary volume (REV), necessary to model any two-scale, multi-constituent, multi-phase material. Additionally, we will discuss a current result stemming from this theory.

Description of the Presentation: A porous medium is one composed of solid particles with liquid/air filled pores. We consider a porous material composed of a liquid and solid phase which swells/shrinks. Examples include: swelling clays, tissues, and polymers/biopolymers such as drug delivery systems. We assume that the material we are modeling has negligible interfacial effects. That is, the interface has no thermodynamic properties and is massless. Thus, no constituent present gains or loses mass, momentum, or energy when crossing an interface. This places special restrictions on each of the field equations. We also assume that the material we are modeling has a representative elementary volume (REV); that is, a volume for which averaged properties will remain the same if the REV is made slightly larger or smaller. In addition we require that the REV size and shape remain the same for all space and time. Such an REV does not exist if the material under question is too heterogeneous. The following theory is applicable to any material meeting these requirements.

The averaging procedure is used to upscale and involves using weighted integration to average the field equations (conservation of mass, momentum, energy, and entropy). HMT uses the indicator function of the α-phase as the weight, and treats the averaged quantities resulting from the weighted integration as distributions. This allows us to bypass the difficulties of defining the derivative of averaged quantities that result from the weighted integration.

The averaged field equations hold for any material. Our choice of constitutive independent variables defines the material being modeled. The entropy inequality is written in a form allowing ease of exploitation, yielding equilibrium, non-equilibrium and near equilibrium results.

Near equilibrium results for the liquid phase stress and momentum are substituted into the conservation of linear momentum equation resulting in a form Darcy's law specific to the material being modeled.

NONLINEAR INTERPOLATION

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My goal is to develop a two-dimensional extension of the Essentially Non-Oscillating (ENO) multiresolution scheme. The ENO scheme uses local error estimates to avoid the oscillations experienced when a polynomial interpolates over a discontinuity. My method extends the concepts in ENO to two dimensions. Due to the shape of the interpolation stencil, I call the method "Spider ENO." The 2D tensor product ENO and other interpolation schemes are compared to Spider ENO in numerical experiments.

AN IMPROVED APPROXIMATION FACTOR FOR DISK COVERING PROBLEM

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General disk covering problem is defined over Euclidean plane as follows, Given a set of points D and a set of points P, find a minimum set S subset of D such that a set of unit disks centered at S covers all of P. This problem is known to be NP complete, however a 108-approximation algorithm is known. This bound can be improved by using the similar technique with a different tiling object. The choice of tiling object will improve the bound to 72.