# Friday, September 26

### Locally Free Abelian Groups of Finite Rank

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A locally free group is determined up to quasi-isomorphism by an equivalence relation on a sequence of integer matrices. Given a prime p, global locally free groups can be constructed from p-adic matrices and p-local torsion-free abelian groups. Included is the answer to an open question of E. L. Lady on the cancellation property for tensor products of locally free groups, as well as a variety of examples.

### On the typeset of a $B(2)$ group

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A $B(n)$-group $G$ is a sum of a finite number of torsionfree Abelian groups of rank 1, subject to $n$ independent linear relations. The types (= isomorphism classes) of said rank one groups are called *base-types* of $G$. While for $n \leq 1$ the base-types of a $B(n)$-group determine the typeset (= set of types of all rank one subgroups) via a simple order-theoretical operation, for $n \geq 2$ the coefficients of the linear conditions affect the computation in a way that does not shape up (as yet?) into a general rule. We show an algorithm which determines whether a given type belongs to all $B(2)$-groups with given base-types and given basic partition, independently from the coefficients of the linear relations; while in the negative case it gives a kind of estimate of the occurrence of the type for particular coefficients of the linear conditions. (The basic partition associates base-elements that are treated equally by the relations).

### Zassenhaus Algebras

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Let $k$ be a commutative ring or field and $1 \in A$ a $k$-algebra. We define $\widehat{A} = \{\varphi \in End_k(A) : \varphi(X) \subseteq X \text{ for all } X \trianglelefteq_\ell A\}$, i.e. $\widehat{A}$ consists of all $k$-linear transformations of the $k$-vector space $A$ leaving all left ideals invariant. The algebra $A$ is called a Zassenhaus algebra if $\widehat{A} = A$. We will present some results on group rings and finite dimensional algebras that are Zassenhaus algebras as well as those which are not. Let $J = \left\{\varphi \in \widehat{A} : \varphi(1) = 0\right\}$. Then $\widehat{A} = A \oplus J$ and $A$ is a natural subring of $\widehat{A}$, which allows for a (transfinite) iteration of the $\widehat{\ }$-construction yielding a sequence $\widehat{A}^\alpha \subseteq \widehat{A}^{(\alpha+1)}$, where $\alpha$ runs through the ordinals. We will discuss the behavior of this chain of algebras.

**Friday, September 26 (continued)**

## Two classic results by Kurosh and by Malcev

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A.I. Malcev has described torsion free abelian groups of finite rank with the help of some matrices in 1938. Unfortunately his result is almost forgotten now. But his approach admits a very interesting generalization. Namely, considering the Malcev matrices as objects of a category, we obtain a duality between this category and the category of torsion-free finite-rank groups. This duality is close to the well-known Pontryagin duality.

A.G. Kurosh has also described p-primitive torsion free groups with the help of matrices (different from the Malcev matrices) in 1937. D. Derry has obtained simultaneously results allowed to generalize the Kurosh description for all torsion free abelian groups of finite rank. The Kurosh-Derry description is well-known but unfortunately it has no serious applications up to now. The p-primitive torsion free groups are quotient divisible in particular. The more natural generalization of the Kurosh theorem is from p-primitive groups to arbitrary quotient divisible groups. It is an equivalence between the category of quotient divisible groups and the mentioned above category of Malcev matrices. The composition of these equivalence and duality is the duality introduced by W. Wickless and the author in 1998.

## To what extent does the endomorphism ring of a mixed module determine the structure of the module?

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The subject of isomorphism theorems began with the celebrated theorem by Baer and Kaplansky which states that two abelian torsion groups are isomorphic if and only if their endomorphism rings are isomorphic rings. Warren May showed that several classes of mixed modules over a complete discrete valuation domain also satisfy a Baer-Kaplansky theorem. The proofs of all of these isomorphism theorems rely heavily on the properties of the ideal generated by primitive idempotents in the endomorphism ring. Is this ideal necessary for an isomorphism theorem to exist? In this spirit, I will explain the extent to which the structure of a mixed module over a discrete valuation domain is reflected in the Jacobson radical of its endomorphism ring.

**Friday, September 26 (continued)**

### Infinitary equivalence of $\mathbb{Z}_p$-modules with nice decomposition bases

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Warfield modules are direct summands of simply presented $\mathbb{Z}_p$-modules or, alternatively, are $\mathbb{Z}_p$-modules possessing a nice decomposition basis with simply presented cokernel. They have been classified up to isomorphism by their Ulm-Kaplansky and Warfield invariants. Taking a model-theoretic point of view and using infinitary languages we give a complete model-theoretic characterization of a large class of $\mathbb{Z}_p$-modules having a nice decomposition basis, which generalizes results by Barwise and Eklof. As a corollary, we obtain the classical classification of countable Warfield modules. This is joint work with R. Göbel, K. Leistner and L. Strüngmann.

### A relationship between Semi-Stable Kernels and Semi-Stable Cokernels and Examples in Modules over Integral Domains

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The concepts of Prebalanced and Precobalanced sequences have been used extensively in the study of pure subgroups (equivalently torsion-free images) of finite direct sums of torsion-free rank-1 Abelian groups (Butler Groups). It is well known that in this case the two concepts are equivalent. But when one considers such modules over integral domains the equivalence is dependent on the relationship between the ring, its integral closure and its lattice of types

In this talk we consider the generalization of prebalanced and precobalanced to arbitrary Preabelian Categories. We start with a class $\mathcal{X}$ of objects closed under finite direct sums. Then letting $\mathcal{R}(\mathcal{X})$ and $\mathcal{P}(\mathcal{X})$ be the closure of $\mathcal{X}$ with respect to semi-stable cokernels and semi-stable kernels, we see that though these classes may not be equivalent or even comparable, $\mathcal{R}(\mathcal{P}(\mathcal{X})) = \mathcal{P}(\mathcal{R}(\mathcal{X}))$. We look at examples of this for the class of finite direct sums of torsion-free rank-1 modules over various integral domains.

**Friday, September 26 (continued)**

## Analytic methods in abelian groups

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Let $K$ be a quadratic number field, let $\overline{E}$ be the ring of algebraic integers in $K$, and for each rational prime $p$ let $E(p) = Z + p\overline{E}$. Let $G(p)$ be a reduced torsion-free rank two abelian group such that $\mathrm{End}(G(p)) = E(p)$, and let $h(G(p))$ be the number of isomorphism classes of abelian groups that are locally isomorphic to $G(p)$. Let $\overline{G(p)}$ denote the integral closure of $G(p)$. Let $L(p)$ be the cardinality of the group $u(\overline{E})/u(E(p))$, where $u(R)$ denotes the group of units of the ring $R$.

THEOREM: Let $K$ be a quadratic number field. The sequence

$$\{L(p)h(G(p))/h(H(p)) \mid p\}$$

of integers is asymptotically equal to the sequence of rational primes.

THEOREM: Let $K$ be a quadratic number field. Then $K$ has unique factorization iff the sequence $\{L(p)h(G(p)) \mid p\}$ is asymptotically equal to the sequence of rational primes.

# Saturday morning, September 27

### von Neumann regular Leavitt path algebras

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[joint work with K.M. Rangaswamy]

Let $E$ be an arbitrary directed graph, and $K$ any field. We show that the von Neumann regular Leavitt path algebras are precisely those for which $E$ is acyclic. We achieve this result by showing that, for $E$ acyclic, $L_K(E)$ is a direct union of subalgebras, each of which is isomorphic to a Leavitt path algebra of a finite acyclic graph. In addition, we show that for Leavitt path algebras, von Neumann regularity is equivalent to both $\pi$-regularity and strong $\pi$-regularity. We conclude by showing how other conditions (unit regularity, strongly clean) can be appended to this list.

### The socle of a Leavitt path algebra

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[joint with D. Martn Barquero, C. Martn Gonzlez and M. Siles Molina]

We characterize the minimal left ideals of a Leavitt path algebra as those which are isomorphic to principal left ideals generated by line points; that is, by vertices whose trees contain neither bifurcations nor closed paths. Moreover, we show that the socle of a Leavitt path algebra is the two-sided ideal generated by these line point vertices. This characterization allows us to compute the socle of certain algebras that arise as the Leavitt path algebra of a row-finite graph. A complete description of the socle of a Leavitt path algebra is given: it is a locally matricial algebra.

**Saturday morning, September 27 (continued)**

**The Loewy Series of a Leavitt Path Algebra over an arbitrary graph**

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[joint work with G. Abrams]

This talk deals with the Loewy series of a Leavitt path algebra $L_K(E)$ over an arbitrary graph $E$ and arbitrary field $K$. Recall that a *left (right) socle* $Soc_l(R)$ ($Soc_r(R)$) of a ring $R$ is the sum of all its minimal left (right) ideals and is a two-sided ideal of $R$. These two ideals coincide when $R = L_K(E)$. The ascending left *Loewy series* of $R$ is a continuous well-ordered ascending chain of ideals

$$0 = S_0 < S_1 < \cdots < S_\alpha < S_{\alpha+1} < \cdots \qquad (\alpha < \tau)$$

where $\tau$ is some fixed ordinal and , for each $\alpha < \tau$, $S_{\alpha+1}/S_\alpha = Soc_l(R/S_\alpha)$ and the chain is continuous in the sense that for any limit ordinal $\gamma < \tau$, $S_\gamma = \cup_{\alpha<\gamma} S_\alpha$. The smallest ordinal $\lambda$ such that $S_\lambda = S_{\lambda+1}$ is called the *Loewy Length* $l(R)$ of $R$ . For any $\alpha$, $S_\alpha$ is called the *$\alpha$-th Socle* of $R$. If $R = S_\alpha$, for some $\alpha$, then $R$ is called a left *Loewy module.* It is shown that for every ordinal $\lambda$ there is a graph $E$ such that the corresponding Leavitt path algebra $L_K(E)$ has Loewy length $\lambda$. The main theorem gives necessary and sufficient graph-theoretical conditions on $E$ in order that $L_K(E)$ becomes a Loewy module. In particular, if $E$ is finite then $L_K(E) = S_n$ for some positive integer $n$ if and only if $n = 1$ and $E$ is acyclic.

**Representations of infinite quivers**

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In the present talk we study injective, projective and flat representations of infinite quivers in terms of local properties of the quiver. We also characterize the existence of injective covers in these categories. Finally we consider the problem of characterizing Gorenstein path algebras and, more generally, when the category of representations by modules of a quiver is a Gorenstein category. Gorenstein categories are the natural framework where Gorenstein homological algebra can be developed.

**Saturday morning, September 27 (continued)**

### On the hierarchy of $h$-divisible modules

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We investigate the class of $h$-divisible $R$-modules over an integral domain $R$, in particular , putting emphasis on the proper subclass consisting of weak-injective $R$-modules which was introduced by the author. We look into the properties of these modules which can be related to the characterization of other classes of modules.

### Finendo modules and pure semisimple rings

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[joint work with José Luis García (Universidad de Murcia)]

Following C. Faith, a module $M$ over an associative ring $R$ (with identity) is called finendo if $M$ is finitely generated as a module over its endomorphism ring. A ring $R$ is left pure semisimple if every left $R$-module is a direct sum of finitely generated modules. In this talk, we discuss rings satisfying the property that every right $R$-module is finendo, and show that if such a ring $R$ is hereditary, then $R$ is of finite representation type. In general, if $R$ is an arbitrary ring with all right $R$-modules finendo, then $R$ is a left pure semisimple ring with a right Morita duality and the quotient ring $R/(J(R))^2$ is of finite representation type.

### Non-Singular Rings of Injective Dimension 1

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A singular module $S$ has the essential extension property if there are a projective module $P$ and an essential extension $M$ of $P$ such that $M/P \cong S$. All singular right and left modules over a semi-prime right and left Goldie ring $R$ have the essential extension property if and only if $R$ is right and left Noetherian and has injective dimension at most 1 as a right and left $R$-module.

# Saturday afternoon September 27

### The Representation of Semi-Hereditary Bezout Semigroups

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[joint work with P.N. Ánh (Math. Institute, Hungarian Academy of Sciences)]

A Bezout semigroup S is called "semi-hereditary" if for each a in S there is an idempotent in S that generates the annihilator of a. Our main result states that a semigroup is a semi-hereditary Bezout semigroup if and only if it is isomorphic to a "semigroup of divisibility," i.e. the semigroup of principal ideals in a semi-hereditary Bezout ring partially ordered by reverse inclusion.

### Formal fibers of Noetherian rings arising from derivations

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[joint work with A. Mimouni]

If $R$ is a commutative ring that is finitely generated as an algebra over a field or the ring of integers, then the integral closure of $R$ is a finitely generated $R$-module, and this implies the the completions of the localizations of $R$ at maximal ideals have no nilpotent elements; i.e., that such localizations are "analytically unramified." However, local Noetherian rings in general need not be analytically unramified, a fact that often poses technical difficulties in dealing with Noetherian rings that are not integrally closed. In this talk we look at a circle of ideas involving derivations, analytically ramified Noetherian rings, and the generic formal fibers of Noetherian rings, and from these relationships we deduce the existence of large classes of analytically ramified Noetherian rings in (arguably) natural settings. Although the main application here is to Noetherian rings, the techniques are mostly non-Noetherian.

**Saturday afternoon, September 27 (continued)**

### Higher derivations and their extensions to rings and modules of quotients

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A *higher derivation* on a ring $R$ is an indexed family $\Delta = \{\delta_n\}$ of additive maps $\delta_n$ such that $\delta_0$ is the identity mapping on $R$ and

$$\delta_n(rs) = \sum_{i=0}^{n} \delta_i(r)\delta_{n-i}(s)$$

for all $r, s \in R$ and all $n$. For such $\Delta$, a *higher* $\Delta$-*derivation* on a right $R$-module $M$ is an indexed family $D = \{d_n\}$ of additive maps $d_n$ such that $d_0$ is the identity mapping on $M$ and $d_n(mr) = \sum_{i=0}^{n} d_i(m)\delta_{n-i}(r)$ for all $m \in M, r \in R$ and all $n$.

We study the conditions under which a higher derivation on a module can be extended to a module of quotients. Then, we study conditions under which a higher derivation extended to a right module of quotients extends also to a larger right module of quotients in such a way that the two extensions agree. Lastly, we prove that the symmetric versions of the results on higher differential (one-sided) modules of quotients hold for symmetric higher derivations and symmetric modules of quotients.

### Elements of minimal primes as zero-divisors

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[joint work with W.D. Burgess and A. Lashgari]

E. Armendariz asked whether, in a general ring $R$, elements of minimal prime ideals were zero-divisors, in some sense. An example shows that the answer is "no" for left or right zero-divisors. An element $a \in R$ is a *weak zero-divisor* if there are $r, s \in R$ with $ras = 0$ and $rs \neq 0$. It is shown that each element of a minimal prime ideal is a weak zero-divisor. Related questions are examined, in particular in rings where the set of nilpotent elements forms an ideal.

**Saturday afternoon, September 27 (continued)**

## Unique Decompositions into ideals for reduced commutative Noetherian Rings

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[joint work with L.C. Klingler, Florida Atlantic U.]

We say that a commutative ring $R$ has the *unique decomposition into ideals* ($UDI$) property if, for any $R$-module $L$ which decomposes into a direct sum of indecomposable ideals, this decomposition is unique up to the order of the ideals. In a recent paper, Goeters and Olberding characterize the $UDI$ for Noetherian integral domains. In this paper, we characterize the $UDI$ for Noetherian reduced rings.

## Minimal Zero-Dimensional Extensions

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The structure of minimal zero-dimensional extensions of one-dimensional rings $R$ with Noetherian spectrum in which zero is a primary ideal is determined. Those rings include Dedekind domains but need not be Noetherian nor integrally closed.

**Saturday afternoon, September 27 (continued)**

### Invariants of Actions on Artin-Schelter Regular Algebras

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[joint with Ellen Kirkman (Wake Forest U.)
and James Zhang (U. Washington)]

This talk is an general survey of the study of invariants of Artin-Schelter regular algebras. In particular it will discuss partial analogs of the Shephard-Todd-Chevalley theorem and Watanabe's theorem. The Shephard-Todd-Chevalley theorem states that if $G$ is a finite group of graded automorphisms of a polynomial ring, then the invariant ring is again a polynomial ring if and only if $G$ is generated by pseudo-reflections (those having invariant subspace of codimension 1). Watanabe's theorem says that if $G \leq SL(n)$, then the invariant ring is Gorenstein. The talk will include numerous examples and questions.

### Designing designs

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Presenting designs as lists of subsets of a set does not reveal any structural properties. Representing a design graphically may make certain properties of a design obvious that otherwise would require tedious reasoning. This holds for symmetries, resolvability, and sub-design inclusion in appropriate cases. We describe how a graphical representation can be achieved in many cases and show several examples.

# Sunday September 28

### Leibniz algebras

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In this talk we will introduce Leibniz algebras which are non-skew symmetric generalizations of Lie algebras. Leibniz algebras were introduced by Jean-Louis Loday as a natural framework for a non-commutative analog of Lie algebra homology. We will explain this and we will also discuss generalizations of several results for Lie algebras to Leibniz algebras. ( 2000 Mathematics Subject Classification: 17A32, 19D55)

### Bidiagonal pairs, the Lie algebra $\mathfrak{sl}_2$, and the quantum group $U_q(\mathfrak{sl}_2)$

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In this talk I will define a linear algebraic object called a bidiagonal pair and provide an example of one. Roughly speaking, a bidiagonal pair is a pair of diagonalizable linear transformations on a finite dimensional vector space that act bidiagonally on each others eigenspaces. I will discuss how to classify bidiagonal pairs by connecting them to the Lie algebra $\mathfrak{sl}_2$ and the quantum group $U_q(\mathfrak{sl}_2)$. In particular, I will describe how the collection of all bidiagonal pairs can be divided into two classes. The first class of bidiagonal pairs can be used to construct all finite dimensional representations of $\mathfrak{sl}_2$. The second class of bidiagonal pairs can be used to construct all finite dimensional representations of $U_q(\mathfrak{sl}_2)$.

### Combinatorial Identities for Yangian

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We introduce analogues of elementary symmetric functions on the generators of Yangian of the Lie algebra $\mathfrak{gl}_n(\mathbb{C})$, and we prove the analogues of classical combiantorial identities - such as Cayley-Hamilton equation, Newton's formula, etc. The applications for quantum integrable systems are discussed.

**Sunday September 28 (continued)**

### Free mappings and factorizations of groups

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[joint work with Nicola Pace (Florida Atlantic U.)]

We say that a collection of subsets $\alpha = [B_1, \ldots, B_k]$ of group $G$ is a *factorization* if $G = B_1 \ldots B_k$ and each element of $G$ is expressed in unique way in this product. Group factorization is a topic, that has besides its theoretical beauty practical use in graph theory, coding theory, number theory and cryptography. By using special type of mappings between groups $A$ and $B$, called *free mappings*, we show a technique for obtaining factorizations of a group $G$, such that $G \cong A \times B$. Moreover, a simple way for constructing free mappings is provided. There is no limitation on the type of groups $A$ and $B$, but we found this approach particularly interesting in the case when $A$ and $B$ are abelian groups. It is also considered an interesting joint of free mappings and Rédei's theorem, with number theoretic implication.

### Hopf algebras, Frobenius-Schur indicators, and the modular group

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Every factorizable Hopf algebra leads to a projective representation of the modular group. In the semisimple case, the kernel of this representation is in fact congruence subgroup whose level is determined by the exponent of the Hopf algebra. We explain how generalized Frobenius-Schur indicators can be used to establish this fact.

**Sunday September 28 (continued)**

### Reduced K-theory for Azumaya Algebras, An overview

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The theory of Azumaya algebras developed parallel to the theory of central simple algebras. However the latter are algebras over fields whereas the former are algebras over rings. One wonders how the K- theory of these objects compare to each other. We look at higher K- theory and reduced K-theory of these objects. We ask nice questions!

### The Clifford Algebra of a Cubic Form

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Let $F$ be a field of characteristic not 2 or 3 and let $f(x, y)$ be a binary cubic form over $F$. Haile proved that the Clifford algebra $C_f$ of the form $f$ is Azumaya of rank 9 and its center is the coordinate ring of the elliptic curve $E$ given by $s^2 = r^3 - 27D_f$, where $D_f$ is the discriminant of $f$. He also proved that there is an induced group homomorphism from $E(F)$ to $B(F)$ with image $B(F(C_0)/F)$, where $F(C_0)$ denote the function field of the curve $C_0 : z^3 = f(x, y)$. In this talk, we shall present an algebra $A_C$ associated to the plane cubic curve $C$ over $F$ given by $z^3 - exyz - f(x, y) = 0$. We prove that $A_C$ is Azumaya of rank 9 and its center is the coordinate ring of the Jacobian of the curve $C$.