

1. Prove the following statements using the ε - δ definition of a limit:

a. $\lim_{x \rightarrow 4} (7 - 3x) = -5$

b. $\lim_{x \rightarrow 3} \frac{x^2 + x - 12}{x - 3} = 7$

2. Given that $\lim_{x \rightarrow a} f(x) = -3$ and $\lim_{x \rightarrow a} g(x) = 0$ and $\lim_{x \rightarrow a} h(x) = 8$, Find each of the following:

a. $\lim_{x \rightarrow a} \frac{f(x)}{g(x)} =$

b. $\lim_{x \rightarrow a} \frac{3f(x)}{h(x) - g(x)} =$

c. $\lim_{x \rightarrow a} \sqrt[3]{f(x)} =$

3. $\lim_{x \rightarrow -1} (x^2 + 1)^3 (x + 3)^5 =$

4. $\lim_{x \rightarrow 2} \frac{x^2 - x + 6}{x - 2} =$

5. $\lim_{x \rightarrow -4} \frac{\frac{1}{4} + \frac{1}{x}}{4 + x} =$

6. Given $2x \leq g(x) \leq x^4 - x + 2$ for all x , find $\lim_{x \rightarrow 1} g(x)$. Justify your conclusion.

7. $\lim_{x \rightarrow -6} \frac{2x + 12}{|x + 6|} =$

8. $\lim_{x \rightarrow 0} \frac{\sin(4x)}{\sin(6x)} =$

9. Sketch the graph of a function that has a jump discontinuity at $x=2$ and has a removable discontinuity at $x=4$ but is continuous elsewhere.

10. Use the definition of continuity and the properties of limits to show that the function $f(x) = x^2 + \sqrt{7-x}$ is continuous at $a=4$.

11. Explain why the following function is discontinuous at $a=1$ and sketch a graph of the

function.
$$f(x) = \begin{cases} 1 - x^2 & x < 1 \\ \frac{1}{x} & x \geq 1 \end{cases}$$

12. Find the numbers at which the function $f(x)$ is discontinuous. At which of these points is $f(x)$ continuous from the right, from the left, or neither? Sketch a graph of $f(x)$.

$$f(x) = \begin{cases} x + 2 & x < 0 \\ 2x^2 & 0 \leq x \leq 1 \\ 2 - x & x > 1 \end{cases}$$

13. Use the Intermediate Value Theorem to show that $f(x)$ has a root in the specified interval.

$$f(x) = 1 - x - \sqrt[3]{x}. \text{ On the interval } (0,1).$$

14. Sketch a graph of a function f that satisfies all of the given conditions:

- a. $\lim_{x \rightarrow 3} f(x) = -\infty$
- b. $\lim_{x \rightarrow \infty} f(x) = 2$
- c. $f(0) = 0$
- d. f is even

$$15. \lim_{x \rightarrow \infty} \frac{x+2}{\sqrt{9x^2+1}} =$$

$$16. \lim_{x \rightarrow \infty} (\sqrt{9x^2 + x} - 3x) =$$

$$17. \lim_{x \rightarrow \infty} (x - \sqrt{x}) =$$

