Review of Limits Worksheet Solutions

1a) Step 1: Find δ $\quad |f(x)-L| = |7-3x-(-5)| = |12-3x| < \varepsilon \Rightarrow$

$3|4-x| < \varepsilon \Rightarrow |x-4| < \frac{\varepsilon}{3} \Rightarrow \delta = \frac{\varepsilon}{3}$

Step 2: State the proof using the definition of a limit

For any $\varepsilon > 0$ there is $\delta = \frac{\varepsilon}{3} > 0$ so that if

$|x-4| < \delta = \frac{\varepsilon}{3} \Rightarrow |3x-12| < \varepsilon \Rightarrow |12-3x| < \varepsilon \Rightarrow$

$|7-3x-(-5)| < \varepsilon \Rightarrow |f(x)-L| < \varepsilon$ ■

1b) Step 1: Find δ $\quad |f(x)-L| = \left| \frac{x^2+x-12}{x-3} - 7 \right| = \left| \frac{x^2+x-12-7(x-3)}{x-3} \right|$

$= \left| \frac{x^2-6x+9}{x-3} \right| < \varepsilon \Rightarrow |x-3| < \varepsilon \Rightarrow \delta = \varepsilon$

Step 2: State the proof using the definition of a limit

For any $\varepsilon > 0$ there is a $\delta = \varepsilon > 0$ so that if $|x-3| < \delta = \varepsilon$

$\Rightarrow \left| \frac{(x-3)^2}{(x-3)} \right| < \varepsilon \Rightarrow \left| \frac{x^2-6x+9}{x-3} \right| < \varepsilon \Rightarrow \left| \frac{x^2+x-12}{x-3} - 7 \right| < \varepsilon$

$\Rightarrow |f(x)-L| < \varepsilon$ ■

2a) since the $\lim_{x \to a} g(x) = 0 \Rightarrow$ limit does not exist

2b) $\dfrac{3 \lim_{x \to a} f(x)}{\lim_{x \to a} h(x) - \lim_{x \to a} g(x)} = \dfrac{3(-3)}{8-0} = \dfrac{-9}{8}$ by Limit Laws

3.) Try direct substitution

$$\left((-1)^2+1\right)^3(-1+3)^5 = (2)^3(2)^5 = 2^8 = 256$$

4.) Try direct substitution

$$\frac{8}{0} \Rightarrow \text{lim does not exist}$$

5.) Try direct substitution: $\frac{0}{0}$ (must work harder)

$$\lim_{x\to -4} \frac{\frac{1}{4}+\frac{1}{x}}{4+x} = \lim_{x\to -4} \frac{\frac{x+4}{4x}}{4+x} = \lim_{x\to -4} \frac{1}{4x} = -\frac{1}{16}$$

6.) $\lim_{x\to 1} 2x = 2$ by direct substitution

$\lim_{x\to 1} x^4 - x + 2 = 2$ by direct substitution

$\Rightarrow \lim_{x\to 1} g(x) = 2$ by squeeze thm.

7.) $\lim_{x\to -6} \frac{2x+12}{|x+6|} = \lim_{x\to -6} \frac{2(x+6)}{|x+6|}$ does not exist

$$\frac{2(x+6)}{|x+6|} = \begin{cases} \frac{2(x+6)}{x+6} & x > -6 \\ \frac{2(x+6)}{-(x+6)} & x < -6 \end{cases}$$

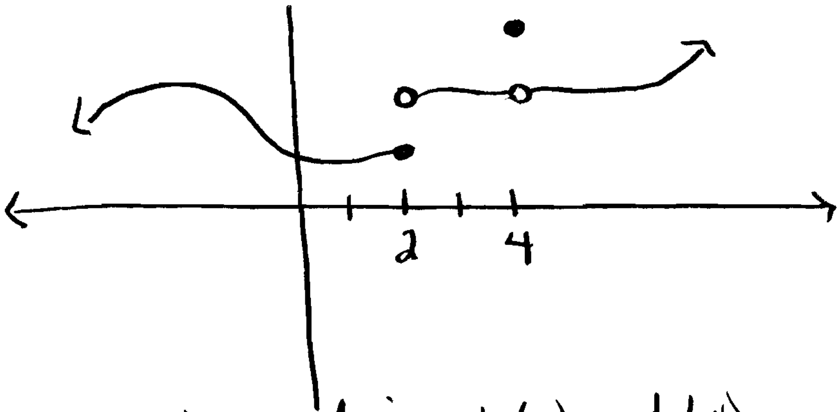
This is a case where you want to investigate right hand vs. left hand limits

$$\lim_{x\to -6^-} \frac{2(x+6)}{-(x+6)} = -2 \neq \lim_{x\to 6^+} \frac{2(x+6)}{(x+6)} = 2$$

8.) $$\lim_{x\to 0} \frac{\sin 4x}{\sin 6x} = \lim_{x\to 0} \frac{x\sin 4x}{x\sin 6x} = \lim_{x\to 0} \frac{x}{\sin 6x} \cdot \lim_{x\to 0} \frac{\sin 4x}{x}$$

$$= \lim_{x\to 0} \frac{6x}{6\sin 6x} \cdot \lim_{x\to 0} \frac{4\sin 4x}{4x} = \frac{1}{6} \cdot 4 = \frac{4}{6} = \frac{2}{3}$$

9)



(there are many examples here)

10.) Must show $\lim_{x\to 4} f(x) = f(4)$

① $\lim_{x\to 4} x^2+\sqrt{7-x} = 16+\sqrt{3}$ by direct substitution

② $f(4) = 16+\sqrt{3}$ and ③ $\lim_{x\to 4} f(x) = f(4)$ therefore

$f(x)$ is continuous at $a=4$.

11.) ① $f(1)=1$ so $f(1)$ is defined

$\lim_{x\to 1^+} f(x) = 1 \neq \lim_{x\to 1^-} f(x) = 0 \Rightarrow \lim_{x\to 1} f(x)$ d.n.e. X

12.) Discontinuous at $x=0$

$\lim_{x\to 0^+} f(x) = 0 \neq \lim_{x\to 0^-} f(x) = 2 \Rightarrow \lim_{x\to 0} f(x)$ d.n.e.

Discontinuous at $x=1$

$\lim_{x\to 1^+} f(x) = 1 \neq \lim_{x\to 1^-} f(x) = 2 \Rightarrow \lim_{x\to 1} f(x)$ d.n.e.

Continuous from the left of $x=1$

$\lim_{x\to 1^-} f(x) = 2 = f(1)$

12 cont) Discontinuous from the right of $x=1$

$$\lim_{x\to 1^+} f(x)=1 \neq f(1)=2$$

Discontinuous from the left of $x=0$

$$\lim_{x\to 0^-} f(x)=2 \neq f(0)=0$$

Continuous from the right of $x=0$

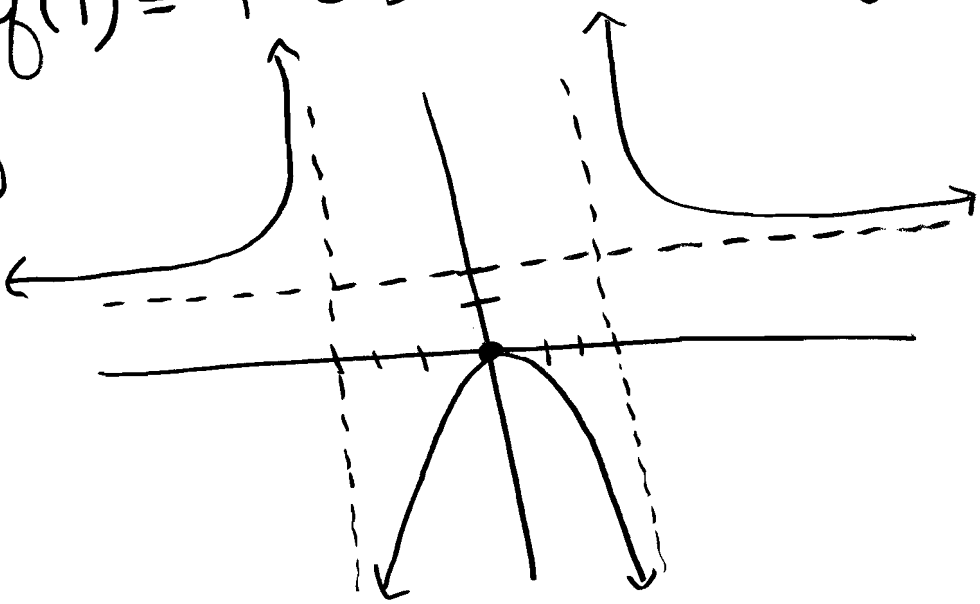
$$\lim_{x\to 0^+} f(x)=0=f(0)$$

13) $f(x)=1-x-\sqrt[3]{x}$ $(0,1)$

$f(x)$ Continuous on the interval from $(0,1)$ because it is the sum or difference of cont. functions

$$\left.\begin{array}{l} f(0)=1>0 \\ f(1)=-1<0 \end{array}\right\} \Longrightarrow$$ by I.V.T. there is a $c\in(0,1)$ so that $f(c)=0$

14)



15.) $\lim_{x\to\infty} \dfrac{\frac{x+2}{x}}{\sqrt{\frac{9x^2+1}{x^2}}} = \lim_{x\to\infty} \dfrac{1+\frac{2}{x}}{\sqrt{9+\frac{1}{x^2}}} = \dfrac{1}{\sqrt{9}} = \dfrac{1}{3}$

16.) $\lim_{x\to\infty} \left(\sqrt{9x^2+x} - 3x\right) \cdot \dfrac{\sqrt{9x^2+x}+3x}{\sqrt{9x^2+x}+3x} =$

$\lim_{x\to\infty} \dfrac{9x^2+x-9x^2}{\sqrt{9x^2+x}+3x} = \lim_{x\to\infty} \dfrac{\frac{\cancel{x}}{x}}{\sqrt{\frac{9x^2+x}{x^2}}+\frac{3x}{x}} =$

$\lim_{x\to\infty} \dfrac{1}{\sqrt{9+\frac{1}{x}}+3} = \dfrac{1}{6}$

17.) $\lim_{x\to\infty} (x-\sqrt{x}) = \lim_{x\to\infty} \sqrt{x}(\sqrt{x}-1) =$ d.n.e

but ∞ is appropriate to describe this behavior