Note Title 12/10/2010

$$f(x) = \frac{x^2-1}{x^2+3x-4} = \frac{(x-1)(x+1)}{(x+4)(x-1)}$$

Domain: $(-\infty,-4)\cup(-4,1)\cup(1,\infty)$

x-intercepts: $0 = \frac{x^2-1}{x^2+3x-4}$ $\quad x^2-1=0$

$x = \pm 1$

1 is out of domain

$(-1,0)$

y-intercept: $f(0) = \frac{-1}{-4} = \frac{1}{4}$

H.A. $\lim\limits_{x\to\infty} \frac{x^2-1}{x^2+3x-4} = 1$ $\quad \boxed{y=1}$

V.A. $\lim\limits_{x\to-4} \frac{x^2-1}{x^2+3x-4} = \lim\limits_{x\to-4^-} \frac{\cancel{(x-1)}(x+1)}{(x+4)\cancel{(x-1)}} = +\infty$

$\lim\limits_{x\to1} \frac{x^2-1}{x^2+3x-4} = \lim\limits_{x\to1} \frac{\cancel{(x-1)}(x+1)}{(x+4)\cancel{(x-1)}} = \frac{2}{5}$

hole $(1,\frac{2}{5})$

$\lim\limits_{x\to-4^+} \frac{x+1}{x+4} = -\infty$ $\quad \boxed{x=-4}$

Increasing / Decreasing

$f(x) = \frac{x^2-1}{x^2+3x-4}$ $\quad f'(x) = \frac{2x(x^2+3x-4)-(2x+3)(x^2-1)}{(x^2+3x-4)^2}$

$$f'(x) = \frac{\cancel{2x^3} + \cancel{6x^2} - 8x - (\cancel{2x^3} - 2x + \cancel{3x^2} - 3)}{(x^2+3x-4)^2}$$

$$= \frac{3x^2 - 6x + 3}{(x^2+3x-4)^2} = 0 \qquad 3(x^2-2x+1) = 0$$

$$3(x-1)^2 = 0$$

$$f'(x) = \frac{3(x-1)^2}{[(x-1)(x+4)]^2} \qquad x = 1$$

$f'(x) = 0$ when $x = 1$

$f'(x)$ undefined when $x = 1$ $x = -4$

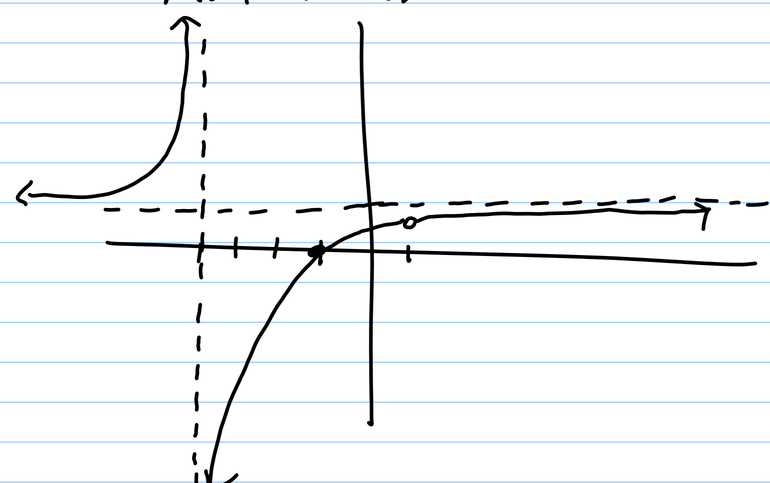
$f'(-5) > 0$ $\quad f'(0) > 0$ $\quad f'(2) > 0$

-4 $\quad 1$

Increasing $(-\infty, -4) \cup (-4, 1) \cup (1, \infty)$

Decreasing nowhere

Relative Max or Min None



39.) $y = xe^{-x}$ $\qquad u = -x \quad v = e^{-x}$

$u' = -1 \quad v' = -e^{-x}$

$y' = e^{-x} - xe^{-x} \qquad y'' = -e^{-x} - (e^{-x} - xe^{-x})$

$$y'' = -2e^{-x} + xe^{-x}$$

<u>Domain</u>: $\mathbb{R}$

x-int: $0 = xe^{-x}$ $\quad x = 0 \quad (0,0)$

y-int: $f(0) = 0e^{-0} = 0 \quad (0,0)$

H.A. $\lim\limits_{x\to\infty} xe^{-x} = \infty \cdot 0$ $\qquad \lim\limits_{x\to-\infty} xe^{-x} = -\infty\infty$ d.n.e

$\lim\limits_{x\to\infty} \frac{x}{e^x} = \frac{\infty}{\infty} \qquad \lim\limits_{x\to\infty} \frac{1}{e^x} = 0$

Inc/Dec $\quad y' = e^{-x} - xe^{-x} = 0$

$e^{-x}(1-x) = 0 \qquad x = 1$

$f'(0) > 0 \qquad f'(2) < 0$

$\nearrow \quad 1 \quad \searrow$

Relative Max at $x = 1 \quad (1, \frac{1}{e})$

No Relative Min

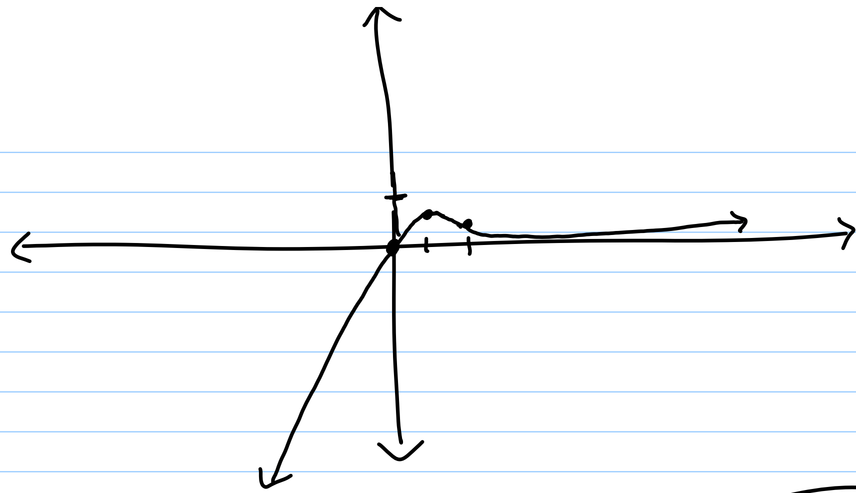
$y'' = -2e^{-x} + xe^{-x} = \cancel{e^{-x}}(-2 + x) = 0 \qquad x = 2$

$f''(0) < 0 \qquad f''(3) > 0$

2

Concave Down $(-\infty, 2)$ $\quad$ Concave Up $(2, \infty)$

Inflection Pt $\quad (2, \frac{2}{e^2})$



$$\int_1^2 x\,\overset{\sqrt{u}}{\sqrt{x-1}}\,\boxed{\overset{du}{dx}} \qquad \begin{aligned} u &= x-1 \Longrightarrow x = u+1 \\ du &= dx \end{aligned}$$

$$\int_{x=1}^{x=2} (u+1)\sqrt{u}\,du$$

$$\int_{x=1}^{x=2} u^{\frac{1}{2}}(u+1)\,du = \int_{x=1}^{x=2}\left(u^{3/2} + u^{1/2}\right)du = \frac{2}{5}u^{5/2} + \frac{2}{3}u^{3/2}\Big|_{x=1}^{x=2}$$

$$\frac{2}{5}(x-1)^{5/2} + \frac{2}{3}(x-1)^{3/2}\Big|_1^2$$

$$\left[\frac{2}{5} + \frac{2}{3}\right] = \frac{16}{15}$$

2.8 Differentials

$y = f(x)$ then $dy = f'(x)\,dx$

$\int \frac{x}{x^2+1}\,dx \qquad u = x^2+1$

$du = 2x\,dx$

$dx = \frac{du}{2x}$

$\int \frac{\cancel{x}}{u} \cdot \frac{du}{2\cancel{x}} = \frac{1}{2}\int \frac{1}{u}\,du$

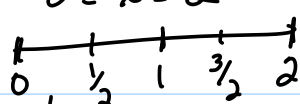
$\int \frac{\boxed{x\,dx}}{x^2+1} \qquad u = x^2+1$

$du = 2x\,dx$

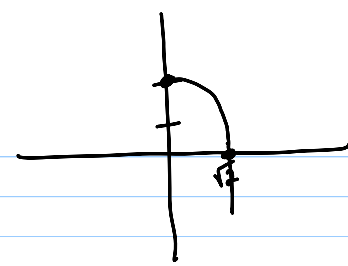
$\frac{1}{2}\,du = x\,dx$

$\frac{1}{2}\int \frac{1}{u}\,du$

$f(x) = 2 - x^2 \qquad 0 \le x \le 2$



$\Delta x = \frac{2-0}{4} = \frac{1}{2}$



$\sum_{i=1}^{4} f(x_i)\,\Delta x = \frac{1}{2}\left[f(x_1) + f(x_2) + f(x_3) + f(x_4)\right]$

$\frac{1}{2}\left[f\left(\frac{1}{2}\right) + f(1) + f\left(\frac{3}{2}\right) + f(2)\right]$

$\frac{1}{2}\left[\frac{7}{4} + \frac{4}{4} - \frac{1}{4} - \frac{8}{4}\right] = \frac{1}{\cancel{2}}\,\frac{\cancel{2}}{4} = \frac{1}{4}$