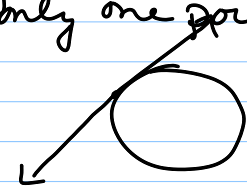
2.1 Derivatives and Rates of Change

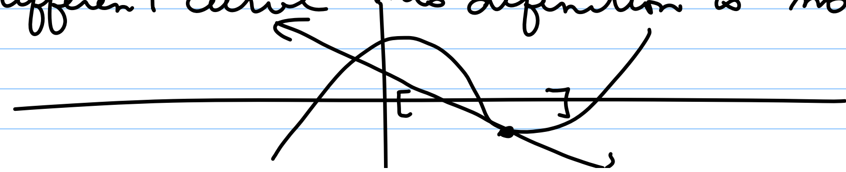
Note Title 9/14/2010

Def. A tangent line to a curve is a line that just touches that curve at one point w/in some interval.

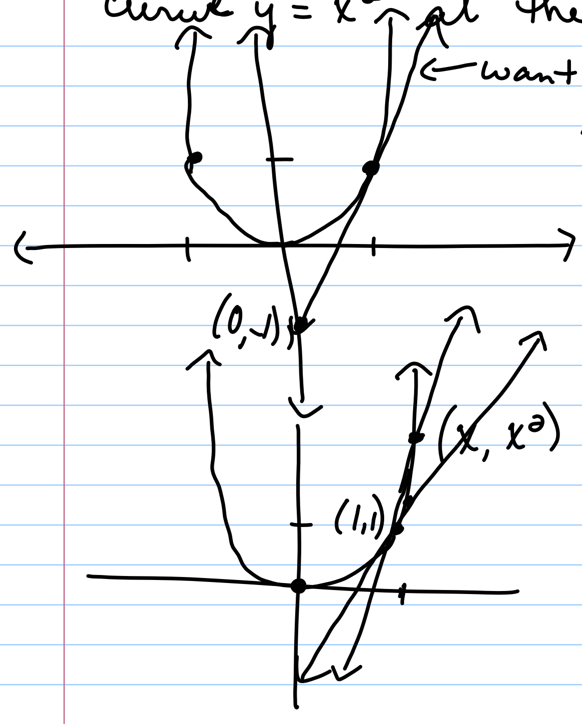
For a circle, we could say that the tangent line touches the circle ~~at~~ only one point



For a different curve this defintion is "no good"



Ex] Find the equation of the tangent line to the curve $y = x^2$ at the point $(1,1)$.



← want the equation of this line.

Problem: we only have one point

the equation of a line

$$y - y_1 = m(x - x_1)$$

$$m = \frac{y - y_1}{x - x_1}$$

$$m = \frac{x^2 - 1}{x - 1}$$ for the secant line

to find the slope of the tangent line we take $x \neq 1$

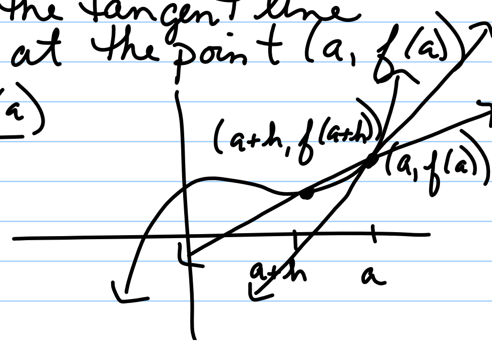
$$\lim_{x\to 1} \frac{f(x)-f(1)}{x-1} = \lim_{x\to 1} \frac{x^2-1}{x-1} = \lim_{x\to 1} \frac{(x-1)(x+1)}{(x-1)} = \lim_{x\to 1} x+1$$

$$= 2$$

$m = 2$ $\quad (1,1) \quad$ $y - 1 = 2(x-1)$

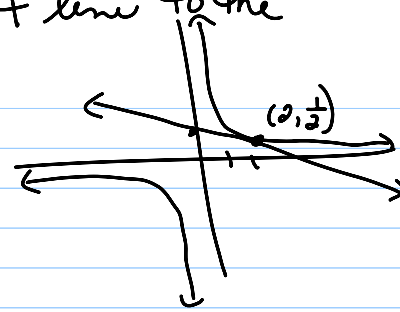
$$\boxed{y = 2x - 1}$$

Def: In general the slope of the tangent line to a given curve $y = f(x)$ at the point $(a, f(a))$ is given by $\lim_{x\to a} \frac{f(x)-f(a)}{x-a}$



Alt Def: $\lim_{h\to 0} \frac{f(a+h)-f(a)}{h}$

Ex) find the equation of the tangent line to the curve $f(x) = \frac{1}{x}$ when $x = 2$



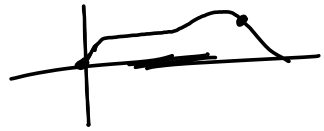
$$\lim_{x\to 2} \frac{\left(\frac{1}{x} - \frac{1}{2}\right)2x}{(x-2)2x} = \lim_{x\to 2} \frac{(2-x)}{2x(x-2)}$$

$x \neq 2$

$$= \lim_{x\to 2} -\frac{\cancel{(x-2)}}{\cancel{(x-2)}2x} = \lim_{x\to 2} -\frac{1}{2x} = -\frac{1}{4} = m$$

$m = -\frac{1}{4}$ $\quad$ point $(2, \frac{1}{2})$ $\quad$ $\boxed{y - \frac{1}{2} = -\frac{1}{4}(x-2)}$ * equation of the line

$\boxed{y = -\frac{1}{4}x + 1}$ equation of the line in slope intercept form the velocity problem:



Average velocity $= \frac{\text{distance}}{\text{time}} = \frac{f(a+h)-f(a)}{h}$

Now, take the velocities over shorter & shorter time intervals. $[a, a+h]$ as $h \to 0$,

Instantaneous velocity $\boxed{\lim_{h\to 0} \frac{f(a+h)-f(a)}{h}} = V(a)$

pg 81 #12 $\quad H = 58t - .83t^2$

a) Find the velocity of the arrow after 1 second.

$$V(1) = \lim_{h\to 0} \frac{H(1+h) - H(1)}{h} = \lim_{h\to 0} \frac{58(1+h) - .83(1+h)^2 - 57.17}{h}$$

$$V(1) = \lim_{t\to 1} \frac{H(t) - H(1)}{t-1} = \lim_{t\to 1} \frac{58t - .83t^2 - 57.17}{t-1}$$

$$V(1) = \lim_{h\to 0} \frac{58 + 58h - .83(1+2h+h^2) - 57.17}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{58} + 58h - \cancel{.83} - 1.66h - .83h^2 - \cancel{57.17}}{h}$$

$h \neq 0$

$$= \lim_{h\to 0} \frac{\cancel{h}(58 - 1.66 - .83h)}{\cancel{h}} = \lim_{h\to 0} (56.34 - .83h)$$

$$= 56.34 \ m/s$$

b) $v(a) = \lim_{h\to 0} \frac{H(a+h) - H(a)}{h} = \lim_{h\to 0} \frac{58(a+h) - .83(a+h)^2 - 58a + .83a^2}{h}$

$= \lim_{h\to 0} \frac{\cancel{58a} + 58h - \cancel{.83a^2} - 1.66ah - .83h^2 - \cancel{58a} + \cancel{.83a^2}}{h}$

$= \lim_{h\to 0} \frac{\cancel{h}(58 - 1.66a - .83h)}{\cancel{h}} = 58 - 1.66a$

The Derivative of f at a is denoted by

$f'(a) = \lim_{h\to 0} \frac{f(a+h) - f(a)}{h}$ = the slope of tangent line

"f prime of a"

<u>if this limit exists</u>

Find the equation of the tangent line to $f(x) = \frac{1}{x}$ when $x = 0$.

$f'(0) = \lim_{h\to 0} \frac{f(0+h) - f(0)}{h} = \lim_{h\to 0} \frac{\frac{1}{h} - \frac{1}{0}}{h}$

D.N.E

#26 $f(x) = \frac{x^2+1}{x-2}$

$(a-2)(a+h-2)$

$f'(a) = \lim_{h\to 0} \frac{\frac{(a+h)^2+1}{a+h-2} - \frac{a^2+1}{a-2}}{h}$

$(a-2)(a+h-2)$

$= \lim_{h\to 0} \frac{((a+h)^2+1)(a-2) - (a^2+1)(a+h-2)}{h(a-2)(a+h-2)}$

$$= \lim_{h \to 0} \frac{(a^2 + 2ah + h^2 + 1)(a-2) - (a^3 + a^2h - 2a^2 + a + h - 2)}{h\,(a-2)\,(a+h-2)}$$

$$=$$