Exam I: Thursday, Sept 23^{rd}

Note Title 9/10/2010

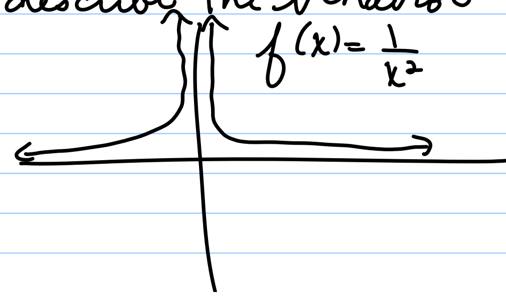
1.6 Limits Involving Infinity

Consider $\lim_{x\to 0} \frac{1}{x^2}$ limit does not exist

Can we describe the behavior of $\frac{1}{x^2}$ as $x \to 0$??

The function values are growing w/out bound therefore we use the notation ∞ to describe the behavior

$\lim_{x\to 0} \frac{1}{x^2}$ d.n.e.



Ex) $f(x) = \frac{2}{x-2}$ $\lim_{x\to 2} f(x) = $ d.n.e

however $\lim_{x\to 2^+} f(x) = \infty$ $\lim_{x\to 2^-} f(x) = -\infty$

Def. The notation $\lim_{x\to a} f(x) = \infty$

means that $f(x) \to \infty$ as $x \to a$

Vertical Asymptotes

The line $x = a$ is a V.A. of f if at least one of the following is true $\lim_{x\to a} f(x) = \pm\infty$ or $\lim_{x\to a^+} f(x) = \pm\infty$

or $\lim_{x\to a^-} f(x) = \pm\infty$

Use your intuition:

$$\lim_{x\to 5} \frac{1}{(x-5)^2} = \infty \qquad \lim_{x\to 3^+} \frac{1}{x-3} = \infty$$

$$\lim_{x\to 3^-} \frac{1}{x-3} = -\infty$$

$$\lim_{x\to 3} \frac{1}{x-3} = \text{d.n.e}$$

Def. the line $y = b$ is a Horizontal Asymptote of f if $\lim_{x\to\infty} f(x) = b$

Ex) $\lim_{x\to\infty} x^3+3x-5 = \infty$ d.n.e.

Ex) $\lim_{x\to\infty} \frac{x^2+2x-3}{x^3+5} = 0$

In general, if n is a positive integer,

$$\lim_{x\to\pm\infty} \frac{1}{x^n} = 0$$

$$\lim_{x\to\infty} \frac{x^2-2x+3}{2x^2-3x+8} \cdot \frac{\frac{1}{x^2}}{\frac{1}{x^2}} = \lim_{x\to\infty} \frac{1-\frac{2}{x}+\frac{3}{x^2}}{2-\frac{3}{x}+\frac{8}{x^2}} = \frac{1}{2}$$

Ex) $\lim_{x\to\infty} \frac{\sqrt{x^2+3}-x}{1} \cdot \frac{\sqrt{x^2+3}+x}{\sqrt{x^2+3}+x}$

$$\lim_{x\to\infty} \frac{x^2+3-x^2}{\sqrt{x^2+3}+x} = \lim_{x\to\infty} \frac{3}{\sqrt{x^2+3}+x} \cdot \frac{\frac{1}{x}}{\frac{1}{x}}$$

$$= \lim_{x\to\infty} \frac{\frac{3}{x}}{\frac{\sqrt{x^2+3}}{x} + 1} = \lim_{x\to\infty} \frac{\frac{3}{x}}{\sqrt{\frac{x^2+3}{x^2}} + 1}$$

$$= \lim_{x\to\infty} \frac{\frac{3}{x}}{\sqrt{1+\frac{3}{x^2}} + 1} = \frac{0}{2} = 0$$

$$\lim_{x\to\infty} x^2 - x = \infty$$

$$\lim_{x\to\infty} x(x-1) = \infty$$

$$\lim_{x\to -2} \sqrt{x^4+3x+6} = \sqrt{16} = 4$$

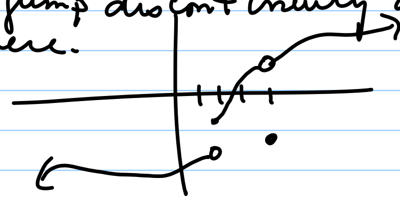
$$\lim_{x\to 1} \left(\frac{1+3x}{1+4x^2+3x^4}\right)^3 = \left(\frac{4}{8}\right)^3 = \left(\frac{1}{2}\right)^3 = \frac{1}{8}$$

$$\lim_{x\to -3} \frac{x^2-9}{2x^2+7x+3} = \lim_{x\to -3} \frac{(x-3)\cancel{(x+3)}}{(2x+1)\cancel{(x+3)}} = \lim_{x\to -3} \frac{x-3}{2x+1} = \frac{6}{5}$$

$$\lim_{x\to 0} \frac{\sqrt{1+x}-1}{x} \cdot \frac{\sqrt{1+x}+1}{\sqrt{1+x}+1} = \lim_{x\to 0} \frac{\cancel{x}}{\cancel{x}(\sqrt{1+x}+1)} = \lim_{x\to 0} \frac{1}{\sqrt{1+x}+1} = \frac{1}{2}$$

$$\lim_{x\to -2}\frac{\left(\frac{1}{x}+\frac{1}{2}\right)2x}{(x+2)\,2x} = \lim_{x\to -2}\frac{\cancel{(2+x)}}{\cancel{(x+2)}\,2x} = \lim_{x\to -2}\frac{1}{2x} = \frac{-1}{4}$$

Sketch the graph of a function w/ a removable discontinuity at $x=4$ a jump discontinuity at $x=2$ but is continuous elsewhere.



Explain why the function $f(x)=\begin{cases}\frac{1}{x-1} & x\neq 1\\ 2 & x=1\end{cases}$

is discontinuous at $x=1$??

Cont $\lim_{x\to a} f(x)=f(a)$ | $\lim_{x\to 1} f(x)$ does not exist

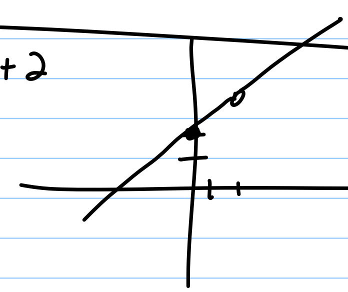
Use the Intermediate Value thm to show that

$f(x)=\sqrt[3]{x}+x-1$ has a root on the interval $(0,1)$.

① $\boxed{f(x)\text{ cont}}$ ② $f(0)=-1<0$

③ $f(1)=1>0$ $\Bigg\}\Longrightarrow$ $f(x)$ has a root on the interval $(0,1)$ by I.V.T.

Ex $f(x)=\dfrac{x^2-4}{x-2}=\dfrac{(x-2)(x+2)}{(x-2)}\overset{x\neq 2}{=}x+2$



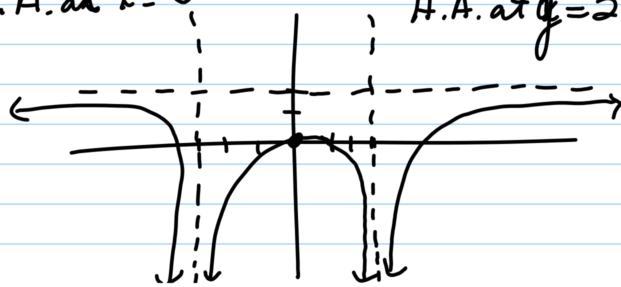
$f(x)=x+2$

$$f(x) = \begin{cases} \dfrac{x^2-4}{x-2} & x \neq 2 \\ 4 & x = 2 \end{cases}$$

Sketch the graph of a function that has the following properties:

$$\underbrace{\lim_{x\to 3} f(x) = -\infty}_{\text{V.A. at } x=3} \quad \underbrace{\lim_{x\to\infty} f(x) = 2}_{\text{H.A. at } y=2} \quad f(0)=0$$

f is even



$$\lim_{x\to\infty} \frac{x+2}{\sqrt{9x^2+1}} = \lim_{x\to\infty} \frac{\sqrt{(x+2)^2}}{\sqrt{9x^2+1}} = \lim_{x\to\infty} \sqrt{\frac{(x+2)^2}{9x^2+1}}$$

$$= \sqrt{\lim_{x\to\infty} \frac{x^2+4x+4}{9x^2+1}} = \sqrt{\frac{1}{9}} = \frac{1}{3}$$

H.W. 1.5 & 1.6