Math 105 - Jen Holmes

Note Title 6/16/2010

Law of Cosines (Solve oblique $\triangle$'s)

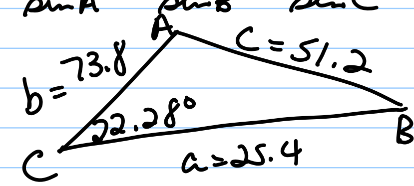
$a^2 = b^2 + c^2 - 2bc\cos A$

$b^2 = a^2 + c^2 - 2ac\cos B$

$c^2 = a^2 + b^2 - 2ab\cos C$

Law of Sines

$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$



$C = 22.28^\circ \quad a = 25.4\text{cm} \quad b = 73.8\text{cm}$

$c^2 = (25.4)^2 + (73.8)^2 - 2(25.4)(73.8)\cos 22.28^\circ$

Kevin $c^2 = 2622.46 \qquad c = 51.2\text{cm}$:)

Rachel $c^2 = - - - \qquad c = 7 - -$:(

Zack $c^2 = - - - - \qquad c = 51.2$:)

Mike $\qquad c = 51.2\text{cm}$:)

$\frac{51.2}{\sin 22.28^\circ} = \frac{25.4}{\sin A}$

$\sin A = \frac{\sin 22.28^\circ \, 25.4}{51.2} =$

$A = 10.84^\circ$

$10.84 + 22.28 + B = 180$

$A = 10.84^\circ \quad a = 25.4\text{cm}$

$B = 146.8^\circ \quad b = 73.8\text{cm}$

$C = 22.28 \quad c = 51.2\text{cm}$

$B = 180 - (10.84 + 22.28)$

$B = 146.8^\circ$

~~Exam 1~~	~~20%~~
Exam 2	20%
Exam 3	20%
HW	15%
Final	25%
Avg	

Exam 1	25%
Exam 2	25%
Exam 3	25%
Final	25%
Avg	*

9.2 # 45

$$4x^2 + y^2 - 8x - 2y + 1 = 0$$

$$4x^2 - 8x + (y^2 - 2y + 1) = -1 + \underline{1}$$

$$4x^2 - 8x + (y-1)^2 = 0$$

$$4(x^2 - 2x + 1) + (y-1)^2 = 0 + \underline{4}$$

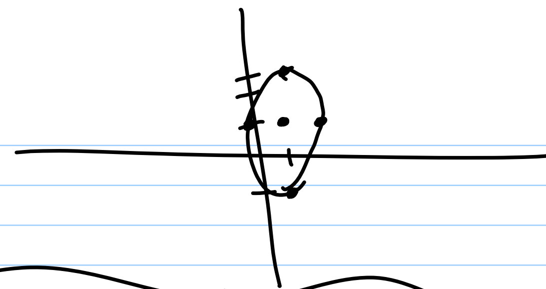
$$\frac{4(x-1)^2}{4} + \frac{(y-1)^2}{4} = \frac{4}{4}$$

$$\frac{(x-1)^2}{1} + \frac{(y-1)^2}{4} = 1$$

Ellipse: Center $(1,1)$

Vertices: $(1,3)\ (1,-1)$

$(2,1)\ (0,1)$

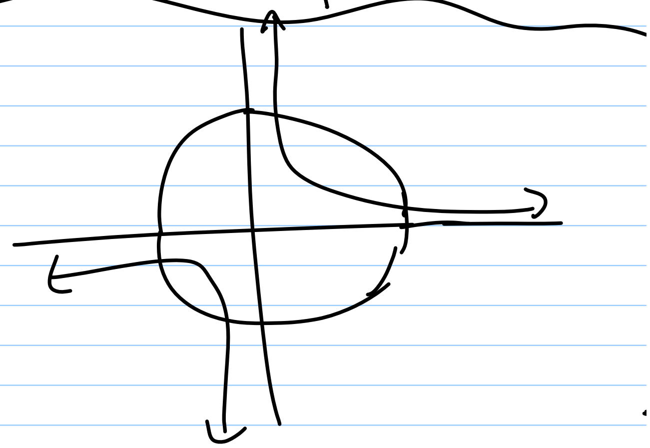


Ex]

$$x^2+y^2=25$$

$$xy=12$$

$$y=\frac{12}{x}$$



$$x^2+\left(\frac{12}{x}\right)^2=25$$

$$x^2\left(x^2+\frac{144}{x^2}\right)=(25)x^2$$

$$x^4+144=25x^2$$

$$x^4-25x^2+144=0$$

$$u=x^2 \Rightarrow u^2=x^4$$

$$u^2-25u+144=0$$

$$(u-9)(u-16)=0$$

$u=9 \qquad u=16$

$x^2=9 \qquad x^2=16$

$x=\pm 3 \qquad x=\pm 4$

144

12 12

3 4 3 4

$$\boxed{(-4,-3)\ (-3,-4)\ (3,4)\ (4,3)}$$

$$\frac{11x^2-8x-7}{(2x^2-1)(x-3)} = \frac{A}{(x-3)} + \frac{Bx+C}{(2x^2-1)}$$

$$11x^2-8x-7 = A(2x^2-1) + (Bx+C)(x-3)$$

$$11x^2-8x-7 = \cancel{2Ax^2} \cancel{-A} + \cancel{Bx^2} - \cancel{3Bx} + \cancel{Cx} - \cancel{3C}$$

$$11x^2-8x-7 = (2A+B)x^2 (-3B+C)x - A - 3C$$

$2A+B=11$

$-3B+C=-8 \Longrightarrow C=3B-8$

$-A-3C=-7 \qquad C=(3(3)-8)=1$

$\boxed{C=1}$

$-A-3(3B-8)=-7$

$-A-9B+24=-7$

$$\begin{cases} -A-9B=-31 \\ 2A+B=11 \end{cases}$$

$-2A-18B=-62$

$-17B=-51$

$\boxed{B=3}$

$2A+3=11$

$2A=8$

$\boxed{A=4}$

$$\frac{11x^2-8x-7}{(2x^2-1)(x-3)} = \frac{4}{(x-3)} + \frac{3x+1}{(2x^2-1)}$$

#66 $\vec{v} = -4i + 2j \qquad \vec{t} = i - 4j$

$\vec{v} = \langle -4, 2 \rangle \qquad \vec{t} = \langle 1, -4 \rangle$

$$\cos\theta = \frac{\vec{v}\cdot\vec{t}}{\|\vec{v}\|\|\vec{t}\|} = \frac{-12}{\sqrt{20}\sqrt{17}} = \frac{-12}{2\sqrt{85}} = \frac{-6}{\sqrt{85}}$$

$$\vec{v}\cdot\vec{t} = -4-8 = -12 \qquad \theta = \cos^{-1}\left(\frac{-6}{\sqrt{85}}\right) = 130.6^\circ$$

$$\|\vec{v}\| = \sqrt{(-4)^2+(2)^2} = \sqrt{20}$$

$$\|\vec{t}\| = \sqrt{(1)^2+(-4)^2} = \sqrt{17}$$

7.1, 7.2 Law of Cosines/Sines (1)

7.5, 7.6. Vector (about 3)

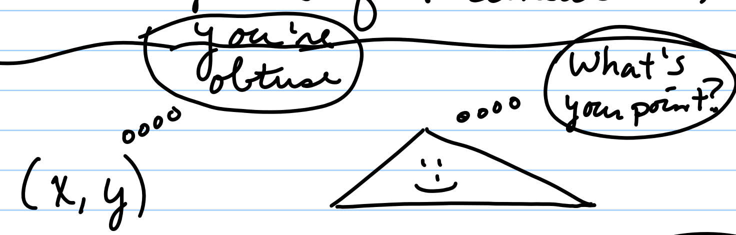
8.1, 8.2 Systems of Linear (1)

8.8 Partial Fractions (1)

9.1 Parabolas (1 sketch)

9.2 Circles & Ellipses (1 of each)

9.4 Systems of Nonlinear (1)



$$\frac{\cot\theta}{\csc\theta-1} = \frac{\csc\theta+1}{\cot\theta}$$

$$\frac{\frac{\cos\theta}{\sin\theta}}{\frac{1}{\sin\theta}-\frac{\sin\theta}{\sin\theta}}$$

$$\frac{\cos\theta}{\cancel{\sin\theta}}\cdot\frac{\cancel{\sin\theta}}{1-\sin\theta} = \frac{\cos\theta}{1-\sin\theta}\boxed{\frac{(1+\sin\theta)}{(1+\sin\theta)}}$$

Crucial Step

$$\frac{\cos\theta(1+\sin\theta)}{1-\sin^2\theta} = \frac{1+\sin\theta}{\cos\theta} = \frac{1+\frac{1}{\csc\theta}}{\frac{1}{\sec\theta}}$$

$$\frac{\frac{\csc\theta+1}{\csc\theta}}{\frac{1}{\sec\theta}} = \frac{\csc\theta+1}{\csc\theta}\cdot\frac{\sec\theta}{1} = \frac{\csc\theta+1\ \frac{1}{\cos\theta}}{\frac{1}{\sin\theta}}$$

$$= \frac{\csc\theta+1}{\cot\theta}$$

$$\frac{\cos^3\beta-\sin^3\beta}{\cos\beta-\sin\beta} = \frac{2+\sin 2\beta}{2}$$

$$(x^3-y^3)=(x-y)(x^2+xy+y^2)$$

$$\frac{\cancel{(\cos\beta-\sin\beta)}(\cos^2\beta+\sin\beta\cos\beta+\sin^2\beta)}{\cancel{(\cos\beta-\sin\beta)}}$$

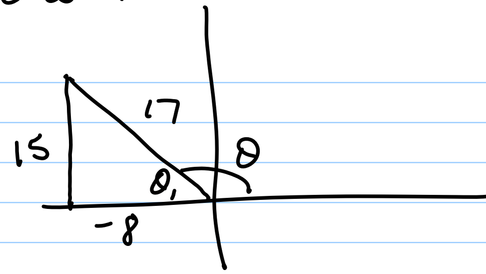
$$1+\sin\beta\cos\beta\left(\frac{2}{2}\right) = \frac{2+2\sin\beta\cos\beta}{2} = \frac{2+\sin 2\beta}{2}$$

$$\frac{1+\cos^2 x}{\sin^2 x} = 2\csc^2 x - 1$$

$$\frac{1+1-\sin^2 x}{\sin^2 x} = \frac{2-\sin^2 x}{\sin^2 x} = \frac{2}{\sin^2 x} - \frac{\sin^2 x}{\sin^2 x}$$

$$\boxed{2\csc^2 x - 1}$$

$\tan\theta = \frac{-15}{8}$ and θ is in Quadrant II



$\sqrt{64+225} = \sqrt{289}$

$\sin\theta = \frac{15}{17}$

$\cos\theta = \frac{-8}{17}$

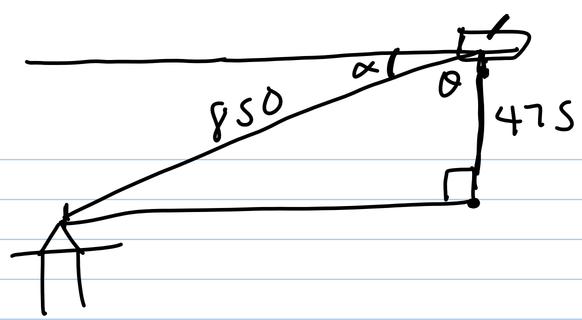
$\sin 2\theta = 2\sin\theta\cos\theta = 2\left(\frac{15}{17}\right)\left(\frac{-8}{17}\right)$
$\underline{-240}$
289

$\cos 2\theta = \cos^2\theta - \sin^2\theta = \left(\frac{-8}{17}\right)^2 - \left(\frac{15}{17}\right)^2$
$\underline{-161}$
289

$\tan 2\theta = \frac{240}{161}$

Where is 2θ?? Quad III

#17 pg 454



$\cos\theta = \frac{475^\circ}{850^\circ}$

$\theta = \cos^{-1}\frac{475^\circ}{850^\circ}$

$\theta = 56.02^\circ$

$\alpha = 90 - 56.02^\circ = 33.98^\circ$

	Domain	Range
$f(x) = \sin x$	$[-\frac{\pi}{2}, \frac{\pi}{2}]$	$[-1, 1]$
$f^{-1}(x) = \sin^{-1}x =$ arcsin x	$[-1, 1]$	$[-\frac{\pi}{2}, \frac{\pi}{2}]$
$f(x) = \cos x$	$[0, \pi]$	$[-1, 1]$
$f^{-1}(x) = \arccos x$ $\cos^{-1}x$	$[-1, 1]$	$[0, \pi]$

$f(x) = \tan x \qquad \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \qquad \mathbb{R}\ (-\infty, \infty)$

$f^{-1}(x) = \tan^{-1}(x) \qquad \mathbb{R} \qquad \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

$= \arctan x$

Find the H.A. of $R(x) = \dfrac{2x^2 - 3x + 4}{x^3 - 8}$

$y = 0$

Find V.A. of $f(x) = \dfrac{(x^2-1)}{(x-1)} = \dfrac{\cancel{(x-1)}(x+1)}{\cancel{(x-1)}} = x+1 \quad x \neq 1$

hole at 1

V.A. at -1

$f(x) = \dfrac{(x-1)}{(x^2-1)} = \dfrac{\cancel{(x-1)}}{\cancel{(x-1)}(x+1)} = \dfrac{1}{x+1} \quad x \neq 1$

$\boxed{I(.2) + II(.2) + III(.2) + IV(.15)} + x(.25) \geq 80$

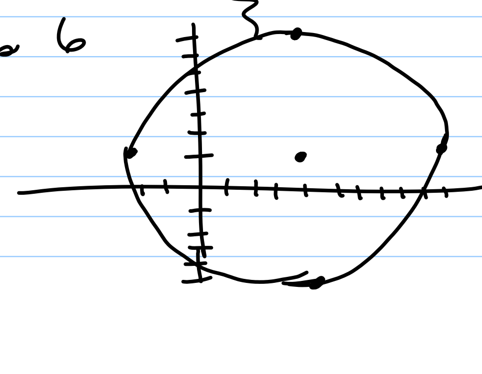
#

$x^2 + y^2 - 8x - 2y - 19 = 0$

$(x^2 - 8x + \underline{16}) + (y^2 - 2y + \underline{1}) = 19 + \underline{16} + \underline{1}$

$(x-4)^2 + (y-1)^2 = 36$

Center $(4, 1)$ Radius 6



$$x^2 + 2y^2 - 10x + 8y + 29 = 0$$

$$(x^2 - 10x + \underline{25}) + (2y^2 + 8y) = -29 + 25$$

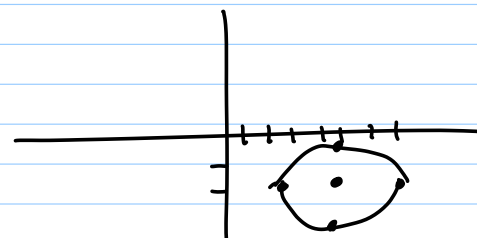
$$(x-5)^2 + (2y^2 + 8y) = -4$$

$$(x-5)^2 + 2(y^2 + 4y + \underline{4}) = -4 + \underline{8}$$

$$\frac{(x-5)^2}{4} + \frac{2(y+2)^2}{4} = \frac{4}{4}$$

$$\frac{(x-5)^2}{4} + \frac{(y+2)^2}{2} = 1$$

$(5, -2)$



$$\frac{(x-h)^2}{r^2} + \frac{(y-k)^2}{r^2} = \frac{r^2}{r^2}$$

$$\frac{(x-h)^2}{r^2} + \frac{(y-k)^2}{r^2} = 1$$

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1 \qquad (h, k)$$

$\sqrt{a}$ x direction from center	$\sqrt{b}$ y direction from center