

Differential Equations
Study Guide for Final
Chapters 1, 2, 3, 5

You may use one half-page (both sides) of notes for this exam.

Concepts and Definitions

Chapter One

- Mathematical models
- Integral curves
- Slope fields
- Initial value problems
- Classification of differential equations

Chapter Two

- Separable equations
- Linear equations and integrating factors
- Exact equations; how to tell if an equation is exact
- Autonomous equations; the logistic equation
- Mathematical models: population, decay, investment, falling body and concentration
- Equilibrium
- Existence and Uniqueness theorem: know what it says

Chapter 3

- Homogeneous equation with constant coefficients
- Characteristic equation
- The Wronskian
- Fundamental set of solutions
- Linear independence/dependence
- Nonhomogeneous equations

Chapter 5

- Power series; radius of convergence; interval of convergence
- Ordinary point
- Singular point; regular and irregular
- Recurrence relation
- Analytic function

Methods and Solutions

Chapter Two

- Verify that a function $y = \phi(x)$ is a solution
- Solve separable, linear and exact equations
- Set up and solve a mathematical model
- Graph the general class of solutions and some particular cases. Find limits of solution curves; find equilibrium solutions and phase lines

Chapter 3

- Solve the second order homogeneous linear differential equation with constant coefficients. (Three cases: real, imaginary and repeated roots of the characteristic equation).
- Solve the second order nonhomogeneous linear differential equation, using either the method of undetermined coefficients or variation of parameters. Know when you can and cannot use the method of undetermined coefficients.
- Show that two functions are linearly dependent or independent. Show that a pair of functions forms a fundamental set of solutions.

Chapter 5

- Determine the coefficients (a_n) of a series solution around an ordinary point, using the recurrence relations
- Find the radius of convergence of the series solution, if it is written in closed form
$$\left(\sum_{n=0}^{\infty} a_n (x - x_0)^n \right)$$
- Determine whether a singularity is regular or irregular
- Solve a second order equation $P(x)y'' + Q(x)y' + R(x)y = 0$ around a regular singular point.

Selected Problems from Text

p. 8: 15-20, 24

p. 17: 12

p. 39: 7, 11, 27, 34, 35

p. 47: 8, 9, 22, 23

p. 59: 2, 7, 12, 13

p. 76: 23, 24 (proof)

p. 89: 11, 13, 14, 15

p. 100: 10, 19, 18 (proof)

p. 142: 5, 9, 28

p. 151: 5, 16, 17, 24

p. 158: 2, 10, 13, 27 (proof)

p. 164: 7, 22, 30 (proof), 32 (proof), 38 (proof)

p. 172: 3, 13, 21 (proof), 28

p. 184: 7, 16

p. 190: 3, 8, 13

p. 259: 10, 23

p. 271: 3, 6, 12, 21 (done in class), 23, 27

You should also look at the previous review problems, and homework.