Review Problems

1. Use Euler's method to find approximate values of the solution of the IVP at $t = 0.5, 1, 1.5, 2, 2.5, \text{and } 3$, with $h = 0.05$

   $y' = -ty + 0.1y^3, y(0) = 1$

2. Solve $y'' + y' - 2y = 0$

3. Solve $2y'' - y' - 3y = 0$, $y(1) = 1$ and $y'(1) = 1$. Sketch the graph and describe $\lim_{t \to \infty} y$

4. Solve $y'' - 2y' + y = 0$

5. Solve $y'' + y' + y = 0$

6. Solve $y'' + 4y = 3t\cos(2t)$

7. Given $(x-1)y'' - xy' + y = 0,\ x > 1$ and $y_1(x) = e^x$, use the method of reduction of order to find $y_2(x)$.

8. Solve $y'' - 3y' = xe^{3x}$

9. Solve $y'' + y = \tan x$

10. Use variation of parameters to solve $y'' + \frac{x}{1-x}y' - \frac{1}{1-x}y = 1 - x$, given that x and e^x are solutions to the homogeneous equation.

11. Prove that $y = t$ and $y = \ln(t)$ are linearly independent on the interval $(0, \infty)$.

12. Define $L[y]$ to be the differential equation $y'' + p(t)y' + q(t)y = 0$. Prove that $L[y_1 + y_2] = L[y_1] + L[y_2]$ and that $L[ay] = aL[y]$.

13. Show that if $y = \varphi(t)$ is a solution of the differential equation $y'' + p(t)y' + q(t)y = g(t)$, where $g(t)$ is not always zero, then $y = c\varphi(t)$, where c is a constant other than one, is not a solution.

14. Prove that if y_1 and y_2 have maxima or minima at the same point in *I*, then they cannot be a fundamental set of solutions on that interval.

15. If the roots of the characteristic equation are real, show that a solution of $ay'' + by' + cy = 0$ can take on the value zero at most once. (Hint: consider the two cases with real roots separately.)