Math 136 Calculus II

Exam Two Review: 7.2, 7.3, 8.1 – 8.4

Volume

Terms to know:
- solid of revolution
- disk, washer
- cylindrical shell
- cross-section

Be able to:
- Set up and evaluate an integral to calculate the volume of a solid of revolution, using either method.
- Describe a solid of revolution, given an integral.
- Describe the differences between the disk/washer method and the shell method

Sequences

Terms to know:
- sequence
- recursive sequence
- limit of a sequence
- increasing sequence, decreasing sequence, monotonic sequence
- bounded above, bounded below

Facts & Properties:
- The sequence $\{r^n\}$ converges to 0 if $-1 < r < 1$, converges to 1 if $r = 1$, and diverges otherwise
- If $\lim_{n\to\infty} |a_n| = 0$ then $\lim_{n\to\infty} a_n = 0$
- The Monotonic Sequence Theorem: A sequence that is bounded above and increasing converges; a sequence that is bounded below and decreasing converges. Every bounded, monotonic sequence converges.
- Squeeze Theorem for sequences

Be able to:
- list the terms of a sequence
- find the rule for a sequence from a list of terms
- show that a sequence is monotonic and/or bounded
- use the Monotonic Sequence Theorem to show that a sequence converges
- determine the limit of a sequence, if if exists
  - correctly apply l'hopital's rule to find the limit of a sequence
  - correctly apply the Squeeze Theorem to find the limit of a sequence
- plot the terms of a sequence

Series

Terms to know:
- series (and how a series is different from a sequence; in particular, know that we do not find the "limit" of a series, but find the sum of a convergent series)

- sequence of partial sums, $\{S_n\}$; be able to list terms of this sequence
- sequence of terms, $\{a_n\}$
- convergent series/divergent series
  - the *sum* of a series is the limit of the partial sums, $s = \lim_{n\to\infty} S_n$
  - **If $\lim_{n\to\infty} S_n$ exists, then the series $\sum_{n=1}^{\infty} a_n$ converges to the same value, and if $\lim_{n\to\infty} S_n$ does not exist then the series diverges.
- geometric series, ratio, leading term, sum
- harmonic series
- telescoping series
- *p*-series
- alternating series
  - absolute convergence, conditional convergence (*only* an alternating series can be conditionally convergent)

Facts & Properties:

- If $\sum a_n$ and $\sum b_n$ are convergent series, then so are $\sum ca_n$ and $\sum (a_n \pm b_n)$, and $\sum ca_n = c\sum a_n$ and $\sum (a_n \pm b_n) = \sum a_n \pm \sum b_n$
- If $\sum a_n$ converges, then $\lim_{n\to\infty} a_n = 0$ (but the converse is *not* true – to show a series converges requires more than showing $\lim_{n\to\infty} a_n = 0$).
- Test for Divergence: if $\lim_{n\to\infty} a_n \neq 0$, then the series $\sum a_n$ diverges.
- If $\sum |a_n|$ converges, then $\sum a_n$ is absolutely convergent
- The error given by the Integral Test: If $f(x)$ is continuous, positive and decreasing, then

  (1) $\int_{n+1}^{\infty} f(x)\,dx < R_n < \int_{n}^{\infty} f(x)\,dx$

  (2) $S_n + \int_{n+1}^{\infty} f(x)\,dx < S < \int_{n}^{\infty} f(x)\,dx + S_n$
- The Alternating Series Estimation Theorem: If $s = \sum (-1)^{n-1} a_n$ is the sum of an alternating series for which $a_{n+1} \le a_n$, then $|R_n| = |S - S_n| \le a_{n+1}$

Be able to:

- identify a geometric series and determine whether it converges or diverges; find the sum of a convergent geometric series
- use partial fractions to rewrite the terms of a series whose n^{th} term is rational; determine the n^{th} partial sum and whether the series converges (or telescopes)
- determine whether the Integral Test applies to a given series; use the Integral Test to determine whether the series converges.
- Use the Integral Test to estimate the sum of a series to which the Integral Test applies
- use the Direct Comparison or Limit Comparison Test to determine the convergence or divergence of a series
- use the Ratio or Root Test to determine the convergence or divergence of a series

- use the Alternating Series Test to determine whether an alternating series that is not absolutely convergent is conditionally convergent
- use the Alternating Series Estimation Theorem to estimate the sum of a convergent alternating series within a stated degree of accuracy

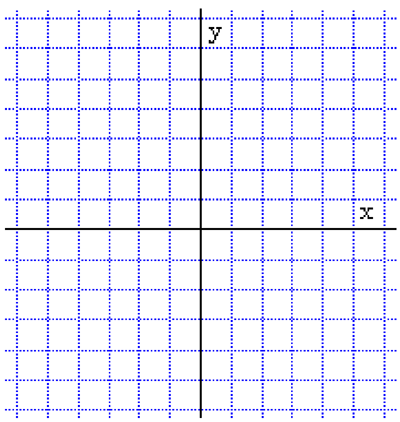
Chapter 7 Review, pp. 407 – 409 Concept Check: 3, 4
Exercises: 5 – 14, 17 - 24

Chapter 8 Review pp. 479 - 481 Concept Check: 1 – 7, True/False: 1, 3, 7, 8, 9, 11, 12, 14, 16 17, Exercises: 1 – 29 odd, 30, 33, 34

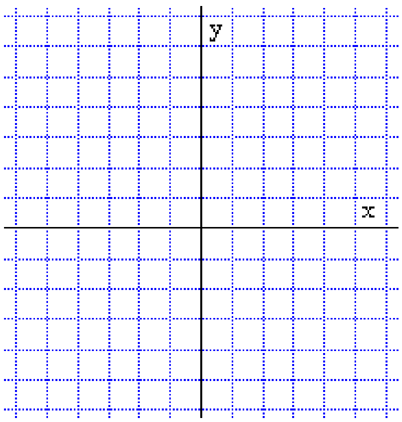
Be sure to study all homework problems (suggested and assigned), as well as quiz problems. Problems like the following may appear on the exam: Section 8.1: 31, 40, Section 8.2: 30, 31, Section 8.3: 35, 36, 37, Section 8.4: 18

Practice Problems

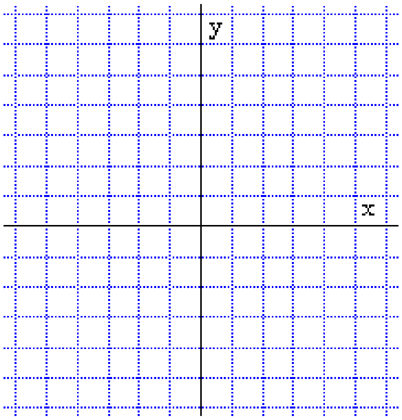
1. Find the volume of the solid obtained by rotating the region bounded by the lines $y = 2x$, $y = 6$, and $x = 0$ about the *y*-axis.



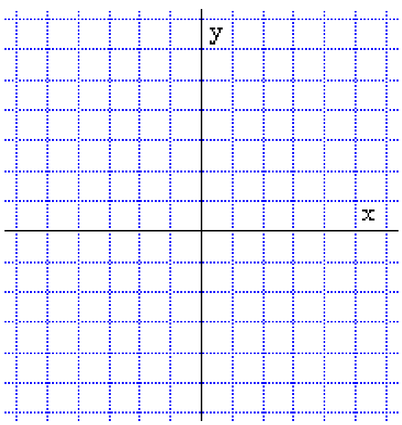
2. Rotate the region from # 1 about the line $x = 3$, then find the volume of the resulting solid.



3. Use cylindrical shells to find the volume of the solid obtained by rotating the region bounded by the graphs of $y = x^2$ and $y = x$ about the line x = -1.



4. Rotate the region bounded by the curves $y = \cos x$ and $y = \sin x$ over the interval $\left[0, \frac{\pi}{4}\right]$ around the x-axis, and find the volume of the resulting solid. (Hint: you may want to use a trigonometric identity to integrate).



5. Determine whether each of the following series converges or diverges:

a. $\sum_{n=0} \frac{n!}{2 \cdot 5 \cdot 8 \cdot \ldots \cdot (3n+2)}$

b. $\sum_{k=1} \frac{2^k k!}{(k+2)!}$

c. $\sum_{n=1} n^2 e^{-n^3}$

d. $\sum_{n=1} \sin(n)$

e. $\sum_{n=1} (-1)^n \frac{\ln n}{\sqrt{n}}$

f. $\sum_{n=1} \tan(1/n)$

g. $\sum_{k=1} \frac{5^k}{3^k + 4^k}$

h. $\sum_{k=1} \frac{k+5}{5^k}$

i. $\sum_{n=1} \frac{e^{1/n}}{n^2}$

j. $\sum_{j=1} (-1)^j \frac{\sqrt{j}}{j+5}$

6. Determine whether the following series converge or diverge. If a series converges, find its sum:

a. $\sum_{n=0}^{\infty} 5\left(\frac{1}{3}\right)^n$

b. $\sum_{n=1}^{\infty} \frac{6}{n(n+3)}$

c. $8+6+\frac{9}{2}+\frac{27}{8}+\cdots$ (Hint: this is a geometric series)

d. $\sum_{n=1}^{\infty} \frac{3\sqrt{n^2+2}}{2n+4}$

e. $\sum_{n=0}^{\infty} 10\left(\frac{2^{2n+1}}{3^n}\right)$

7. Determine whether the following sequences converge or diverge. If a sequence converges, find its limit.

a. $\left\{\frac{n!}{(n+2)!}\right\}$

b. $\left\{\frac{\cos^2 n}{2^n}\right\}$

c. $\left\{\frac{n}{1+\sqrt{n}}\right\}$

8. Determine whether the sequence $a_n = \frac{3n^2}{n^2+2}$ is monotone, and whether it is bounded.

9. Estimate $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n!}$ to four decimal places.