Section 2.6
Solutions

4. $x^2 - 2xy + y^3 = C$ Find $\frac{dy}{dx}$.

$2x - 2y - 2xy' + 3y^2 y' = 0 \quad \Rightarrow \quad \frac{dy}{dx} = \frac{2y - 2x}{3y^2 - 2x}$

5. $x^2 y + xy^2 = 3x$

$2xy + x^2 y' + y^2 + 2xyy' = 3 \quad \Rightarrow$

$y' = \frac{3 - 2xy - y^2}{x^2 + 2xy}$

$y^5 + x^2 y^3 = 1 + x^4 y$

6. $5y^4 y' + 2xy^3 + 3x^2 y^2 y' = 4x^3 y + x^4 y' \Rightarrow$

$y' = \frac{4x^3 y - 2xy^3}{5y^4 + 3x^2 y^2 - x^4}$

$y \sin(x^2) = x \sin(y^2)$

10. $y' \sin(x^2) + 2xy \cos(x^2) = \sin(y^2) + 2xyy' \cos(y^2) \Rightarrow$

$y' = \frac{\sin\left(y^2\right) - 2xy\cos\left(x^2\right)}{\sin\left(x^2\right) - 2xy\cos\left(y^2\right)}$

20. Write the equation of the tangent line to $x^{2/3} + y^{2/3} = 4$ at the point $\left(3\sqrt{3}, 1\right)$.

Find y': $\frac{2}{3}x^{-1/3} + \frac{2}{3}y^{-1/3}y' = 0 \Rightarrow y' = -\frac{y^{1/3}}{x^{1/3}}$, so the slope is $-\left(\frac{1}{\left(-3\sqrt{3}\right)^{1/3}}\right) = \frac{1}{\sqrt{3}}$.

Equation of the tangent line: $y - 1 = \frac{1}{\sqrt{3}}(x + 3\sqrt{3})$

or $y = \frac{1}{\sqrt{3}}x + 4$

26. $x^4 + y^4 = a^4$

$4x^3 + 4y^3 y' = 0 \Rightarrow y' = \frac{-x^3}{y^3}$. Then $y'' = \frac{-3x^2 y^3 + 3y^2 x^3 y'}{y^6} = \frac{-3x^2 y^3 + 3y^2 x^3\left(\frac{-x^3}{y^3}\right)}{y^6}$

$y'' = \frac{-3x^2\left(y^4 - x^4\right)}{y^7}$.

28. (a) Write the equation of the tangent line to $y^2 = x^3 + 3x^2$ at the point (1,-2).

$2yy' = 3x^2 + 6x$, so $-4y' = 9,$ so $y' = -\frac{9}{4}$. Then $y + 2 = -\frac{9}{4}(x-1),$ or $9x + 4y = 7$

(b) Curve has horizontal tangent lines when $y' = 0$, so $0 = 3x^2 + 6x$, or $x = 0$ or $x = -2$. When $x = 0$, $y = 0$, which makes the derivative undefined. When $x = -2$, $y^2 = 4,$ or $y = \pm 2$, thus (-2,2) and (-2,-2) are the points where the tangent lines are horizontal.

(c)