

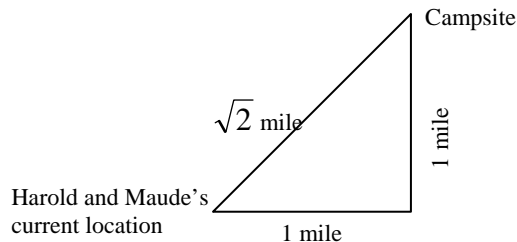
Sample Project Solution

The Shortest Path

In this project, we will consider different ways of calculating distance along a curved path.

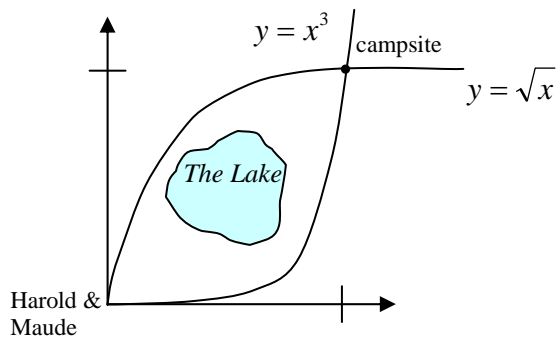
1. Harold and Maude are hiking and they want to make camp at a campsite that is one mile east and one mile north of their current location. How far (as the crow flies) are they from the campsite?

We can represent the situation with the following diagram:



From the diagram, we can see that the length of each of the sides is one mile. Thus the Pythagorean Theorem gives us the length of the hypotenuse as $\sqrt{2}$, and we know that the “crow flies” distance is $\sqrt{2}$ miles.

2. When Harold and Maude look at their map, they see that there is a lake between the campsite and their current location. If they follow the path $y = \sqrt{x}$ (the Radical Road) they will skirt the northern edge of the lake. If they follow the path $y = x^3$ (the Cubic Camp Trail) they will skirt the southern edge of the lake.
 - (a) Draw a picture of the situation.
 - (b) Explain why both paths will reach the campsite.
- (a) If we locate Harold and Maude’s current location at the origin, the campsite is at the point (1,1). The graphs of the functions $y = \sqrt{x}$ and $y = x^3$ correspond to the two paths:



- (b) Since both of the functions pass through the point (1,1), we know both paths will go to the campsite.
3. Deciding to estimate the length of each path, they do the following. Each path has a rest stop located at a point on the path which is $\frac{1}{2}$ mile east and some distance north of the starting point.
- (a) How far north is each rest stop from their starting position?
 For both paths, the rest stops are located at the point $(\frac{1}{2}, y)$. On the Cubic Camp Trail, $y = x^3$, so the point is $(\frac{1}{2}, \frac{1}{8})$. Thus the rest stop is $\frac{1}{8}$ or .125 mile north. On the Radical Road, $y = \sqrt{x}$, so the point is $(\frac{1}{2}, \frac{1}{\sqrt{2}})$. This rest stop is approximately .707 miles north.
- (b) How far (as the crow flies) is each rest stop from their starting position?
 Using the distance formula, we find that the distance for the Cubic Camp Trail is $d = \sqrt{(\frac{1}{2})^2 + (\frac{1}{8})^2} = \frac{\sqrt{17}}{8}$ or approximately .515 mile. On the Radical Road, the distance is $d = \sqrt{(\frac{1}{2})^2 + (\frac{1}{\sqrt{2}})^2} = \frac{\sqrt{3}}{2}$ or approximately .8660 mile.
- (c) How far (as the crow flies) is each rest stop from the campsite?
 Again using the distance formula, we can calculate the distance from the rest stops (at $(\frac{1}{2}, y)$) and the camp site at (1,1). On the Cubic Camp Trail, this distance is $\sqrt{(1 - \frac{1}{2})^2 + (1 - \frac{1}{8})^2} = \frac{\sqrt{65}}{8}$ or approximately 1.01 miles. On the Radical Road, the distance is $d = \sqrt{(1 - \frac{1}{2})^2 + (1 - \frac{1}{\sqrt{2}})^2} \approx \sqrt{.25 + .0858} = .5794$ mile.
- (d) Now give an estimate for the total distance along each path to the campsite. Is your estimate larger than the actual distance along the path (an upper estimate) or smaller than the actual distance along the path (a lower estimate) or neither? Explain.

We can estimate the total distance along each path by adding the two distances found above for each path. Along the Cubic Camp Trail the total distance is approximately $.515 + 1.01 = 1.525$ miles. Along the Radical Road it is $.8660 + .5794 = 1.4454$ miles. Since all of the estimated distances are distances along line segments rather than the curves, they are both underestimates.