Math 135
Calculus I

Exam One Extra Review Problems - Answers

1. a. $\lim_{t\to 2}\frac{t^2-3t+2}{t^2-4}=\lim_{t\to 2}\frac{(t-2)(t-1)}{(t-2)(t+2)}=\frac{1}{4}$

b. $\lim_{x\to\pi} x\sin x=\pi\sin\pi=0$

c. $\lim_{x\to 3} 2x^2-2x+4=16$

d. $\lim_{x\to 4^+}\sqrt{16-x^2}=0$

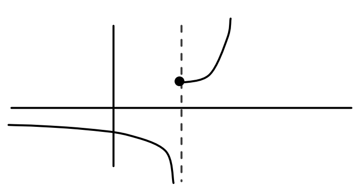
e. $\lim_{x\to 0}\frac{\sin 3x}{\tan 5x}=\lim_{x\to 0}\sin 3x\cot 5x=\lim_{x\to 0}\frac{\sin 3x}{\sin 5x}\frac{\cos 5x}{1}=\lim_{x\to 0}\frac{\sin 3x}{\sin 5x}\lim_{x\to 0}\cos 5x=$
$\lim_{x\to 0}\frac{\sin 3x}{\sin 5x}\left(\frac{3\cdot 5\cdot x}{3\cdot 5\cdot x}\right)\cdot 1=\frac{3}{5}\lim_{x\to 0}\frac{\sin 3x}{3x}\cdot\frac{5x}{\sin 5x}=\frac{3}{5}\lim_{x\to 0}\frac{\sin 3x}{3x}\cdot\lim_{x\to 0}\frac{5x}{\sin 5x}=\frac{3}{5}\cdot 1\cdot 1=\frac{3}{5}$

f. $\lim_{x\to 3}\frac{2\sin(x-3)}{3(x-3)}=\lim_{x\to 3}\frac{2}{3}\cdot\frac{\sin(x-3)}{x-3}=\frac{2}{3}\cdot 1=\frac{2}{3}$

g. $f(x)=\begin{cases}2x & x<2\\ x^2 & x\ge 2\end{cases}$ $\qquad \lim_{x\to 1} f(x)=2$

h. $\lim_{x\to 0}\frac{\sqrt{x+5}-\sqrt{5}}{x}=\lim_{x\to 0}\frac{\sqrt{x+5}-\sqrt{5}}{x}\left(\frac{\sqrt{x+5}+5}{\sqrt{x+5}+5}\right)=\lim_{x\to 0}\frac{x}{x\left(\sqrt{x+5}+\sqrt{5}\right)}=$
$\lim_{x\to 0}\frac{1}{\sqrt{x+5}+\sqrt{5}}=\frac{1}{2\sqrt{5}}$

i. $f(x)=\begin{cases}\frac{1}{x-1} & x<1\\ x^2+1 & x\ge 1\end{cases}$



(i) $\lim_{x\to 1^+} f(x)=2$ (ii) $\lim_{x\to 1^-} f(x)=-\infty$ (iii) $\lim_{x\to 1} f(x)$ does not exist b/c the left-hand and right-hand limits are not the same.

2. $f(x)=\begin{cases} \dfrac{1}{\sqrt{1-x^2}} & 0\le x<1 \\ x^2 & x\ge 1 \\ 1 & x<0 \end{cases}$ Is f continuous at x = 1? No. Checking the three conditions: (i) $f(1)=1$, (ii) $\lim_{x\to 1^-} f(x)$ does not exist. Since the function fails condition (ii), it is not continuous at x = 1.

f is continuous at $x=0$ since $\lim_{x\to 0^-} f(x)=\lim_{x\to 0^+} f(x)=1=f(0)$

3. $f(x)=\sqrt{1-x^2}$ is continuous on its domain (according to one of our continuity theorems). To find the domain, solve:

$$\begin{aligned}1-x^2&\ge 0\\ x^2-1&\le 0\\ (x-1)(x+1)&\le 0\end{aligned}$$

. Testing x-values in the intervals $(-\infty,-1)$, $(-1,1)$ and $(1,\infty)$, we find the domain is $(-1,1)$. (Note that even though x = -1 and x = 1 are in the domain, f is not continuous at those values. Why not?)

4. $f(x)=\sin(x^2+2)$ is continuous for all values of x.

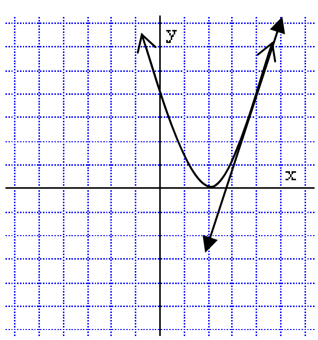
5. a. $f(x)=\dfrac{x}{4-x^2}=\dfrac{x}{(2-x)(2+x)}$ has discontinuities at x = 2 and x = -2. Both are vertical asymptotes.

b. $f(x)=\dfrac{x^2-6x-7}{x+1}=\dfrac{(x-7)(x+1)}{x+1}$ has a removable discontinuity at x = -1.

6. a. Using $m_{\tan}=f'(4)=\lim_{h\to 0}\dfrac{f(4+h)-f(4)}{h}$, we find m = 4.

b. $y-4=4(x-4)$, or $y=4x-12$

c.



7. a. $f'(x) = \lim_{h \to 0} \dfrac{(1-(x+h))^2 - (1-x)^2}{h} = \cdots = -2(1-x)$

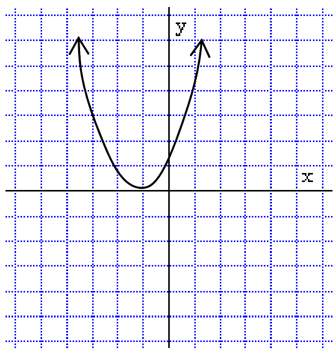
b. $f'(x) = \lim_{h \to 0} \dfrac{\dfrac{3}{x+h+1} - \dfrac{3}{x+1}}{h} = \cdots = -\dfrac{3}{(x+1)^2}$

c. $f'(x) = \lim_{h \to 0} \dfrac{\sqrt{3(x+h)+1} - \sqrt{3x+1}}{h} = \cdots = \dfrac{3}{2\sqrt{3x+1}}$

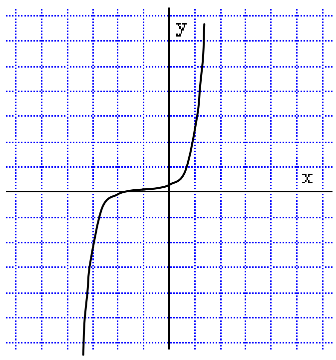
8. The function is not differentiable at $x = 2$ or $x = -3$.

9.

$y = f'(x)$

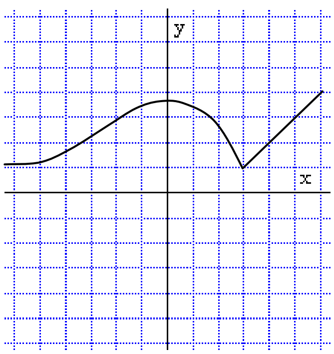


$y = f(x)$



10.

$y = f(x)$



$y = f'(x)$