Extra Review Problems for
Exam One

1. Find the following limits:

   (a) $\lim_{t\to 2}\dfrac{t^2-3t+2}{t^2-4}$ (b) $\lim_{x\to\pi} x\sin x$ (c) $\lim_{x\to 3} 2x^2-2x+4$

   (d) $\lim_{x\to 4^+}\sqrt{16-x^2}$ (e) $\lim_{x\to 0}\dfrac{\sin(3x)}{\tan(5x)}$ (f) $\lim_{x\to 3}\dfrac{2\sin(x-3)}{3(x-3)}$

   (g) $\lim_{x\to 1} f(x)$, where $f(x)=\begin{cases}2x & \text{if } x<2\\ x^2 & \text{if } x\ge 2\end{cases}$ (h) $\lim_{x\to 0}\dfrac{\sqrt{x+5}-\sqrt{5}}{x}$

   (i) Graph $f(x) = \begin{cases}\dfrac{1}{x-1}; & x<1\\ x^2+1; & x\ge 1\end{cases}$

   (i) find $\lim_{x\to 1^+} f(x)$

   (ii) find $\lim_{x\to 1^-} f(x)$

   (iii) does $\lim_{x\to 1} f(x)$ exist? Why or why not?

2. Given $f(x)=\begin{cases}1 & x<0\\ \dfrac{1}{\sqrt{1-x^2}} & 0\le x\le 1\\ x^2 & x>1\end{cases}$

   Is $f(x)$ continuous at $x=1$? at $x=0$? Why or why not? Justify your answer using the definition of continuity.

3. Find all x-values where $f(x)=\sqrt{1-x^2}$ is not continuous.

4. Determine the intervals over which $f(x)=\sin(x^2+2)$ is continuous.

5. At what x-values do the following rational functions have discontinuities? For each such value, decide whether the discontinuity is a vertical asymptote or a removable discontinuity.

   (a) $f(x)=\dfrac{x}{4-x^2}$

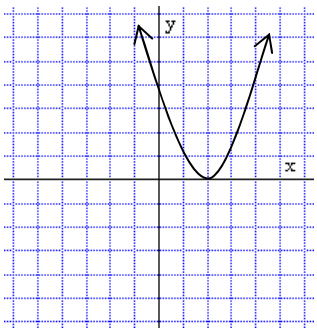
   (b) $f(x)=\dfrac{x^2-6x-7}{x+1}$

6. Let $f(x)=x^2-4x-4$.

   (a) Find the slope of the tangent line to the graph of $y=f(x)$ at $x=4$.

   (b) Write the equation of the tangent line to $y=f(x)$ at $x=4$.

(c) Sketch the tangent line below:

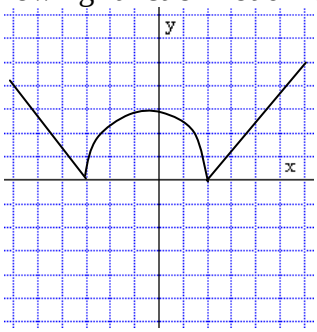


7. Find the derivatives of the following functions, using the *limit definition* of the derivative.

(a) $f(x) = (1-x)^2$ (b) $f(x) = \dfrac{3}{x+1}$

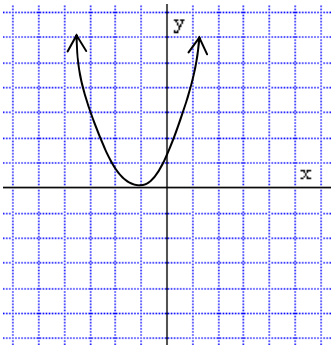
(c) $f(x) = \sqrt{3x+1}$

9. At what point(s) is the following function not differentiable?

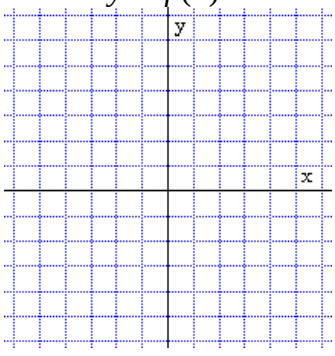


10. Use the given graph of $f'(x)$ to sketch a possible graph of $f(x)$.

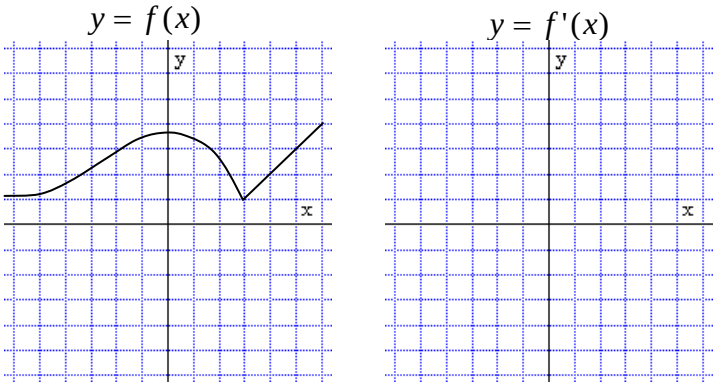
$y = f'(x)$



$y = f(x)$



11. Use the given graph of $y = f(x)$ to sketch a graph of $y = f'(x)$.



Remember to review homework and quiz problems too – they are fair game for test questions.