Extra Review Problems
Exam Two

1. Find the derivative of each of the following functions, using the appropriate derivative rule(s).

a. $f(t) = 3t^3 - 2\sqrt{t}$

b. $h(x) = \dfrac{10}{\sqrt{x}} - 2x$

c. $f(x) = \dfrac{3x-2}{5x+1}$

d. $f(x) = \dfrac{x^2}{3} + \dfrac{5}{x^2}$

e. $f(x) = x\sin x$

f. $f(x) = \dfrac{\tan x}{x^2}$

g. $y = \sin^2(2x)$

h. $f(x) = x^{2/3}(x^2 - 2)(x^3 - x + 1)$

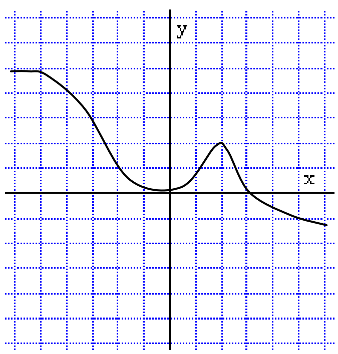
i. $f(x) = \sqrt{x^2 + 4}$

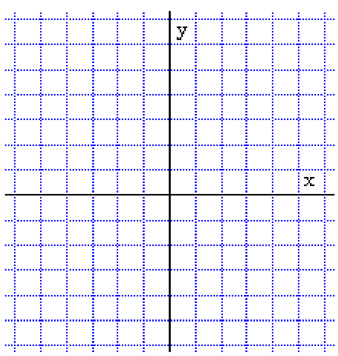
j. $f(x) = \sec(4x)\tan(4x)$

k. $f(x) = \cos\sqrt{x^2 + 1}$

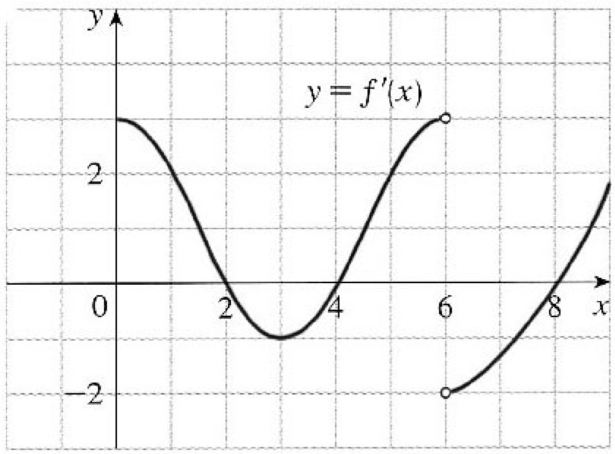
2. Find $f''(x)$ and $f'''(x)$ for $f(x) = x^3 - 6x + \dfrac{2}{x}$.

3. Use the given graph of $f(x)$ to sketch a possible graph of $f'(x)$.





4. The graph of $f'(x)$ of a continuous function $f(x)$ is shown.
   (a) On what intervals is *f* increasing? How do you know?
   (b) On what intervals is *f* decreasing? How do you know?
   (c) Using the graph of $f'(x)$, is it possible to tell where *f* crosses the *x*-axis? Explain why or why not.



5. A boat is pulled in to a dock by means of a rope with one end attached to the bow of the boat, the other end passing through a ring attached to the dock at a point 4 feet higher than the bow of the boat. If the rope is pulled in at the rate of 2 ft/sec, how fast is the boat approaching the dock when 10 feet of rope are out?

6. A balloon is 200 feet off the ground and rising vertically at the constant rate of 15 feet per second. An automobile passes beneath it traveling along a straight road at the constant rate of 45 mph (or 66 feet per second). How fast is the distance between them changing one second later?

7. When air changes volume adiabatically (without any heat being added to it), the pressure *p* and the volume *v* satisfy the relationship $pv^{1.4}$ =constant. At a certain instant the pressure is 50 lb/in^2 and the volume is 35 in^3 and is decreasing at the rate of 4 in^3/sec. How rapidly is the pressure changing at that instant?

8. The height of a ball (in feet) *t* seconds after it is thrown is given by $h(t) = -16t^2 + 38t + 74$.
   (a) Find the velocity of the ball at time *t*.
   (b) What was the ball's initial velocity? Was it thrown up or down? How can you tell?
   (c) Was the ball's height increasing or decreasing at time $t = 2$?
   (d) How long was the ball in the air?
   (e) What is the acceleration of the ball at time *t*?

9. Find $\frac{dy}{dx}$ for each of the following:

(a) $x^3 + y^3 = 3xy$

(b) $4x^2 - 9y^2 = 36$

(c) $(x^2 + y^2)^2 = 2xy$

10. Given the curve $x^2 + xy + y^2 = 1$,

(a) Find $\frac{dy}{dx}$.

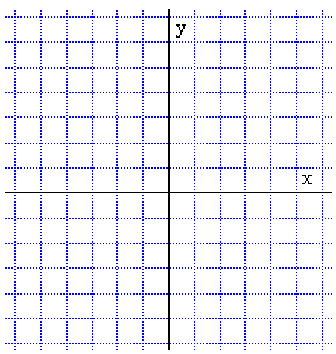
(b) Write the equation of the tangent line to the curve at the point (0,1).

(c) Find all points at which the curve has horizontal tangent lines.

(d) Find all points at which the curve has vertical tangent lines.

(e) Find all points at which the curve has tangent lines with slope one.

11. (a) Sketch the graph of $f(x) = 4 - e^{-x}$, by transforming the graph of $y = e^x$.



(b) Find $\lim_{x \to \infty} 4 - e^{-x}$.

12. Animal populations are not capable of unrestricted growth because of limited habitat and food supplies. Under such conditions, the population follows what is called a *logistic growth model*

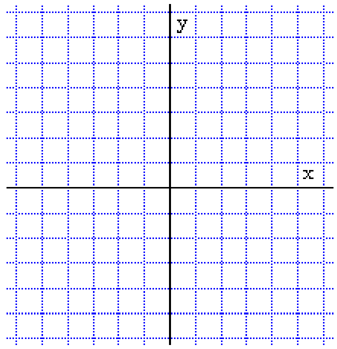
$$P(t) = \frac{d}{1 + ke^{-ct}}$$

where *c*, *d*, and *k* are positive constants. For a certain fish population in a small pond $d = 1200$, $k = 11$, $c = 0.2$ and *t* is measured in years. The fish were introduced into the pond at time *t* = 0.

(a) How many fish were originally put into the pond?

(b) What is the limiting value of the number of fish in the pond? (*i.e.* is there a number the population will never exceed?)

13. Sketch the graph of $y = e^x$:



Use your graph to sketch the graph of the derivative of $y = e^x$: