Math 481/581 Mathematical Statistics I Review for Test 2 Fall 2007

1. Let X be a random variable with probability density $f(x) = \alpha x^{-\alpha-1}$, $x \geq 1$, for some $\alpha > 2$.
(a) Find the mean and variance of X.
(b) Calculate $E(X^k)$ for each postive integer k for which the expectation exists. For which k (in terms of α) does this expectation exist?

2. A stick has unit length. We break the stick at $X \in (0, 1)$ according to the probability density

$$f(x) = 6x(1 - x), \quad 0 < x < 1.$$

Find the expected value of the ratio of the longer piece to the shorter piece.

3. Let X and Y have the joint density $f(x, y) = 3x$, $0 \leq y \leq x$, $0 \leq x \leq 1$, and $f(x, y) = 0$ otherwise.
(a) Find the marginal density $f_X(x)$.
(b) Compute $E(Y|X = x)$.
(c) Compute $Var(Y|X = x)$.
(d) Show directly that $E(Y) = E(E(Y|X))$.

4. The number of offspring of an organism is a non-negative integer valued random variable with mean μ and variance σ^2. Each offspring reproduces in the same mnanner. let X_1 be the number of offspring in generation 1 from a single individual. So $E(X_1) = \mu$ and $Var(X_1) = \sigma^2$. Next let X_2 be the number of offspring coming from the individuals arising in generation 1. Assume that the individuals of generation 1 reproduce independently. Thus compute:
(a) $E(X_2|X_1 = k)$ and $Var(X_2|X_1 = k)$, $k = 0, 1, 2, \dots,$ and
(b) $E(X_2)$ and $Var(X_2)$.

5. The volume of a bubble is estimated by measuring its diameter D and using the relationship $V = \pi D^3/6$. Suppose that the true diameter is $\mu = 2.0$ mm and that the measurement has mean μ and standard deviation 0.1 mm.
(a) What is the approximate standard deviation of the estimated volume?
(b) What is the approximate volume when corrected for the variability in the measurement of the diameter?

6. Suppose that $X_1,$, X_2, $\dots$, X_{20} are independent random variables each with the density function $f(x) = 2x$, $0 < x < 1$. Let $S = X_1 + X_2 + \cdots + X_{20}$. Use the central limit theorem to approximate $P(S \leq 10)$.