Math 425/525 Chaos Test I Summer 2007

Due: June 28

1. Let $r > 0$ and $d > 0$. (a) Solve the difference equation $x(n+1) = (1+r)x(n) - d$, $x(0) = A$, for all $n \geq 0$. Show your work.

(b) Solve approximately for r such that if $d = .50$ and $A = 20$ then $x(48) = 0$. Here r is the monthly interest rate corresponding to \$500 payments on a 48 month loan of \$20,000. (Hint: start with $r = .010$ and let r decrease until an approximate solution is obtained.)

(c) Let $r = .005$. Compute approximate values of d and n such that $x(n) = 0$ and $nd = 22$. (Suggestion: start with $d = .5$ and $n = 44$ and increase d)

2. Let $D(x) = 2x (\text{mod } 1)$ for $0 \leq x < 1$.

(a) Show that if $x = \frac{k}{2^n}$ for some $0 \leq k \leq 2^n - 1$, then $D^n(x) = 0$.

(b) Find a $5-$cycle for D.

3. Let $f(x) = 1 - 1.25x^2$, $-1 \leq x \leq 1$. Let $g(x) = f(f(x))$.

(a) Graph $y = g(x)$, $y = f(x)$ and $y = x$ on the same plot.

(b) Find an exact expression for the 2-cycle $\overline{x}_1, \overline{x}_2$ of f and show that $g'(\overline{x}_1) = g'(\overline{x}_2) = -1$.

(c) Calculate $Sg(\overline{x}_1)$

(d) Is the $2-$cycle stable? Justify your answer.

4. (a) Establish the formula $Sf = (\ln(f\,'))'' - \frac{1}{2}([\ln(f\,')]')^2$

(b) Let $G_\mu(x) = \mu \arctan(x)$. Use (a) to show that $S(G_\mu)(x) < 0$ for all x.

(c) Show graphically that for $\mu > 1$ the map $G_\mu : (-\infty, \infty) \to (-\infty, \infty)$ has two attracting fixed points. Does this contradict Singer's Theorem?

5. (Grad) Use 4(a) to show that $Sf(x) = 0$ for all x if and only if $f(x) = (ax+b)/(cx+d)$.