

**Math 381 Probability and Statistics Review for Final Exam Spring 2008**

**Final Exam:** Tuesday, May 13, 1:40-4:10 p.m.

The following tables will be included: A.1, A.3, A.4, A.5, A.6.

1. Suppose that a number from the set  $\{1, 2, 3, 4, 5, 6, 7\}$  is chosen at random each of 100 times. Approximate the probability that the sum of the 100 numbers so chosen falls between 360 and 440 inclusive.

2. Given a normal random variable with mean 20 and variance 9, and a random sample of size  $n$  taken from the distribution, what sample size is necessary in order that

$$P(19.9 \leq \bar{X} \leq 20.1) = 0.95?$$

3. A random sample of 16 CitiBank VISA cardholder accounts indicated a sample mean debt of \$1220 with a sample standard deviation of \$540.

(a) What assumption must you make to apply the Student method to construct a confidence interval for the population average debt per cardholder,  $\mu$ ?

(b) Under the assumption in (a), find a 96% confidence interval for  $\mu$ .

4. A tobacco company claims that the amount of nicotine in cigarettes of a certain type is a random variable with mean 1.2 mg. The standard deviation in the nicotine amounts is known to be 0.2 mg. To test the hypothesis  $\mu = 1.2$  against the alternative  $\mu > 1.2$ , a random sample of 25 cigarettes will be tested. The critical region is defined to be  $\bar{x} > 1.3$ .

(a) Find the probability  $\alpha$  of committing a type I error.

(b) Evaluate the probability  $\beta$  of a type II error for the alternative  $\mu = 1.4$ .

5. The mean breaking strength of a certain type of fiber is required to be at least 200 psi. An inspection is done to see whether the fiber is unacceptable. Assume that the breaking strengths are approximately normally distributed.

(a) State the null and alternative hypotheses.

(b) An independent random sample of 8 pieces of fiber will be tested for breaking strength. The population standard deviation of the breaking strengths is known to be 3 psi. What is the standard error of the sample mean?

(c) A sample of 8 pieces of fiber yielded breakage at the following pressures (measurements in psi)

202 198 196 204 197 196 200 195

Would you conclude at the 5% level of significance that the fiber is unacceptable? What about the 10% level of significance? Report a  $P$ -value.

6. A statistics student wants to test the hypothesis that a certain coin is equally likely to land on either heads or tails when it is flipped.

(a) State the null and alternative hypotheses. What is the exact distribution under the null hypothesis of the total number of heads in 225 flips of the coin?

(b) The student flips the coin 225 times, obtaining 130 heads and 95 tails. Compute a  $P$ -value to summarize your findings. Explain the meaning of this  $P$ -value. What conclusion should be reached at the 5% level of significance?

7. It is “well-known” that the average automobile commuter in the Los Angeles area drives more miles daily than does a commuter in the San Francisco Bay area. To see whether this “fact” is indeed true, a random sample of 15 Los Angeles area commuters and 15 San Francisco Bay area commuters were randomly chosen and their driving habits monitored. The following data relating to the average number and standard deviation of miles driven resulted.

Los Angeles commuter	$\bar{x}_1 = 57.4$	$s_1 = 12.4$
San Francisco commuter	$\bar{x}_2 = 52.8$	$s_2 = 13.8$

Assume that the miles driven by the two types of commuters are normally distributed with equal variances.

(a) What is the pooled estimate of variance in the miles driven by commuters in these two cities? How many degrees of freedom are associated with this estimate?

(b) If  $\mu_1$  and  $\mu_2$  denote the full commuting population averages of mileages in the two cities, test  $\mu_1 = \mu_2$  against the alternative  $\mu_1 > \mu_2$ . What conclusion should be drawn at each of the following levels of significance: 10%, 5%, 1%?

8. A manufacturer claims that the average tensile strength of thread A exceeds the average tensile strength of thread B by at least 12 kg. To test this claim, 50 pieces of each type of thread are tested under similar conditions. Type A had a sample average tensile strength of 86.7 kg. with known population standard deviation of  $\sigma_A = 6.28$  kg., while type B had a sample average tensile strength of 77.8 kg. with a known population standard deviation of  $\sigma_B = 5.61$  kg.

(a) Test the manufacturer's claim at  $\alpha = 0.01$ . What is the P-value?

(b) (extra credit). Assume equal sample sizes  $n_A = n_B = n$  in another test of the manufacturer's claim. How large should  $n$  be so that the power of the test is 0.95 when the difference of population means is actually  $\mu_A - \mu_B = 8$  kg. Assume the critical region is defined by  $\alpha = .01$ .

9. To study the effectiveness of a certain commercial liquid protein diet, the Food and Drug Administration sampled nine individuals who were entering the program. Their weights both immediately before they entered and 6 months after they completed the 2-week program were recorded. The following data resulted.

Subject	1	2	3	4	5	6	7	8	9
Weight Before	197	212	188	226	170	194	233	166	205
Weight After	185	220	180	217	185	197	219	170	202

Suppose we want to determine if these data prove that the diet is effective, in the sense that the expected weight loss after 6 months is positive.

(a) What are the null and alternative hypotheses?

(b) Do the data prove that the diet works? Use the 5% level of significance.

10. A manufacturer turns out a product item that is labeled either “defective” or “nondefective”. In order to estimate the proportion defective, a random sample of 100 items is tested and 10 are found to be defective. Following the implementation of a quality improvement program, the experiment was conducted again. A new sample of 100 items is taken and this time only 6 are found defective. Let  $p_1$  denote the population proportion defective before improvement, and  $p_2$  the proportion defective after improvement. Test  $p_1 = p_2$  against the alternative  $p_2 < p_1$  at the 0.05 level of significance.

11. (extra credit) two types of instruments for measuring the amount of sulfur monoxide in the atmosphere are being compared in an air -pollution experiment. It is desired to determine whether the two types of instruments yield measurements having the same variability. The following readings were recorded for the two instruments.

A	0.86	0.82	0.75	0.61	0.89	0.64	0.81	0.68	0.65
B	0.87	0.74	0.63	0.55	0.76	0.70	0.69	0.57	0.53

Assuming the populations to be approximately normally distributed, test the hypothesis  $\sigma_A = \sigma_B$  against the alternative  $\sigma_A \neq \sigma_B$  at the 2% level of significance.