

Math 381 Probability and Statistics Review for Test 2 Morrow, Spring 2008

The following tables will be provided: Table A.3 and Table A.24.

1. The joint probability mass function $f(x, y)$ of a pair (X, Y) of random variables is given:

$f(x, y)$	x	0	1
y	0	.35	.25
	1	.25	.15

Compute (i) $E(X)$, (ii) $E(Y)$, (iii) $E(X + Y)$, (iv) $E(XY)$, and (v) $Cov(X, Y)$.

2. Let X and Y be random variables with joint density function

$$f(x, y) = \begin{cases} 4xy, & 0 \leq x \leq 1 \text{ and } 0 \leq y \leq 1. \\ 0, & \text{elsewhere} \end{cases}$$

Find the (a) mean of the product $Z := XY$, and (b) variance of the product $Z := XY$.

3. Let X be the number showing when a standard (6-sided) red die is tossed and let Y be the number showing when a standard green die is tossed.

(a) Show that (i) $Var(X) = 35/12$, and (ii) $Var((X + Y)/2) = 35/24$.

(b) Use Chebyshev's theorem to estimate $P(1 < (X + Y)/2 < 6)$.

(c) What is the actual probability in part (b)?

4. A fair coin is tossed independently n times. Find the probability that there are between 40% and 60% heads inclusive among the n tosses when

(a) (i) $n = 10$, and (ii) $n = 20$. Here, write out the exact probabilities. You do not need to find decimal expressions.

(b) Use normal approximation including a continuity correction to find (decimal) estimates of the answers to parts (a)(i) and (ii).

5. A lot of 20 items contains exactly 2 that are defective. If 3 items are selected at random without replacement, and the lot is accepted if none of the sampled items is defective,

- (a) What is the probability that the lot is accepted?
- (b) What is the expected number of defective items in the sample?
- (c) What is the probability that both defective items are in the sample?

6. The average number of field mice per acre in a wheat field is 10. Find the probability that:

- (a) there are at most 6 field mice on a given acre,
- (b) there are at most 6 field mice on exactly 2 of the next 3 acres inspected.
- (c) Use normal approximation to estimate the probability that there are a total of at most 20 field mice in the next 3 acres. For this part recall that the variance of a Poisson random variable is numerically equal to its mean.

7. The annual precipitation in a certain region is normally distributed with a mean of 25 inches and a standard deviation of 5 inches.

(a) What is the chance that next year there will be at least 30 inches of precipitation?

(b) What is the chance that in each year of the next four years there will be between 15 and 25 inches of precipitation?

8. The life in months of a certain type of battery has an exponential distribution with an average lifetime $\beta = 4$ months. Ten such batteries will be installed in sequence as they expire.

(a) What is the probability that at most 2 of these 10 batteries will fail in their first month of use?

(b) What is the probability that the sum of the battery lifetimes will exceed 36 months (i) using an exact calculation with a gamma distribution, and (ii) using normal approximation?