Math 381 Probability and Statistics Review for Test 1 Morrow, Spring 2008

1. An urn contains 2 red, 3 green, and 4 yellow marbles. Three marbles are drawn one at a time at random without replacement from the urn. The order of the colors of marbles is observed. What is the probability that

(a) All three green marbles are drawn.

(b) Both red marbles are drawn.

2. A very large lot of electronic components contains 10% that are defective. If four components are chosen at random from the lot, what is the probability that

(a) at least one of the components is defective?.

(b) exactly two of the components are defective?

3. An elevator is going up from ground level carrying 5 people. The elevator may open at any of 10 floors above. We assume that each person is equally likely to get off at any floor and the people get off independently of each other. In effect each person picks a number at random from 1 to 10 to determine which floor their self will get off.

(a) Calculate the size of the sample space.

(b) What is the probability that each person gets off at a different floor?

4. A bag contains three marbles numbered 2, 3, 4, respectively. A single marble is drawn at random. If the marble drawn is numbered x, then a fair coin is tossed x times. Assuming independent tosses, find

(a) the probability that exactly two heads appear.

(b) the conditional probability that the marble numbered 2 was drawn given that exactly two heads appear.

5. An urn contains 3 red and 2 green balls. We consider two different sampling procedures. In procedure (a) the balls are drawn at random one at a time without replacement. In this case let X denote the number of the draw on which a red ball is first chosen. So $X = 1$ if the first ball is red, $X = 2$ if the first ball is green while the second ball drawn is red, etc. In procedure (b) the balls are drawn one at a time at random with replacement. In this case let Y denote the the number of the draws on which a red ball is first chosen.

(a) Compute $P(X = i)$, $i = 1, 2, 3$.

(b) compute $P(Y = i)$, $i = 1, 2, 3, \ldots$.

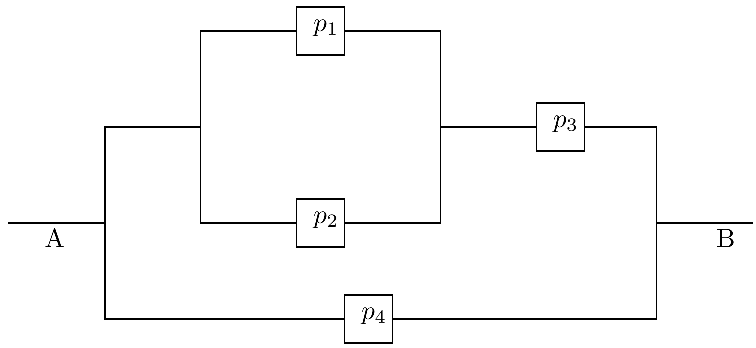
6. The life span in hours of an electrical component is a random variable X with cumulative distribution function

$$F(x) = \left\{ \begin{array}{cc} 1 - \exp(-x/1000), & x > 0. \\ 0, & \text{elsewhere} \end{array} \right\}$$

(a) Determine the probability density function of X.

(b) Determine the probability that the lifespan of such a component will exceed 1500 hours.

(c) Exactly 1% of the life spans of such components exceed x hours. What is x?



7. The probability that the i^{th} component in the circuit shown is working is $p_i = 0.8$, $i = 1, 2, 3, 4$. If all components function independently, what is the probability that the system is working (current flows from A to B)?

8. A service facility operates with two service lines. On a randomly selected day, let X denote the the proportion of time that the first line is in use and let Y denote the proportion of time that the second line is in use. Suppose that the joint probability density function for (X, Y) is

$$f(x) = \left\{ \begin{array}{cc} (3/2)(x^2 + y^2), & 0 \leq x \leq 1 \text{ and } 0 \leq y \leq 1. \\ 0, & \text{elsewhere} \end{array} \right\}$$

(a) Verify that $f(x)$ is indeed a probability density.

(b) Find (i) $P(X \leq .50 \text{ and } Y \leq .50)$ and (ii) $P(X \leq .50)$.

(c) Are X and Y independent random variables? Why or why not?