Math 313.002 Linear Algebra Review for Test2 Fall 2007

1. Show that the range of the linear operator defined by the equations is not all of $\mathbb{R}^3$, and find a vector that is not in the range.

$$\begin{aligned} w_1 &= x_1 - 2x_2 + x_3 \\ w_2 &= 5x_1 - x_2 + 3x_3 \\ w_3 &= 4x_1 + x_2 + 2x_3 \end{aligned} .$$

2. Which of the following subsets W of P_3 (the polynomials of degree 3 or less) are in fact subspaces of P_3? Why or why not?
(a) all polynomials $ax + 1$ where a is a real number.
(b) all polynomials $ax^2 + bx + c$ where $a + b = 0$ and c is an arbitrary real number.
(c) all polynomials $e^a x$ where a is a real number.
(d) all polynomials $a + ax + bx^3$ where a and b are real numbers.

3. (a) Express $\mathbf{w} = (4, 3, 2)$ as a linear combination of $\mathbf{u} = (1, 0, -1)$ and $\mathbf{v} = (2, 1, 0)$.
(b) Is it possible to express any vector in $\mathbb{R}^3$ as a linear combination of $\mathbf{u}$ and $\mathbf{v}$? Why or why not?

4. Determine whether the following vectors span $\mathbb{R}^3$.
$\mathbf{v}_1 = (1, 4, 7)$, $\mathbf{v}_2 = (2, 5, 8)$, $\mathbf{v}_3 = (3, 6, 10)$.

5 (a) Show that $\mathbf{v}_1 = (1, 2, 3)$, $\mathbf{v}_2 = (4, 5, 6)$, and $\mathbf{v}_3 = (7, 8, 9)$ are linearly dependent in $\mathbb{R}^3$.
(b) Express each vector as a linear combination of the other two.

6. Determine bases of each of the following subspaces of $\mathbb{R}^4$. State the dimension of each subspace.
(a) all vectors of the form $(a, a + b, 0, 0)$.
(b) all vectors of the form (a, b, b, b).
(c) all vectors of the form (a, a, b, c) where $a + b + c = 0$.

7. Suppose A is a 3×5 matrix.
(a) Are the columns of A linearly independent, linearly dependent, or can you tell?
(b) What is the smallest the nullity of A can be? Explain.
(c) If the column space of A is a plane through the origin in $\mathbb{R}^3$, what are the rank and nullity of A? Explain.

8. Let the inner product on $\mathbb{R}^2$ be given $\langle \mathbf{u}, \mathbf{v} \rangle = u_1v_1 + 4u_2v_2$ for $\mathbf{u} = (u_1, u_2)$ and $\mathbf{v} =(v_1, v_2)$. Let $\mathbf{u} = (1, 1)$ and $\mathbf{v} = (0, 2)$.
Compute:
(a) $\|\mathbf{u}\|$,
(b) $\|\mathbf{v}\|$,
(c) the cosine of the angle between $\mathbf{u}$ and $\mathbf{v}$, and,
(d) the distance $d(\mathbf{u}, \mathbf{v})$.

9. let $A = \begin{bmatrix} 1 & 2 & 3 & 1 \\ -2 & -3 & -5 & -1 \\ 1 & 1 & 2 & 0 \end{bmatrix}$.
(a) Find a basis for the row space of A.
(b) Find a basis for the nullspace of A.
(c) Verify that every vector in the row space is orthogonal to every vector in the nullspace (for the Euclidean inner product =dot product on $\mathbb{R}^4$).