Math 313.002 Linear Algebra Review for Test 1 Fall 2007

1. In each case the *augmented* matrix for a system of linear equations has been reduced by row operations to the given form. For each matrix do the following: (i) determine whether the matrix is in reduced row echelon form, row-echelon form only, or neither, (ii) determine whether the linear system is consistent or inconsistent, and (iii) tell whether the system has no solution, exactly one solution or infinitley many solutions.

(a) $\begin{bmatrix} 1 & 0 & 2 & 1 \\ 0 & 1 & 3 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 2 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 2 & 0 & 2 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 3 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$ (d) $\begin{bmatrix} 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \end{bmatrix}$

2. Solve for u, v, w, x, y, and z in the following matrix equation.

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 1 \\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} u & x \\ v & y \\ w & z \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}$$

3. Let $A = \begin{bmatrix} 1 & 2 & 0 & 1 \\ 2 & 3 & 1 & 1 \\ 0 & -5 & 5 & 6 \\ -2 & 1 & 4 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 & 0 & 1 \\ 2 & 3 & 1 & 1 \\ 0 & -5 & 5 & 6 \\ 0 & 5 & 4 & 3 \end{bmatrix}$.

(a) Write down an elementary row matrix E such that $EA = B$.

(b) What is $\det(E)$?

4. (a) Compute $\det(A)$. (b) Find all values of t such that A is not invertible.

$$A = \begin{bmatrix} t & 0 & 0 & t & 0 \\ 0 & 0 & 2 & 0 & 0 \\ 0 & t-1 & 0 & 0 & t-2 \\ 0 & 0 & 0 & 0 & t \\ 1 & 0 & 0 & 3 & 0 \end{bmatrix}$$

5. Let $A = \begin{bmatrix} 1 & -1 & 3 & -3 \\ 0 & 1 & -1 & 3 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$. Compute A^{-1}.

6. Find conditions that the values $b1, b2, b3, b4$, and $b5$ must satisfy for the system to be consistent.

$$\begin{array}{lllll} x & +y & -z & & = b1 \\ & 2y & -z & -w & = b2 \\ x & & +z & +w & = b3 \\ & y & -2z & -w & = b4 \\ & y & +z & & = b5 \end{array}$$

7. Evaluate the determinant. $\begin{vmatrix} 1 & -1 & 1 & 6 \\ 0 & 1 & -4 & 3 \\ 1 & 1 & 0 & 5 \\ -2 & 1 & -1 & -2 \end{vmatrix}$

8. Given that $\|u\| = 3$, $\|v\| = 4$, and $\|u - v\| = 2$, find $u \cdot v$.

9. Find two unit vectors in $\mathbf{R}^3$ that are orthogonal to each of $(1, -1, 2)$ and $(1, 1, 4)$. (You may use the cross product only to check your answer; the cross product does not work in higher dimernsions.)

10. Find the standard matrix for the stated composition of linear operators in R^2.
(a) A counterclockwise roation of 270° followed by a reflection about the x-axis.
(b) An orthogonal projection on the y-axis, followed by reflection across the line $y = x$.
Does the order of application matter in part (a)? What about part (b)?

Math 313.002 Linear Algebra Ans. to Review Test 1 Fall 2007

1. (a) row echelon, inconsistent, no solution
(b) reduced row echelon, consistent, infinitely many solutions
(c) row echelon, consistent, unique solution
(d) neither, consistent, infinitely many solutions

2. $\begin{bmatrix} 1 & 2 & 3 & 1 & 2 \\ 0 & 1 & 1 & 3 & 4 \\ 0 & 1 & 2 & 5 & 6 \end{bmatrix} \to \begin{bmatrix} 1 & 2 & 3 & 1 & 2 \\ 0 & 1 & 1 & 3 & 4 \\ 0 & 0 & 1 & 2 & 2 \end{bmatrix}$. Thus by back substitution, $w = 2, v = 3 - 2 = 1, u = 1 - 6 - 2 = -7$; $z = 2, y = 4 - 2 = 2, x = 2 - 6 - 4 = -8$

3. (a) $E = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 2 & 0 & 0 & 1 \end{bmatrix}$. (b) $\det(E) = 1$

4. $\det(A) = 4t^2(t-1)$. A invertible for $t \neq 0, 1$.

5. $A^{-1} = \begin{bmatrix} 1 & 1 & -2 & -2 \\ 0 & 1 & 1 & -2 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

6. (1): $b_4 = b_1 - b_3$ and (2): $b_5 = b_3 - b_1 + b_2$ (or (2): $b_5 = -b_4 + b_2$ from (1))

7. $det = 70$

8. $u \cdot v = 10.5$

9. $(x, y, z) = \frac{1}{\sqrt{11}}(-3, -1, 1)$ or $\frac{-1}{\sqrt{11}}(-3, -1, 1)$

10. (a) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$; order matters. (b) $\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$; order matters.