Math 131 Fall 2009 Abrams / Holmes **Practice Exam 1 - SOLUTIONS**

Show all your work. No credit will be given for answers which are not accompanied by supporting computations. Only a non-graphing calculator with no QWERTY keyboard is allowed. Use the back of the sheet if you need more space. **Circle answers when appropriate**. Good luck !!

1. (2 pt) Find the equation of the line having slope 2 which passes through the point $(5,-2)$. Give your answer in $y = mx + b$ form.

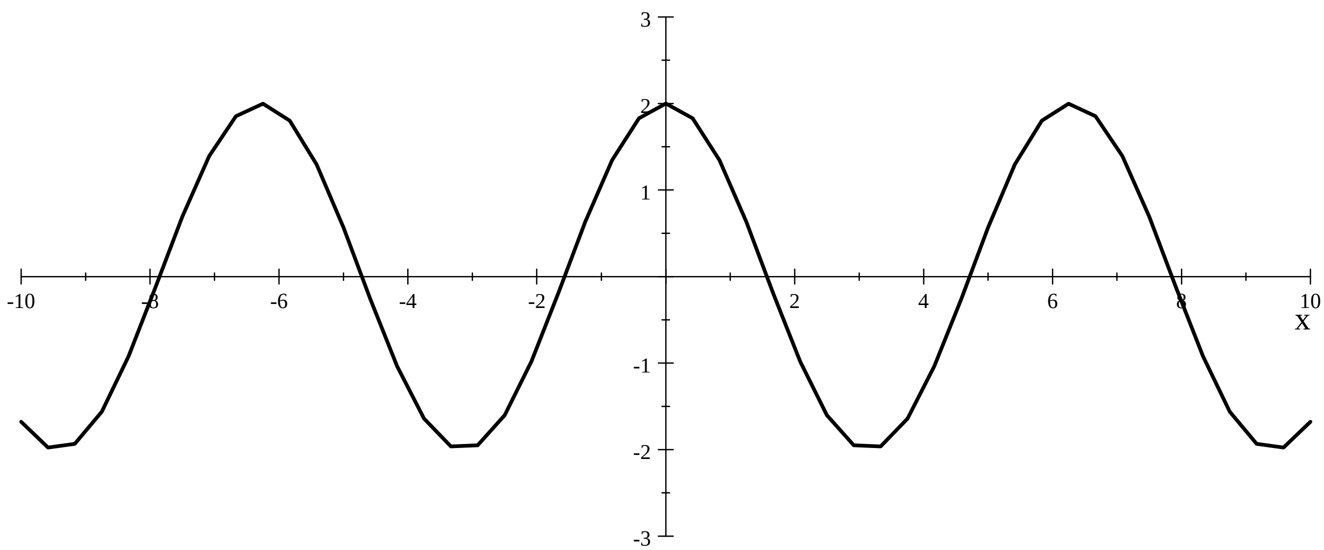
Use the point-slope formula to get $y - (-2) = 2(x - 5)$. So $y + 2 = 2x - 10$, so $y = 2x - 12$.

2. (3 pt total) (a) Solve for x: $\sin(x) = 1/2$

We know from the table of basic angles that $x = \pi/6$ and that $x = 5\pi/6$ work. But then so does adding 2π to any of these. So ALL answers are: $\pi/6 + 2\pi k$ and $5\pi/6 + 2\pi k$ (where k is any integer).

(b) $\sin^{-1}(1/2) = \pi/6$ because $\pi/6$ is the angle between $-\pi/2$ and $\pi/2$ whose sine is $1/2$.

3. (3 pt) Carefully draw the function $y = 2\cos x$ on these axes. Label each of the x-intercepts.



The x-intercepts are at (in order) $-5\pi/2, -3\pi/2, -\pi/2, \pi/2, 3\pi/2,$ and $5\pi/2$.

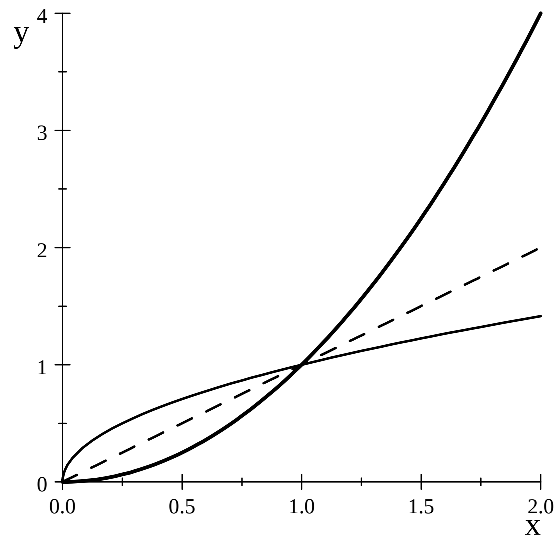
4. (1 pt each) (a) Simplify to an expression of the form e^a : $\dfrac{e^3e^{-7}}{e^5e^2} = \dfrac{e^{-4}}{e^7} = e^{-11}$

(b) $\log_8(2) = 1/3$ since $8^{1/3} = 2$

5. (3 pt) Find the inverse function for the function $f(x) = x^3 - 7$.

Write $y = x^3 - 7$. Switch x with y: $x = y^3 - 7$. Now solve for y : $x + 7 = y^3$, so $y = \sqrt[3]{x+7}$. So $f^{-1}(x) = \sqrt[3]{x+7}$

6. (3 pt) The graph of the function f is shown here. (The domain of f is $\{x \mid x \geq 0\}$.) On the same axes, sketch a graph of $f^{-1}(x)$.



7. (4 pt total) Short answer

(a) For any angle α, define $\tan(\alpha)$ in terms of $\sin(\alpha)$ and $\cos(\alpha)$. $\tan(\alpha) = \dfrac{\sin(\alpha)}{\cos(\alpha)}$

(b) Compute the exact value of $\tan(\frac{\pi}{6}) = \dfrac{\sin(\frac{\pi}{6})}{\cos(\frac{\pi}{6})} = \dfrac{\frac{1}{2}}{\frac{\sqrt{3}}{2})} = \dfrac{1}{\sqrt{3}} = \dfrac{\sqrt{3}}{3}.$

(c) Find the exact value of $e^{\ln(2)} = 2..$ (e^x and $\ln(x)$ are inverse functions.)

(d) Find the exact value of $\cos(\cos^{-1}(\frac{1}{3})) = \frac{1}{3}$ ($\cos(x)$ and $\cos^{-1}(x)$ are inverse functions.)

8. (2 pt) Give PRECISELY the meaning of the statement: $\lim_{x\to a} f(x) = L$.

$\lim_{x\to a} f(x) = L$ if we can make the values of $f(x)$ arbitrarily close to L (as close to L as we like) by taking x to be sufficiently close to a (on either side of a) but not equal to a. (See DEFINITION, page 25 of the text.)

9. (1 pt each) (a) (Circle the correct answer): The statement $\lim_{x\to a} f(x) = L$ intuitively means:

i. as the values of x get closer and closer to a, then eventually there is an input value whose output value equals L.

ii. the values of $f(x)$ never equal L as the values of x get closer and closer to a.

iii. Some of the values of $f(x)$ get arbitrarily close to L as the values of x get arbitrarily close to a.

iv. All of the values of $f(x)$ get arbitrarily close to L as the values of x get arbitrarily close to a.

(b) **FALSE** For every function f, if $\lim_{x\to a} f(x) = L$ then $f(a) = L$.

(c) **FALSE** For every function f, if $\lim_{x\to a} f(x) = L$ then a is in the domain of f.

For both parts (b) and (c), keep in mind that the limit does NOT at all depend on what is happening **AT** the value a.

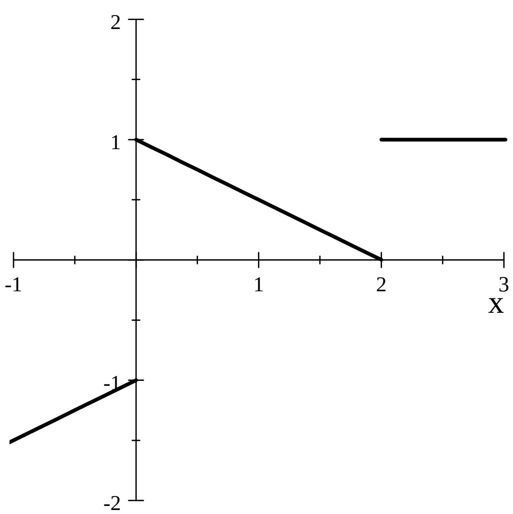
10. (4 pt) On the left hand axes below for this problem, sketch CLEARLY a graph of a function f which has:

$\lim_{x\to 0^-} f(x) = -1$, $\lim_{x\to 0^+} f(x) = +1$, $\lim_{x\to 2^-} f(x) = 0$, $\lim_{x\to 2^+} f(x) = 1$, $f(2) = 1$, and $f(0)$ is undefined.

(Use open circles as appropriate.)

11. (2 pt) On the right hand axes below for this problem, sketch CLEARLY a graph of a function f which has:

$\lim_{x\to 2} f(x) = 1$ and $f(2) = -1$.



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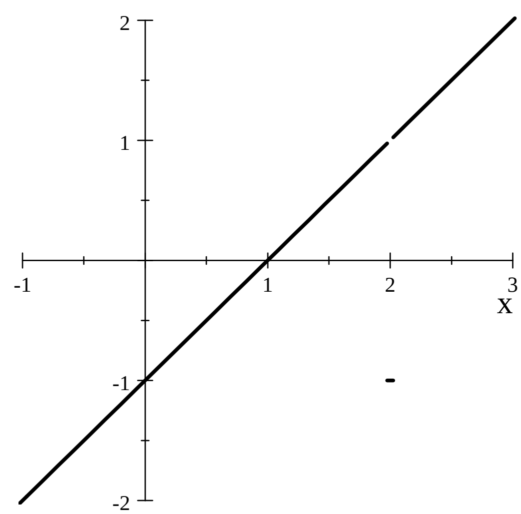

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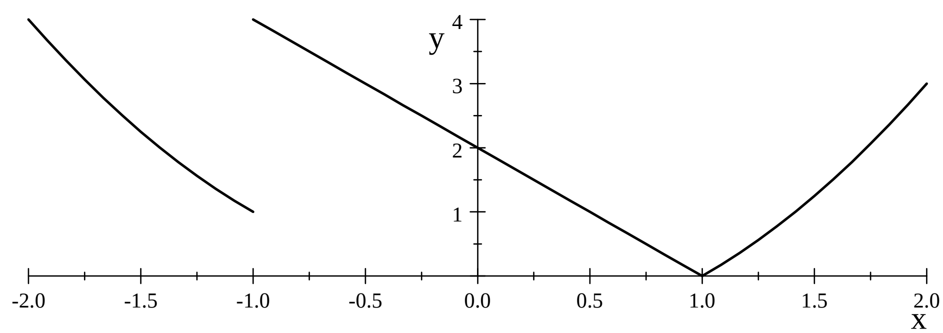
12. (1/2 pt each) Using the graph of the function f given here, find the indicated limits.



(a) $\lim_{x\to -1^+} f(x) = 4$ (b) $\lim_{x\to -1^-} f(x) = 1$ (c) $\lim_{x\to -1} f(x)$ does not exist (d) $\lim_{x\to 0} f(x) = 2$

(e) $\lim_{x\to 1^+} f(x) = 0$ (f) $\lim_{x\to 1^-} f(x) = 0$ (g) $\lim_{x\to 1} f(x) = 0$ (h) $f(0.5) = 1$

13. (6 pt total) Find the indicated limit. **Use the most informative of the phrases 'does not exist', ∞, or $-\infty$ whenever appropriate**. Remember to circle your answer.

(a) $\lim_{x\to -1} \frac{x^2+5x+4}{x+1} = \lim_{x\to -1} \frac{(x+1)(x+4)}{x+1} = \lim_{x\to -1}(x+4)$ (we can cancel $x+1$ since $x \neq -1$) $= (-1)+4 = 3$

(b) $\lim_{x\to -1} \frac{x^2+3x+4}{x+2} = \frac{(-1)^2+3(-1)+4}{(-1)+2}$ (direct substitution) $= \frac{2}{1} = 2.$

(c) $\lim_{x\to\infty} \frac{x^2+x+4}{9-3x^2} = \frac{1}{-3}$ (the degrees of the numerator and denominator are equal, so this is the quotient of the leading coefficients) $= -\frac{1}{3}$

(d) $\lim_{x\to 0} \frac{2\sin(x)}{x} = 2 \cdot \lim_{x\to 0} \frac{\sin(x)}{x} = 2 \cdot 1 = 2.$ ($\lim_{x\to 0} \frac{\sin(x)}{x} = 1$ is a basic formula that we derived in class.)

(e) $\lim_{x\to 5^-} \frac{1}{x-5} =$ Does Not Exist; but we can say more, since when $x < 5$ then the expression is always negative, so in fact $\lim_{x\to 5^-} \frac{1}{x-5} = -\infty.$

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14. (3 pt total). (a) Suppose $f, g,$ and k are functions which have $f(x) \leq g(x) \leq k(x)$ for all values of x. If $\lim_{x\to a} f(x) = L = \lim_{x\to a} k(x)$, then

$\lim_{x\to a} g(x) = L$ (in other words, the same limit.)

(b) What is the name of the result you used to get your answer to part (a)? (Hint: there are two possible correct answers.) The Sandwich Theorem or the Squeeze Theorem.

(c) Which one of the five limits in Problem 13 were we able to compute by using this result? Part (d), $\lim_{x\to 0} \frac{\sin(x)}{x} = 1$ because of the sandwich theorem.

15. (4 pt each) Compute the indicated limit. **Show all your work**, **justify each step**, **and make sure your notation is clear and precise**. Remember to circle your answer.

(a) $\lim_{h\to 0} \frac{(-3+h)^2+5(-3+h)+6}{h} = \lim_{h\to 0} \frac{(9-6h+h^2)-15+5h+6}{h}$

$= \lim_{h\to 0} \frac{-h+h^2}{h} = \lim_{h\to 0} \frac{h(-1+h)}{h} = \lim_{h\to 0}(-1+h)$ (we can cancel the h since $h \neq 0$)

$= -1+0 = -1.$

(b) $\lim_{x\to 16} \frac{\sqrt{x}-4}{x-16} = \lim_{x\to 16} \frac{\sqrt{x}-4}{x-16} \cdot \frac{\sqrt{x}+4}{\sqrt{x}+4}$

$= \lim_{x\to 16} \frac{x-16}{(x-16)(\sqrt{x}+4)} = \lim_{x\to 16} \frac{1}{\sqrt{x}+4}$ (we can cancel the $x-16$ since $x \neq 16$)

$= \frac{1}{\sqrt{16}+4} = \frac{1}{4+4} = \frac{1}{8}.$

16. (6 pt total) (a) If a ball is thrown into the air with velocity 100 ft/sec, its height above the ground in feet after t seconds is given by $y = 100t - 16t^2$. Find the average velocity of the ball for the time period beginning when $t = 4$ and lasting

(i) 0.1 second

Average velocity is change in position over change in time. So

$$\frac{y(4.1) - y(4)}{0.1} = \frac{100(4.1) - 16(4.1)^2 - [100(4) - 16(4)^2]}{0.1} = \frac{141.04 - [144]}{0.1} = \frac{.-2.96}{0.1} = -29.6 \; ft/\sec.$$

So on average, on the time interval $[4, 4.1]$, the object is moving DOWN at $29.6 ft/\sec$

(ii) 0.05 second

Average velocity is change in position over change in time. So

$$\frac{y(4.05) - y(4)}{0.05} = \frac{100(4.05) - 16(4.05)^2 - [100(4) - 16(4)^2]}{0.05} = \frac{142.56 - [144]}{0.05} = \frac{.-1.44}{0.05} = -28.8$$

$ft/\sec$. So on average, on the time interval $[4, 4.05]$, the object is moving DOWN at $28.8 ft/\sec$

(b) Using limits, find the instantaneous velocity of the ball when $t = 4$.

Instantaneous velocity is the limit of the average velocities ...

$$v(4) = \lim_{t \to 4} \frac{y(t) - y(4)}{t - 4} = \lim_{t \to 4} \frac{100(t) - 16(t)^2 - [100(4) - 16(4)^2]}{t - 4} = \lim_{t \to 4} \frac{-16(t)^2 + 100t - 144}{t - 4} = \lim_{t \to 4}$$

(we can cancel since $t \neq 4$) $= -16(4) + 36 = -28 \; ft/\sec.$ So the instantaneous velocity at time $t = 4$ is $28 ft/\sec$, going downward.