Math 132 Spring 2008 Abrams **Exam 2** Name ______________________

Show all your work. No credit will be given for answers which are not accompanied by supporting computations. Circle your answer when appropriate. Use the back of the sheet if necessary. As usual, notation counts! Be clear and precise with your answers, justifying responses appropriately. Make sure to include the units in your answer when warranted. You may use a standard calculator, one which does not have graphing capability or a QWERTY keyboard. Good luck!

1. (7 pt total) a. A farmer wants to fence a rectangular pasture adjacent to a river. The side of the pasture adjacent to the river will require no fencing. The farmer has available 6,000 feet of barbed wire fencing for the pasture. The length and width of the fenced pasture must each be at least 100 feet. Draw a picture of the configuration of the pasture which will enclose the maximum amount of area. Make sure to indicate the length of each side of the pasture, as well as show where the river is, in your picture.

b. How much total area is enclosed in the pasture you have pictured above?

2. (5 pt total) A video camera manufacturer knows from past experience that the relationship between p (the price per camera) and x (the number of cameras the manufacturer can sell per week) is given by $p = 700 - \frac{x}{10}$. Also, the weekly cost function to produce and sell x cameras is given by $C(x) = \$468,000 + 150x$.

(a) How many cameras should the manufacturer sell per week in order to maximize its weekly profit?

(b) What is the maximum weekly profit?

3. (3 pt each) Antiderivatives. (**Suggestion: it is a good idea to check your answers on these**.)

a. Find the most general antiderivative of $f(x) = 2 + \sqrt{x} - \dfrac{4}{x^2}$.

b. Find the most general antiderivative of $f(x) = 5e^x + 3\sin x$

c1. Assume $n \neq -1$. Find the most general antiderivative of $f(x) = x^n$.

c2. Then find the most general antiderivative of $f(x) = x^{-1}$.

4. (4 pt) Find the position function $s(t)$ of an object whose velocity function is given by $v(t) = \sin t - \cos t$, and for which $s(0) = 0$.

5. (5 pt) Find f if $f''(\theta) = \sin\theta + e^{\theta}$, $f(0) = -4$, $f'(0) = 5$.

6. (5 pt) A stone was dropped off a bridge and hit the river below with a speed of 90 ft/sec. What is the height of the bridge? (Reminder: acceleration due to gravity is $32 ft/sec^2$ downward)

7. (4 pt total) The speed of a runner increased steadily during the first three seconds of a race. Her speed at half-second intervals is given in the table. Find lower and upper estimates for the distance that she traveled during these three seconds. (Indicate how you got your answers, don't just give the numbers without supporting calculations. Make sure to give the correct units in your answers.)

t (in seconds)	0	0.5	1.0	1.5	2.0	2.5	3.0
$v(t)$ (in feet/second)	0	6.7	10.1	15.0	18.3	19.7	21.2

Lower estimate = __________ Upper estimate = __________

8. (1 pt each) True / False

T **F** a. If $F(x)$ and $G(x)$ are antiderivatives of the function $f(x)$, then $F(x) = G(x) + C$ for some constant C.

T **F** b. Using more rectangles to approximate an area gives a better estimate of the actual area than using fewer rectangles.

T **F** c. Using more rectangles to approximate an area gives a larger estimate of the actual area than using fewer rectangles.

T **F** d. The process of finding antiderivatives is more difficult than the process of finding derivatives.

9. (10 pt total) In this problem you will estimate the area under the graph of $f(x) = x^2 + 1$ (and above the x –axis) over the interval $[0, 4]$.

a. Carefully draw the region in question on the axes below. (Note that the scales on the x and y axes are different.)

b. On the graph above, draw four approximating rectangles, using right hand endpoints of the subintervals.

c. What is the estimated area of the region using your rectangles from part (b)?

d. Is your answer to part (c) an overestimate or an underestimate of the actual area of the region? Explain.

e. On the same graph, draw eight approximating rectangles, using right hand endpoints of the subintervals.

f. Let R_8 denote the number that you would get if you computed the estimated area using your rectangles from part (e). (You need NOT compute R_8 !) Let R_4 denote your answer to part (c). Let A denote the exact area of the region. List, IN ORDER FROM SMALLEST TO LARGEST, the three numbers R_8, R_4, and A.