Math 131   Fall 2009 Abrams **Exam 2** Name ______________________

Show all your work. No credit will be given for answers which are not accompanied by supporting computations. Circle your answer when appropriate. Use the back of the sheet if necessary. As usual, notation counts! Be clear and precise with your answers, justifying responses appropriately. **Make sure to include the units in your answer when warranted**. You may use a standard calculator, one which does not have graphing capability or a QWERTY keyboard. Good luck!

1. (2 pt) If f is a function of the variable x, give the **PRECISE** definition of the *derivative* of f. (I am looking for the definition of the derivative $f'(x)$ in terms of some sort of limit. Make sure your notation is correct.)

2.. (7 pt total) In this question, the function f will always denote $f(x) = \frac{1}{x+2}$

(a) **By directly using the definition of derivative given in the previous question**, compute the derivative $f'(x)$ for this function. Justify all your steps, and make sure your notation is clear and correct. (No credit on this problem if all you do is use the derivative formulas without the limit computation.)

(b) Find the equation of the tangent line to the curve $f(x) = \frac{1}{x+2}$ at the point $(2, \frac{1}{4})$.

3. (2 pt total) In this question, f is the function which describes the temperature (in degrees Fahrenheit) of a baked turkey t minutes after it has been taken out of a 400° oven and set on the dinner table.

(a) What are the units on $f'(t)$?

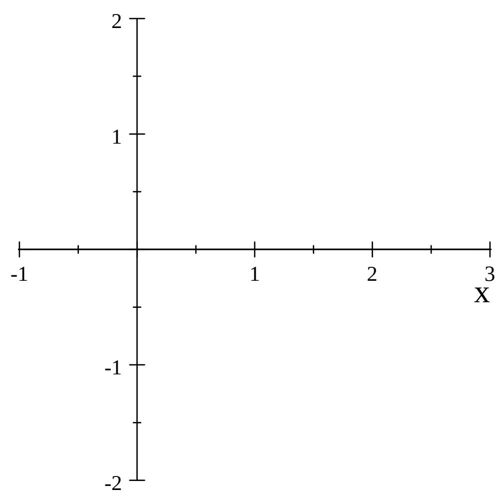
(b) Circle the correct inequality: $f'(t) > 0$ $f'(t) < 0$

4. (5 pt total) (a) If an arrow is shot upward on the moon with a velocity of 48 meters / second, its height (in meters) after t seconds is given by $s(t) = 48t - 0.83t^2$. Find the instantaneous velocity of the arrow at time $a = 30$ seconds. (Use the definition of derivatives in terms of limits. No credit on this problem if all you do is use the derivative formulas without the limit computation. As always, make sure to include the appropriate units in your answer.)

(b) Circle ONE: For the arrow described in part (a), at time 60 seconds the arrow is:

(i) travelling upwards (ii) momentarily stopped (iii) travelling downwards

5. (3 pt) On the axes below, **carefully** sketch the graph of a function f whose domain is $-1 \leq x \leq 3$, and for which: $f(0) = 0,\ f'(0) = -2,\ f'(1) = 1,$ and $f'(2) = 0.$



6. (2 pt) (Circle ALL that apply.) Suppose there is a tangent line to the function f at the point $(a, f(a))$. Which of these is an appropriate description for the slope of the tangent line?

a. The limit of the slopes of the secant lines between $(x, f(x))$ and $(a, f(a))$, as x gets closer and closer to a.

b. $\dfrac{f(x) - f(a)}{x - a}$

c. $\lim_{a \to 0} \dfrac{f(a+h) - f(a)}{h}$

d. The slope of the 'direction line' of $f(x)$ at the point $(a, f(a))$.

7. (2 pt total) For any function $g(x)$, and any input value a at which g has a derivative, which one of these phrases is always an appropriate interpretation of $g'(a)$? (Circle ALL that apply).

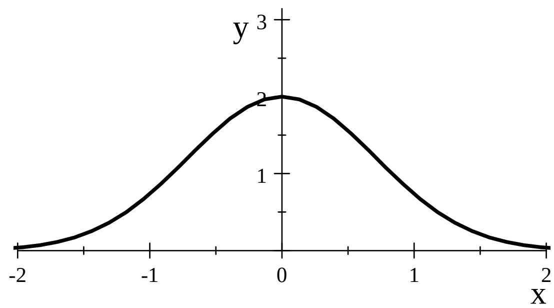
(i) Instantaneous rate of change of g at $x = a$.

(ii) Average rate of change of g at $x = a$.

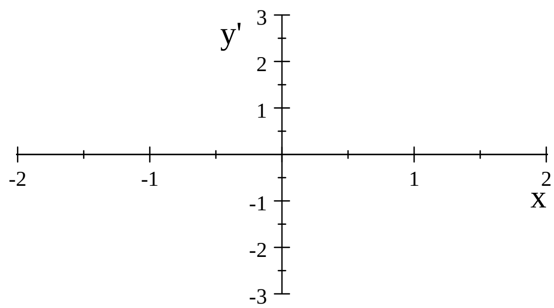
(iii) Slope of the secant line to the graph of g at the point $(a, g(a))$.

(iv) Slope of the tangent line to the graph of g at the point $(a, g(a))$.

8. (2 pt) Here is a graph of the function $g(x)$. On the axes directly below it, sketch a graph of the function $g'(x)$.



The function $g(x)$



The function $g'(x)$

9. (1 pt) **T** **F** The formulas in Table 1 and Table 2 come from the definition of derivative as a limit.

10. (2 pt each) For each part, fill in the first blank with the correct formula, and fill in the second blank with either "Table 1" or "Table 2".

(a) $\frac{d}{dx}(\cos x) =$ ____________ This formula appears in ________________.

(b) $\frac{d}{dx}(f(x) + g(x)) =$ _______________________ This formula appears in ________________.

11. (3 pt) If $f(t) = \sin(1 - t^3)$, then $f'(t) =$

12. (4 pt) Find the equation of the normal line to the curve $y = x^4 - 3x^3$ at the point $(1, -2)$.

13. (3 pt) The position of a particle on a vibrating string is given by the equation $s(t) = 4 + \frac{1}{10}\sin(8\pi t)$ where s is measured in centimeters and t in seconds. Find the velocity of the particle after 5 seconds. (Give your answer as a decimal expression correct to two decimal places. Make sure to include the correct units in your answer.)

14. (3 pt each) Differentiate each function.

(a) $g(t) = \cos^3 t.$ (Remember, this means the same as $(\cos t)^3$.)

(b) $g(x) = x^2 \sin x$

(c) $g(x) = \dfrac{3x}{x^2 + 1}$

15. (3 pt total) A student who took Calculus 1 during a recent semester was asked to compute the derivative of the function $g(x) = \frac{\sin x}{\cos x}$. The student's answer was $g'(x) = \frac{\cos x}{-\sin x}$

(a) What is the name of the derivative formula that the student mis-used?

(b) Give the correct computation of $g'(x)$. Simplify your answer as much as possible.

16. (3 pt) If $y = \sqrt{1 - x^2}$, find the derivative dy/dx.

17. (4 pt) Find the equation of the tangent line to the curve $y = (1 - 10x)^2$ at the point $(0, 1)$.

18. (4 pt) The equation of motion of a particle is $s(t) = \cos(2t) + 3\sin(6t)$, where s is in meters and t is in seconds. Find the acceleration of the particle at time $t = \pi$. (Make sure to include appropriate units in your answer.)

19. (5 pt total) If a cylindrical tank holds 750 gallons of water, which can be drained from the bottom of the tank in an hour, then Torricelli's Law gives the volume V of water remaining in the tank after t minutes as $V(t) = 750(1 - \frac{1}{60}t)^2 \quad (0 \le t \le 60)$.

(a) Find the rate at which the water is flowing out of the tank (the instantaneous rate of change of V with respect to t) as a function of t.

(b) What is the flow rate when $t = 10$? (Make sure to include the correct units in your answer)

(c) What is the flow rate when $t = 30$? (Make sure to include the correct units in your answer)

(d) If you are filling up a bucket using the water which is flowing out of this tank, would your bucket be filled faster if you filled your bucket around time $t = 10$, or around time $t = 30$?

Circle one: $t = 10$ $t = 30$